This exam corresponds to learning objective 1.

Solve the following problems. The number of points for each problem is shown in the table below.

Use only the space provided to solve each problem, and copy the answers to the space marked "Answer:" Do not forget to specify units.

Show all the steps of your solution. Final answers alone will not be considered.

Non-integer answers can be written as \( \frac{a}{b} \) or as \( c.d \) where \( d \) can be rounded to 2-3 digits.

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(a) Using KCL, express $V_1$ as a function of $I_{in}$.

\[ I_{in} + 3V_1 - \frac{V_1}{1} = 0 \]
\[ I_{in} = -2V_1 \]

(10 points) Answer: $V_1 = -0.5I_{in}$

(b) Using KVL, find $R_{eq} = \frac{V_{in}}{I_{in}}$.

\[ 1I_{in} + V_1 - V_{in} = 0 \]
\[ I_{in} - 0.5I_{in} - V_{in} = 0 \]
\[ V_{in} = 0.5I_{in} \]

(10 points) Answer: $R_{eq} = 0.5 \Omega$
Problem 2

Answer the questions below with respect to the following circuit. Use the space on the next page for computations.

(a) Using only voltages and currents that are marked on the circuit, write the variables for **nodal** analysis as learned in class.

(5 points) Answer (Variables only): $V_a, V_b, I_2$

(b) Using only voltages and currents that are marked on the circuit, write the variables for **loop** analysis as learned in class.

(5 points) Answer (Variables only): $I_1, I_2$

(c) Write equations for **nodal** analysis as learned in class. Do not simplify the equations for this part.

(10 points) Answer (Equations):

\[
\begin{align*}
\frac{V_a - 2}{20} + \frac{V_a}{50} + I_2 &= 0 \\
\frac{V_b}{100} + 0.02V_a - I_2 &= 0 \\
V_b - V_a &= 1
\end{align*}
\]
(d) Write equations for loop analysis as learned in class. Do not simplify the equations for this part.

\[ I_3 = 0.02V_a = 0.02 \cdot 50(I_1 - I_2) = I_1 - I_2 \]

(10 points) Answer (Equations):

\[ 50(I_1 - I_2) - 2 + 20I_1 = 0 \]
\[ -1 + 100(I_2 - (I_1 - I_2)) + 50(I_2 - I_1) = 0 \]

(e) Solve either the equations you wrote in part (c), or the equations you wrote in part (d). Then find \( I_1, I_2, I_3, V_a \) and \( V_b \).

Note: you do not need to solve both sets of equations.

(10 points) Answer:

\[
\begin{align*}
I_1 &= 0.055\text{A} \\
I_2 &= 0.037\text{A} \\
I_3 &= 0.018\text{A} \\
V_a &= 0.9\text{V} \\
V_b &= 1.9\text{V}
\end{align*}
\]

Space for computations:

\[
\begin{align*}
50I_1 - 50I_2 - 2 + 20I_1 &= 0 \\
-1 + 100I_2 - 100I_1 + 100I_2 + 50I_2 - 50I_1 &= 0 \\
70I_1 - 50I_2 &= 2 \\
-150I_1 + 250I_2 &= 1
\end{align*}
\]

\[
\begin{align*}
200I_1 &= 11, \quad I_1 = 0.055\text{A} \\
I_2 &= (70I_1 - 2)/50 = 0.037\text{A} \\
V_a &= 50(I_1 - I_2) = 0.9\text{V} \\
I_3 &= 0.02V_a = 0.018\text{A} \\
V_b &= 100(I_2 - I_3) = 1.9\text{V}
\end{align*}
\]
Problem 3

For the following circuit, find the value of $R_L$ for maximum power transfer. Using this value of $R_L$, find the power $P_L$ absorbed by $R_L$, the current $I_L$ through $R_L$, and the power $P_{I1}$ delivered by the current source $I_1$.

\[ R_{th} = 2 + 3 = 5 \Omega \]
\[ v_{oc} = 2 \cdot 0.5 + 3 \cdot 1 = 4 V \]
\[ P_L = \frac{v_{oc}^2}{4R_{th}} = \frac{4^2}{4 \cdot 5} = 0.8 W \]
\[ I_L = \frac{v_{oc}}{R_{th} + R_L} = \frac{4}{5 + 5} = 0.4 A \]
\[ V_{2\Omega} = 2 \cdot (0.5 - 0.4) = 0.2 V \]
\[ P_{I1} = V_{2\Omega}I_1 = 0.2 \cdot 0.5 = 0.1 W \]

Answer:

(10 points) $R_L = 5 \Omega$

(10 points) $P_L = 0.8 W$

(10 points) $I_L = 0.4 A$

(10 points) $P_{I1} = 0.1 W$