ECE608 chapter 35 problems

1) 35.1-1
2) 35.1-2
3) 35.1-4
4) 35.2-3
5) 35.2-5
6) 35.3-1
7) 35.3-2

(35.1-1) Consider the graph shown in Fig. 1. This will always terminate with two vertices as part of the vertex cover, while the optimal solution has only one vertex in the vertex cover.

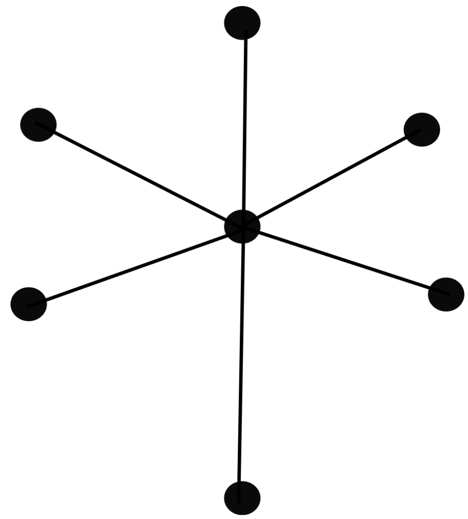

Figure 1: Example for 35.1-1

(35.1-2) A maximal matching M is a matching such that adding one more edge to the matching would make the new set of edges cease to be a matching. The edges that we pick in the APPROX-VERTEX-COVER are such that the corresponding vertices form a vertex cover. If this set of edges is denoted by M, then M is a maximal matching. This is because adding any new edge (u, v) to M would be between two vertices that are part of a vertex cover and M would have atleast one edge of the form (u, i) or (v, j). This would make $M \cup (u, v)$ cease to be a matching.

(35.1-4) In a vertex cover, there has to be at least one vertex for each edge. For each leaf, there is always an edge adjacent to it and we can pick the corresponding non-leaf vertex as part of the cover. Once we pick these vertices, we can trim off the covered vertices and repeat the process until the tree has no edges or just one edge. If the tree has just on edge, we can pick any one of the two vertices. This process can be done usning a DFS in linear time.

(35.2-3)When using the closest-point heuristic, the nodes are added to the cycle such that there is an MST within the cycle. This is because the way in which nodes are added is according to Prim's algorithm. The length of the MST will be less than the length of the full walk which will walk through each edge of the MST twice. So, c(closest-point heuristic) $\leq 2c(H^*)$, where H^* is the optimal tour.

(35.2-5) Consider the graph shown in Fig. 2. The edges that are part of the tour are shown as solid lines. Since the distances obey triangle inequality, we have $z \leq x + y$ and $w \leq u + v$. Combining the inequalities, we get that $z + w \leq x + y + u + v$. So instead of having the edges that cross each other as part of the tour, we could have the dotted edges and reduce the cost. Therefore, in the optimal solution to the TSP with Euclidean distances, the tour never crosses itself.

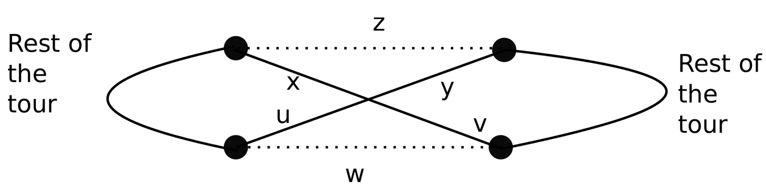

Figure 2: Figure for 35.2-5

(35.3-1) In this case we have $X = \{a, d, e, h, i, l, n, o, r, s, t, u\}$. The letters of each word are from the family of subsets of X. The cover obtained from GREEDY-SET-COVER when ties are broken by dictionary order, the words that form the smallest cover are: thread, lost, drain, shun.

(35.3-2) The decision problem corresponding to the SET-COVER problem is as follows:

$$\text{SET-COVER} = \{X, F, k | X \text{ has a set cover of size k } \}$$

To show that the problem is NP-complete, we need to first show that it is in N. If we are given a set X, a cover C and an integer k. We can verify whether the number of sets in C is equal to k and whether each element of X belongs to atleast one set in C in polynomial time.
Secondly, we can reduce the SET-COVER to VERTEX-COVER in polynomial time by a direct mapping. This can be got by observing that in a graph formulation $X = E$. In other words, the elements of X in SET-COVER are the edges $(u, v) \in E$ in VERTEX-COVER. Also, $|F| = |V|$, or the number of subsets in the entire family of subsets in SET-COVER is equal to the number of vertices in VERTEX-COVER. So it is a direct mapping to say that a vertex cover of size k is equivalent to a SET-COVER of size k in the respective problems.