Multi-period Equilibrium Modeling Planning Framework for Tradable Credit Schemes

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Abstract

This study proposes the concept of multi-period tradable credit scheme (TCS) for a planning context. In it, travelers determine their actions in terms of consumption or sale of credits in the current period or transfer to future periods. In the first scheme, travelers can transfer credits to future periods without penalty. In the second scheme, the effects of two regulatory instruments are investigated on the market behavior. Study insights suggest that a multi-period TCS dampens credit price volatility. It allows the central authority to develop TCSs with stable credit prices in which travelers can hedge against potential monetary losses.

Keywords: multi-period tradable credit scheme; equilibrium credit price; transfer fee; reservation credit price;

1. Introduction

Traffic congestion is one of the most important issues for metropolitan areas. It has led to increased productivity losses over the past few decades in the U.S. and elsewhere (Schrank et al., 2012; Nash et al., 2008). Pricing of the road network is a strategy to address the congestion problem. It can be an effective market-based instrument to manage demand by influencing traveler decisions related to travel mode, departure time and route.

There is substantial literature on congestion pricing and price-based instruments. Pigou (1920) proposed the concept of marginal cost pricing to maximize social welfare by charging the marginal external cost that travelers impose on society. It has been applied to the general traffic network with different assumptions such as link flow interactions, heterogeneity of travelers and multiple modes of transportation (Dafermos and Sparrow, 1971; Dafermos, 1973; Yang and Huang, 1998). While simple and appealing in principle, it has not been well-received in practice due to issues of equity and traveler reluctance to pay the central authority to use roads due to perceptions of indirect taxation. It also does not account for the variability in driver ability to pay, and can cause an inequitable distribution of social welfare. Lawphongpanich and Yin (2010) propose a "pareto-improving" congestion pricing scheme in which travelers perform at least as well as under the no-pricing scheme. Thereby, the link tolls are computed to reduce the total system travel time while ensuring that all travelers experience the same or lesser travel time compared to the no-pricing scenario. While the approach promotes greater equity, the issue of transference of wealth from the drivers to the central authority still exists in the proposed scheme.

To overcome the aforementioned issues, the idea of using a tradable credit scheme (TCS) has received more attention recently. It has been used in different sectors such as air pollution control, managing water resources and land use control (Leggett et al., 2012; OECD, 1997). The application of a TCS in the transportation sector to control traffic congestion can be traced to Verhoef et al. (1997) who propose a "tradable road-pricing smart card" to manage road transportation externalities. Nagurney et al. (1998) investigate a system of link-based marketable

pollution permits where the central planner aims to achieve certain environmental objectives. Later, it is extended to path-based and origin-destination (O-D) pollution permits (Nagurney, 2000a, 2000b). Viegas (2001) develops the notion of tradable smart card to allocate "mobility credits" to all tax-payers, and provides the flexibility to trade credit and use it to pay transit fare in addition to paying toll. Along this thread, Yang and Wang (2011) formulate the tradable mobility credit scheme as a variant of the standard user equilibrium (UE) model with additional constraints on the network credit feasibility condition. In this scheme, a central planner determines the total endowment of travelers and the subsequent link tolls to charge them. Thereby, in a tradable mobility credit scheme travelers need to pay credits in order to travel in the network. This can provide a central authority control mechanisms to manage the traveler demand in the network so as to achieve some system-level goals. Travelers are able to trade credits amongst themselves in the market. Since there is no transfer of wealth between the central authority and the travelers while considering the equity effects of this scheme, there could potentially be less societal objection to its implementation in practice.

Numerous efforts have sought to improve the rationality of the assumptions adopted in the Yang and Wang (2011) scheme. The associated literature can be classified into three groups based on its focus on the different stages of the TCS. The first group deals with the different assumptions of the market. Nie (2012) investigates the effect of the transaction cost in two different types of markets (negotiable and auction). He demonstrates that the transaction cost can lead to the desired equilibrium solution only in an auction market with certain assumptions rather than in a negotiated one. Shirmohammadi et al. (2013) establish linkages between TCS and congestion pricing to address different goals in the traffic network. They propose a "safety valve" policy in a cap-and-trade scheme to resolve the issue of price volatility under uncertainty related to a regulation. The second group of studies analyzes the effects of traveler characteristics in determining link tolls and total allocated credits. Wang et al. (2012) propose a TCS formulation by considering heterogeneous travelers with a discrete set of values of time. They investigate the relationship between the uniqueness of the aggregate UE link flow pattern and the equilibrium credit price. He et al. (2013) study the effect of the mixed behaviors of UE-following and self-optimizing oligopoly Cournot players in the optimal design of TCS with transaction costs. Bao et al. (2014) develop a more realistic TCS by considering travelers' loss aversion behaviors in their route choice. Given the market with transaction cost for buying and selling credits, they demonstrate that the system optimum link flow pattern may not be achievable when travelers' loss aversion behavior is considered. Zhu et al. (2015) investigate the UE condition under TCS where travelers are assumed to be heterogeneous with continuous distribution of value of time. It is shown that optimal allocation of credits can make travelers better off compared to the no-pricing scenario. Xu and Grant-Muller (2016) propose a simulation framework to analyze the mode-choice of travelers in the traffic network before and after implementation of TCS. This framework is applied to the case of Beijing, China where it is demonstrated that TCS is a promising policy to reduce the total vehicle-miles traveled in the traffic network. The third group of studies deals with the effects of the implementation of TCS on traffic dynamics. Nie and Yin (2013) propose a scheme to manage morning commute choices with no initial allocation of credits. First, the central authority divides the planning horizon into peak and off-peak periods. Then, the authority rewards the commuters traveling in the off-peak period and charges the commuters traveling in the peak period. While they develop a general analytical framework to determine the traveler departure choices under the proposed TCS, the credit price is constant through the planning horizon.

