The optimization problem called Vertex Cover is the following:
Given a graph \( G(V,E) \), find a minimum subset \( V' \subseteq V \) such that, for every edge \( (u,v) \in E \), at least one of \( u \) and \( v \) belongs to \( V' \).
The subset \( V' \) is called a minimum vertex cover.

(1) Mark a minimum vertex cover for the following graph.

(2) Write the decision problem corresponding to Vertex Cover.

Decision problem Vertex Cover:

Given a graph \( G(V,E) \), and a positive integer \( K \leq |V| \), is there a vertex cover of size \( K \) or less for \( G \), i.e., a subset \( V' \subseteq V \) such that \( |V'| \leq K \), and such that for each edge \( \{u,v\} \), at least one of \( u \) and \( v \) belongs to \( V' \)?

(3) Explain why the optimization problem Vertex Cover is at least as difficult as the decision problem Vertex Cover.

Answer:

Solving the optimization problem yields a minimum vertex cover \( V' \). It is possible to verify in linear time that \( |V'| \leq K \), and provide a solution to the decision problem. There may be more efficient solutions to the decision problem. In any case, the decision problem is not more difficult than the optimization problem.
The optimization problem called Independent Set is the following:
Given a graph $G(V,E)$, find a maximum subset $V' \subseteq V$ such that no two vertices in $V'$ are connected by an edge in $E$.
The subset $V'$ is called a maximum independent set.

(1) Mark a maximum independent set for the following graph.

(2) Write the decision problem corresponding to Independent Set.

Decision problem Independent Set:
Given a graph $G(V,E)$, and positive integer $K \leq |V|$, does $G$ contain an independent set of size $K$ or more, i.e., a subset $V' \subseteq V$, such that $|V'| \geq K$ and such that no two vertices in $V'$ are joined by an edge in $E$?

(3) Explain why the optimization problem Independent Set is at least as difficult as the decision problem Independent Set.

Answer:
Solving the optimization problem yields a maximum independent set $V'$. It is possible to verify in linear time that $|V'| \geq K$, and provide a solution to the decision problem. There may be more efficient solutions to the decision problem. In any case, the decision problem is not more difficult than the optimization problem.