ECE608, Fall 2013, Quiz 13

First Name: ___________________ Last Name: ____________________

Use only the space provided on this page to answer the following question.

$D_1, D_2$ and $D_3$ are decision problems.

(1) Prove the following claim:
If $D_1 \propto D_2$ and $D_2 \propto D_3$ then $D_1 \propto D_3$.

(2) Suppose that $D_1 \propto D_2$ and $D_2 \propto D_3$, and that $D_3$ has a polynomial time complexity algorithm $A_3$ with complexity $p_3(n_3)$. Write an expression for the complexity of solving $D_1$. Define the notation that you use to write the expression.

Solution:

(1) From $D_1 \propto D_2$, there exists a function $f_1(I_1)$ such that the answer to $I_1$ with respect to $D_1$ is yes if and only if the answer to $f_1(I_1)$ is yes with respect to $D_2$, and there exists a polynomially bounded algorithm to compute $f_1(I_1)$. Let the polynomial be $q_1(n_1)$.

From $D_2 \propto D_3$, there exists a function $f_2(I_2)$ such that the answer to $I_2$ with respect to $D_2$ is yes if and only if the answer to $f_2(I_2)$ is yes with respect to $D_3$, and there exists a polynomially bounded algorithm to compute $f_2(I_2)$. Let the polynomial be $q_2(n_2)$.

Consider the function $f_2(f_1(I_1))$. The answer to $I_1$ with respect to $D_1$ is yes if and only if the answer to $f_1(I_1)$ is yes with respect to $D_2$, and the answer to $f_1(I_1)$) with respect to $D_2$ is yes if and only if the answer to $f_2(f_1(I_1))$ is yes with respect to $D_3$. Therefore, the answer to $I_1$ with respect to $D_1$ is yes if and only if the answer to $f_2(f_1(I_1))$ is yes with respect to $D_3$.

The complexity of computing $f_2(f_1(I_1))$ is $q_2(q_1(n_1))$, which is a polynomial.

(2) Using the notation from (1), $q_1(n_1)+q_2(q_1(n_1))+p_3(q_2(q_1(n_1)))$. 