Categorized into the third group of studies, Ye and Yang (2013) capture the evolution of

network traffic and market performance using the notion of day-to-day traffic dynamics. They consider the traveler learning process, the credit demand-supply interaction, and the proposed TCS to analyze the day-to-day evolution of the credit price. Since day-to-day models describe the evolution of travel choice decisions over time, such models can be used to capture the credit price variation until the equilibrium condition is reached. Ye and Yang develop a continuous dynamic model in a finite planning horizon and prove that the credit price reaches equilibrium over a few to several days under a sufficiently long planning horizon. In terms of the TCS, they assume that the central authority distributes credits to travelers at a certain rate at the beginning of the planning horizon itself; hence, there is no distribution of credits to travelers in the future. In deriving the credit price and traffic congestion evolution, they assume that the current credit price is not connected to the traffic network demand on future days. They employ a sequential approach to obtain the credit price by assuming that the credit price on the next day is a function of only the current credit price and the current excess credit demand in the market.

Planners/decision-makers (referred to as "central authority" in this study) have used TCS in different sectors to achieve steady progress toward system-level goals over a long planning horizon. For example, the European Union has implemented a TCS to reduce emissions in the aviation industry over a 15-year period, where the emission cap declines from 97% of the average of the 2005-2007 values in 2012 to 95% in 2020 (Leggett et al., 2012). In a similar way, since the road transportation sector is a major source of emissions, a TCS can be a tool for a central authority to meet some goals associated with traffic-related emission standards over a long-term planning horizon. To achieve such long-term goals, the planning horizon can be divided into multiple periods in which the central authority uses the credit supply (the total number of credits to be allocated to travelers) and the credit rate charged per link for each period as control parameters, labeled TCS parameters hereafter. Then, a TCS can be used to enable a mobility credit market in which the traveler actions also depend on the traffic network demand and supply. However, due to the long-term nature of the TCS design, the traffic network demand and supply can vary over the horizon. The TCS parameters in a multi-period TCS can be used by the central authority to control the evolution of the traffic system to ensure steady progress towards the long-term system-level goals while factoring the projected long-term traffic network demand and supply changes. This leads to the planning problem addressed in this study in which the central authority seeks to determine the equilibrium credit price for all periods of the planning horizon simultaneously given the credit supply, the link credit rate charging scheme, and the projected traffic network demand/supply for each period at the beginning of the planning horizon. Hence, the equilibrium credit prices reflect the outcomes of traveler actions related to credit consumption, sale or transfer across periods. It is this traveler behavior and the interactions of TCS parameters and traffic network demand/supply across periods that the central authority seeks to understand in the planning context so as to determine the control decisions (in terms of the TCS parameter values for each period) in an operational context to achieve the long-term system-level goals. By contrast, the TCS models in the literature discussed heretofore can be labeled as "single-period" schemes in that credit supply and traffic network demand rates are constant. The day-to-day model of Ye and Yang (2013) would represent a single-period TCS in which the credit price may fluctuate for some days initially and the goal is to capture the credit price fluctuation before convergence to equilibrium. In the other single-period TCS models, the credit price itself is time invariant. This paper is the first study to model the multi-period TCS to address the long-term planning problem of the central authority.

The main contribution of this study is to develop the multi-period TCS framework from

the long-term planning perspective of the central authority to foster consistency between the credit price variability in the market across periods and the long-term goals of the central authority, and manage the traffic network demand during the planning horizon to achieve these goals. Also, this framework allows the central authority to evaluate the progress toward the long-term system-level goals at the end of each period. That is, the central authority would solve this planning problem from the current time to the end of the planning horizon at the end of each period, to reflect the current system-level conditions related to achieving the long-term goals as well as more recent forecasts of future traffic network demand and supply. Using the updated future equilibrium credit prices and the current conditions/forecasts, the central authority updates the TCS parameters for the rest of the planning horizon. Two multi-period tradable credit schemes are proposed, in which the traffic network demand (for example, the inverse travel demand function) and supply (for example, the link travel cost functions) can vary across periods but are unchanged within each period. The credit price and network flows in a period depend on the interactions between the credit and travel markets in the current and future periods. As the length of each period can be sufficiently long (for example, in the order of a few months) due to the multi-year planning horizon of interest, the link flows and credit price are assumed unchanged within each period, and are viewed as the manifestation of equilibrium conditions achieved under a multi-period TCS based on generalized travel costs that include the travel times and the credits charged for the trip.

The study analyzes the impact of the multi-period TCS design on the evolution of the equilibrium credit prices over the planning horizon. While the proposed framework factors various dimensions for consistency with the real world, its planning context entails three specific assumptions that need to be relaxed when operationalizing this framework. First, it is assumed that travelers have perfect information of future credit prices in each period of the planning horizon, solved for using the proposed framework. Second, travelers are assumed to be homogeneous in terms of how they value travel time and credit price, and their valuations are assumed to be constant through the planning horizon. Third, the discount rate, which is the interest rate set by the central authority for the future value of credits, is assumed to be zero during the planning horizon.

In this study, the design of the multi-period TCS entails the following assumptions. The central authority allocates credit equally among travelers free of charge for each period. The total allocated credit satisfies the total credit demand of travelers during the planning horizon. Further, travelers are assumed to trade the credits in the market with negligible transaction cost that can be ignored. In both schemes considered in this study, credits expire only at the end of planning horizon, implying that they can be transferred to future periods. In the single-period TCS, credits are either consumed or sold. Another scheme, labeled the multi single-period TCS, is where the central authority can regulate that the unused credits at the end of each period of a planning horizon are discarded at the end of that period without any gainful value. That is, in the multi single-period TCS, credits cannot be transferred to future periods. It can be viewed as a special case of the multi-period TCS where travelers do not transfer credits across periods.

From the central authority's perspective, understanding the evolution of credit price under the proposed multi-period TCS is beneficial since the credit price stability, that is, the reduction of credit price fluctuation, is important for public acceptance of the TCS. Hence, the planning problem seeks to determine the equilibrium credit prices for all periods of the planning horizon. The travelers consider the future credit prices in their credit usage decision-making to transfer credits to future periods. Thereby, the multi-period TCS leads to better credit price stability and the reduction of credit price fluctuations in the market through the planning horizon. This effect

has been observed in the European Union aviation industry emissions TCS. Further, the European Union has used reservation credit price as another successful instrument to mitigate price volatility. Thereby, the European Union buys credits back at a predetermined "reservation" price from aviation industries. Synergistically, this instrument also helps the European Union to reduce the total emissions during the planning horizon as the total number of credits in the market decrease (Ellerman and Joskow, 2008). We use the reservation price as an instrument in the second TCS proposed in this study, as discussed next.

Two multi-period tradable credit schemes are analyzed in this study. In the first scheme, travelers can transfer credits to future periods without penalty. So, in a period, credits are either consumed by the traveler or unused. Unused credits in a period are stored and transferred to future periods. Any remaining credits at the end of planning horizon do not have any residual value, and are hence discarded. Since tradable credits represent market instruments, they are subject to the risk of market manipulation and the artificial control of credit price through mechanisms such as hoarding and collusion over time. The mitigation of these risks entails a suitably designed multi-period TCS. For this purpose, the second proposed TCS investigates the effects of two regulatory instruments, transfer fee and reservation credit price, on the market behavior. In it, as in the first scheme, credits are either consumed in a period or unused. Unused credits can be stored and transferred to a future period or sold back to the central authority at a reservation price in the current period; hence, some credits may be transferred and others may be sold back. For unused credits that are stored and transferred to future periods, travelers need to pay transfer fees that can vary with the period. Thereby, the central authority can mitigate credit hoarding by travelers using transfer fees as a market instrument. For unused credits that are sold back in the current period, it is assumed that the central authority (or, for example, environmental groups) buys back these credits at predetermined reservation prices that can vary with the period. If the central authority regulates the reservation credit price to be higher than a certain level, this policy can lead the market to increase the credit price to the level of the reservation credit price. The central authority can choose to discard the bought-back credits to reduce the total amount of travel during the planning horizon depending on the system-level goals. The first scheme is a special case of the multi-period TCS with transfer fee and reservation credit price (the second scheme) in which the reservation credit price and transfer fee are set to zero.

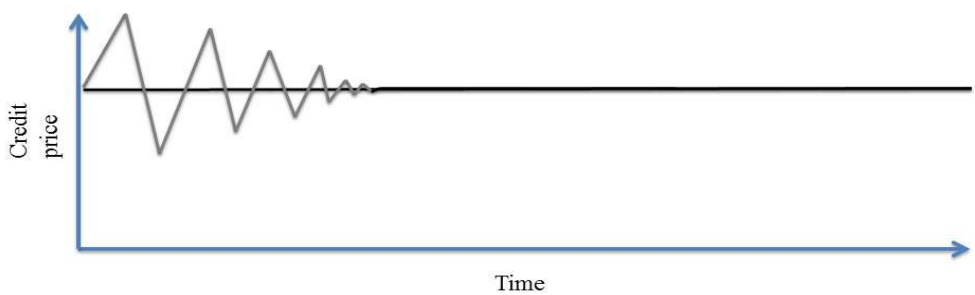
The remainder of the paper is organized as follows. Section 2 compares the various types of tradable credit schemes. Section 3 discusses the formulation of the first multi-period TCS, in which there is no penalty for transfer of credits to future periods. Section 4 discusses the second multi-period TCS, in which transfer fee and reservation credit price are considered. Section 5 describes numerical experiments and discusses associated insights. Section 6 provides some concluding comments.

2. Types of tradable credit schemes

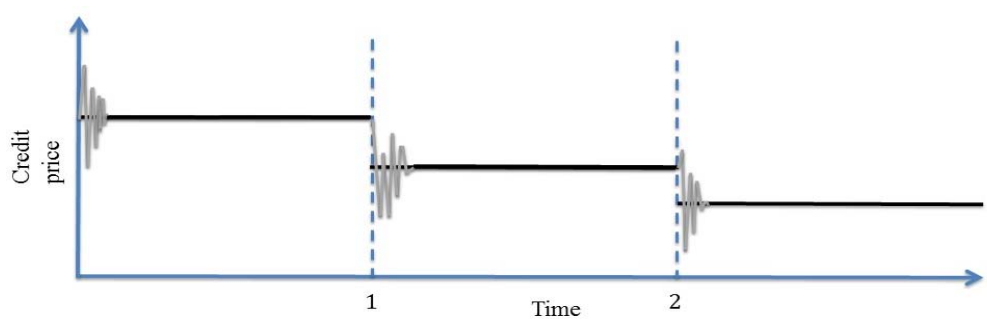
In the single-period TCS, the central authority distributes a certain number of credits at a predetermined rate at the beginning of the planning horizon. It is assumed that the traffic network demand/supply, the credit supply rate and the link credit rate charging scheme are unchanged throughout the planning horizon. Fig. 1(a) illustrates plausible credit price fluctuations (shown in grey color) with time under a single-period TCS. Such fluctuations can occur initially due to traveler behavior and market dynamics, before an equilibrium credit price is reached. By assuming that the time duration to reach equilibrium is small relative to the length of the planning horizon,

single-period schemes focus on the rest of the planning horizon under the equilibrium state. In this study, this equilibrium credit price is referred to as the "single-period equilibrium credit price", and is determined based on the interaction between the credit supply determined by the central authority and traffic network demand. Ye and Yang (2013) examine the evolution of the market credit price and the dynamics of traffic congestion until equilibrium. They propose a day-to-day dynamic model to capture the credit price fluctuation before the convergence to equilibrium. They illustrate that if the planning horizon is sufficiently long, the credit price and network state (path/link flows) under different initial conditions converge to the same stable equilibrium. The planning horizon for the proposed multi-period scheme is much larger than that for a single-period scheme. This is illustrated in Fig. 1(b), where the planning horizon is in the order of a few years and divided into time periods of equal length in the context of the proposed multi-period TCS.

The primary study objective is to determine the equilibrium credit price for each time period in the planning context under the proposed multi-period TCS. Hence, equilibrium path/link flows and travel times in the context of equilibrium credit price can be interpreted as representing network steady-state conditions in the absence of longer-term demand/supply changes in the network within a period. The study focus is on the evolution of equilibrium credit prices over the planning horizon rather than the credit price within a period. Consequently, the proposed multi-period schemes cannot capture potential day-to-day credit price and link flow fluctuations at the beginning of a period, but can be combined with day-to-day models (e.g., Ye and Yang (2013)) to do so. In contrast to day-to-day models, due to the interactions across periods, the multi-period scheme cannot be solved by considering a series of disjointed single-period schemes (e.g., Ye and Yang (2013)) and sequentially determining the credit price and network state in each period separately by transferring only unused credits from the current period to the next.



(a) Single-period TCS over its (short-term) planning horizon.



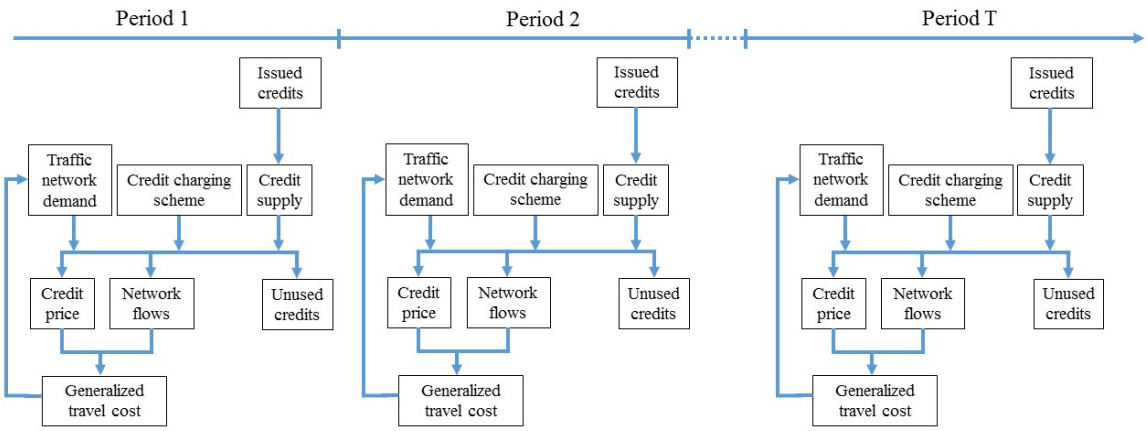
(b) Multi-period TCS over the long-term planning horizon.

Fig. 1. Credit price evolution under single-period and multi-period tradable credit schemes.

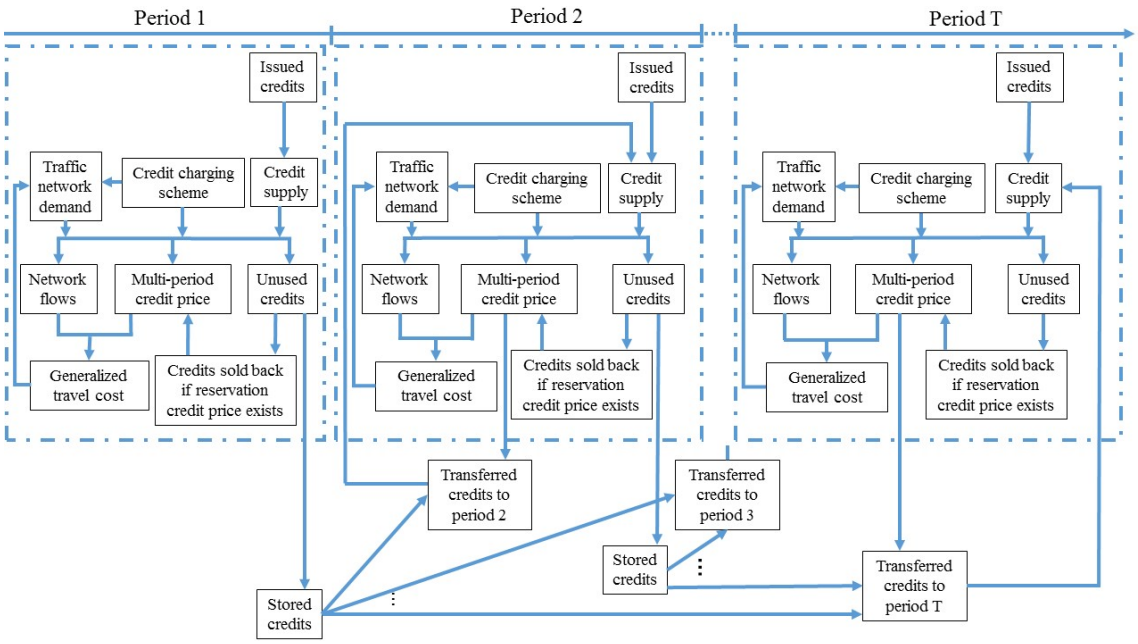
As discussed earlier, in the multi-period TCS, the central authority distributes a certain number of credits at a predetermined rate in each period of the planning horizon. Also, the central authority predetermines the charging scheme (that is, credit charging rate) for each link in each period. If the central authority divides the planning horizon into multiple independent periods without allowing the transfer of credits, travelers have only the options of selling or consuming credits within a period; this would correspond to a multi single-period TCS. Fig. 2(a) illustrates the structure of the multi single-period TCS. In each period, the interactions between traffic network demand and supply and TCS parameters determine the equilibrium credit price and network flows. The generalized travel cost of each trip, which includes travel time and credit charge, affects the traffic network demand. Because each period can be treated independently without considering transferred credits from other periods, the equilibrium credit price of a period in this scheme is equal to its single-period equilibrium credit price. In other words, as the TCS in each period can be analyzed independently, the credit price in a period for the multi single-period TCS is equal to its single-period credit price. Credit supply of each period is either consumed to pay travel charges or remains unused due to lack of demand in the market in the current period. Travelers discard unused credits at the end of each period without obtaining any benefit. So, the current credit price is not impacted by the past/future TCS parameters and traffic network demand.

In the multi-period TCS, as shown in Fig. 2(b), credits can be transferred to any future period. Travelers decide how many credits they would consume in the current period depending on several factors such as the current traffic network demand/supply, the current credit charging scheme, the current credit supply and the credit prices in the future periods. The unused credits are transferred to future periods if reservation price is not used as an instrument. If reservation price exists for the current period, travelers may either sell back unused credits to the central authority or store them for transfer to future periods based on the aforementioned factors and the reservation price. If credits are sold back to the central authority in the current period, the credit price of current period would be affected.

As stated earlier, the motivation for the planning problem addressed in this study is to determine the equilibrium credit prices for all periods of the planning horizon so that the central authority can provide them as forecasts to travelers. The credit price in a period under the multi-period TCS is labeled the "multi-period credit price", and its equilibrium credit price is referred to as the "multi-period equilibrium credit price" for that period. That is, the multi-period equilibrium credit price in a period can be different from the corresponding single-period equilibrium credit price. In each period, travelers transfer the stored credits from previous periods based on the credit price in that period. So, the credit supply in a period includes the new credits issued by the central authority in that period and the stored credits from previous periods transferred by travelers. A key feature of the proposed multi-period equilibrium modeling framework is the consideration of the effects of the past and future TCS parameters and traffic network demand/supply on the current credit price. This is because interactions can occur between the credit price, TCS parameters, and travel/credit demand through the planning horizon due to the effect of: (1) TCS parameters in the current period on the future credit price, and (2) future traffic network demand/supply and TCS parameters on the current credit price.



(a) Multi single-period TCS; transfer of credits is not allowed.



(b) Multi-period TCS; transfer of credits is allowed.

Fig. 2. Structure of the multi-period TCS with and without transfer of credits allowed.

3. Multi-period tradable credit scheme without penalty

Let $G(N, A)$ be a directed traffic network where N is the set of nodes and A the set of directed links. Let t denote a time period in the planning horizon and Γ denote the set of time periods with cardinality of T. It is assumed that traffic demand and supply are constant within each period. Thereby, day-to-day or within day fluctuations of traffic conditions through each period are ignored in the planning context where the interactions between network traffic demand and credit supply are analyzed per unit of time (for example, an hour) in each period as an average state of the traffic network for that time period. Hereafter, all rates are defined per the same unit of time in each period.

The central authority distributes credits among the network travelers at the rate of ξ^t during period t, which can be interpreted as the average number of issued credits to the travelers per unit

of time. Let $z^{t',t}$ denote the rate of transfer of the credits issued in time period t' to time period t by the travelers. Let $u = \{u_a^t, a \in A\}$ denote the charging scheme used by the central authority, where u_a^t is the number of credits that travelers have to pay to use link a at any time in time period t. Each link $a \in A$ is associated with a separable, nonnegative, convex and monotonically increasing travel time function c_a^t that depends on the link flow v_a^t of period t. Let W be the set of O-D pairs. The traffic network demand rate for O-D pair w is denoted by d_w^t in period t. It is assumed to be elastic and a function of the minimum generalized travel cost for that O-D pair for period t. The generalized travel cost for a path is the sum of two components: (i) the path travel time, and (ii) charged credit cost for that path. Based on the assumption of homogeneity across travelers, one unit of credit cost is valued as one unit of travel time. Hence, the generalized travel cost is in time units. Each O-D pair w is associated with a continuous and monotonically decreasing inverse demand function $D_w^{-1}(.)$. Let R_w denote the feasible path set for O-D pair w. $f_{r,w}^t$ denotes the flow on path $r \in R_w$ for O-D pair w in period t. Let $\Omega_{(f,v,d)}$ denote the feasible set of path flows for all O-D pairs, link flows and O-D demand rates, defined by:

$$\Omega_{(f,v,d)} = \{(f,v,d) | \sum_{w\in W}\sum_{r\in R_w} f_{r,w}^t \delta_{a,r,w} = v_a^t, d_w^t = \sum_{r\in R_w} f_{r,w}^t, f_{r,w}^t \geq 0, d_w^t \geq 0, \forall t \in \Gamma, \forall w \in W, \forall r \in R_w\} \tag{1}$$

where $\delta_{a,r,w} = 1,$ if link a is on path r for O-D pair w, and 0 otherwise. Given the TCS, the multi-period equilibrium problem can be formulated as follows:

$$\min \sum_{t\in\Gamma}\left(\sum_{a\in A}\left(\int_o^{v_a^t} c_a^t(\omega)d\omega\right) - \sum_{w\in W}\left(\int_o^{d_w^t} D^{-1}(\omega)\, d\omega\right)\right) \tag{2}$$

$$\sum_{j=2}^{T} z^{1,j} + \sum_{a\in A} u_a^1 v_a^1 = \xi^1 \tag{3}$$

$$\sum_{j=t+1}^{T} z^{t,j} + \sum_{a\in A} u_a^t v_a^t = \xi^t + \sum_{j=1}^{t-1} z^{j,t} \qquad 1 < t < T \tag{4}$$

$$\sum_{a\in A} u_a^T v_a^T + Y^T = \xi^T + \sum_{j=1}^{t-1} z^{j,T} \tag{5}$$

$$z^{t',t} \geq 0 \qquad \forall t' < T, \forall t > t' \tag{6}$$

$$Y^T \geq 0 \tag{7}$$

$$(f,v,d) \in \Omega_{(f,v,d)} \tag{8}$$

Model (2)-(8) is the elastic-demand UE model with additional constraints on credit conservation and credit availability. Constraints (3)-(5) denote the credit conservation constraints. Constraint (3) states that the issued rate of credits to the travelers in the first period by the central authority is equal to the sum of the credit consumption rate for that period and the transfer rate of

credits to the future periods. Constraint (4) states that the credit supply rate, which is the sum of the issued credit rate in the current period and the transfer credit rate from previous periods, is equal to the sum of the credit transfer rate to future periods and the rate of credit consumption in the current period. Constraint (5) states that the credit supply rate is equal to the sum of the consumption and discarding rates of credits by travelers in the last period, where Y^T denotes the discarding rate of unused credits in the last period. Constraints (6) and (7) ensure the non-negativity of the credit transfer rate and discarding rate of credits, respectively. Constraint (6) also indicates that travelers cannot transfer credits from the last period of the planning horizon. Constraint (8) ensures that the O-D demand rate, link flows and path flows satisfy the traffic network conditions. With the assumed properties of the demand and the link travel time functions, the objective function and the constraints of model (2)-(8) are convex in link flows and realized demand. Hence, it yields unique optimal link flows and realized demand. Since the optimal link flows are unique and the credit charging scheme is known, the credit consumption rate of each period is also unique. Let Δ^t denote the net credit transfer rate of period t which is the consumption rate of transferred credits. The net credit transfer rate of period t is given by:

$$\Delta^t = \sum_{j=1}^{t-1} z^{j,t} - \sum_{j=t+1}^{T} z^{t,j} \qquad T > t > 1 \tag{9}$$

If the net credit transfer rate is positive in period t, travelers transfer fewer number of credits from period t to future periods than those transferred to period t. The net credit transfer rate is unique based on constraint (4) because the issued credit rate is given and the credit consumption rate is unique. However, the credit transfer rate may not be unique. Assume that travelers need to consume the credits issued in period t in period $t+2$. Then, they can either transfer credits directly to period $t+2$ from period t or transfer them to period $t+1$ and thereafter, transfer them from period $t+1$ to consume in period $t+2$. So, the credit transfer rates may not be unique.

Let $\boldsymbol{p}$ denote the set of Lagrange multipliers for the credit conservation constraints (3)-(5), and $\boldsymbol{\mu}$ denote the set of Lagrange multipliers for the path flow conservation constraint (1). Then, the first order (KKT) conditions for this convex nonlinear program are as follows:

$$0 \le \sum_{a \in A} (((c_a^t(v_a^t) + p^t u_a^t)\delta_{a,r,w}) - \mu_w^t) \perp f_{r,w}^t \ \ge 0 \qquad \forall r, w, t \tag{10}$$

$$0 \le (\mu_w^t - D_w^{-1}(d_w^t)) \perp d_w^t \ \ge 0 \qquad \forall w, t \tag{11}$$

$$0 \le p^T \perp Y^T \ge 0 \tag{12}$$

$$0 \le \left(p^{t'} - p^t\right) \perp z^{t',t} \ \ge 0 \qquad \forall t' < T, \forall t > t' \tag{13}$$

$$(f, v, d) \in \Omega_{(f,v,d)} \tag{14}$$

(3)-(5)

where μ_w^t is the minimum generalized travel cost for O-D pair *w*. Mathematically, $\perp$ means "perpendicular", i.e., vectors $x \perp y$ if and only if $x^T y = 0$. Complementarity constraints (10) represent the user equilibrium conditions for the planning horizon. They state that for each time period t, the generalized travel costs of all travelers on the used paths for each O-D pair *w* should be equal to μ_w^t where the Lagrange multipliers p^t of constraints (3)-(5) denote the multi-period credit price in time period t. Constraint (11) states that if the traffic network demand for O-D pair w is positive, the inverse demand function is equal to the minimum generalized travel cost of O-D pair w. Constraint (12) states that if travelers discard credits, the multi-period equilibrium credit price is equal to zero in the last period. So, travelers discard unused credits at the end of the planning horizon without gaining benefit. Constraint (13) states that the multi-period equilibrium credit price in a future period is less than or equal to the multi-period equilibrium credit price in the current period. Constraint (13) states that if travelers transfer credits from period t' to period $t > t'$, the multi-period equilibrium credit prices are equal in these periods. Travelers transfer credits from the current period to future periods with higher single-period equilibrium credit prices until their multi-period equilibrium credit prices are equal. If they were to transfer additional credits beyond the point where the multi-period equilibrium credit prices are equal, it would lead to monetary losses. Hence, an equilibrium condition is reached only when no traveler can benefit by unilaterally storing extra credits to transfer to future periods. So, the total monetary value of credits in the aforementioned periods is preserved after transfer to future time periods at the equilibrium condition. Constraint (13) also indicates that the multi-period equilibrium credit prices decline over the planning horizon, but not necessarily strictly so; that is, the multi-period equilibrium credit prices can be equal across some periods. In the latter periods of the planning horizon, there are fewer opportunities to transfer unused credits and sell them at higher multi-period equilibrium credit prices compared to those in the earlier periods. Consequently, as unused credits have no value at the end of the planning horizon, the multi-period equilibrium credit price decreases monotonically over the periods. In summary, the multi-period equilibrium credit price in the market decreases through the planning horizon. Constraint (14) is identical to constraint (8).

As discussed earlier, the multi single-period TCS is a special case of the multi-period TCS in which there is no transfer of credits between periods. Let $z^{t',t} = 0$; then the multi-period TCS model (2)-(8) reduces to the multi single-period TCS model in which there is no linkage between the multi-period credit prices in different periods. Since the credit supply rate in each period is then equal to the issued credit rate by the central authority, the Lagrange multipliers of constraints (3)-(5) denote the single-period equilibrium credit prices in this case. Constraint (13) is satisfied irrespective of the single-period credit prices in different periods, and hence the single-period equilibrium credit prices can increase or decrease over the planning horizon. In the multi-period TCS, travelers are able to store credits to transfer to future periods. Thereby, Constraint (13) illustrates that the multi-period TCS can reduce price volatility by enabling only a decreasing trend in the credit price value through the planning horizon. This effect has been observed in the European Union emission TCS where the credit price volatility is dampened by issuing credits for longer trading periods (Ellerman and Joskow, 2008). It enables the central authority to develop the TCS with stable equilibrium credit prices during the planning horizon. It increases public acceptance because it allows travelers to hedge against the potential monetary losses associated

with consuming credits. The next proposition shows how the multi-period equilibrium credit price in each period is different from the single-period equilibrium credit price under the multi-period TCS (that is, the multi single-period equilibrium credit price) because of the ability of travelers to store and transfer credits to future periods.

Proposition 1. In the multi-period TCS, travelers transfer credits issued in a time period t' (that is, $\Delta^{t'} < 0$) with lower single-period equilibrium credit price for possible consumption in a period t (that is, $\Delta^{t} > 0$) with higher single-period equilibrium credit price.

Proof. Suppose travelers transfer credits issued in period t' to potentially consume in period t in the multi-period TCS. The credit supply rate in the market would increase for consumption in period t and decrease in period t'. Based on constraint (13), the multi-period equilibrium credit prices in periods t and t' are equal. So, it is sufficient to prove that the multi-period equilibrium credit price is higher than the single-period equilibrium credit price in period t' and lesser than the single-period equilibrium credit price in period t. We first prove the latter. Let $(\tilde{v}^t, \tilde{d}^t)$ and $(\bar{v}^t, \bar{d}^t)$ be the equilibrium link flows and traffic network O-D demand rates under the single-period TCS and multi-period TCS in period t, respectively. The single-period TCS can be formulated as the following optimization model:

$$\min_{v^t, d^t} U_1(v, d) = \sum_{a \in A} \left(\int_o^{v_a^t} c_a^t(\omega) d\omega \right) - \sum_{w \in W} \left(\int_o^{d_w^t} D^{-1}(\omega) d\omega \right) \tag{15}$$

$$\sum_{a \in A} u_a^t v_a^t \leq \xi^t \tag{16}$$

$$(f, v, d) \in \Omega_{(f,v,d)} \tag{17}$$

Inequality (16) implies that the credit supply in period t is either consumed or discarded under the single-period TCS. Let p_1^t be the Lagrange multiplier of the credit conservation constraint (16); it is the single-period equilibrium credit price $\tilde{p}^t$ under the equilibrium condition. Using the Lagrangian function, the optimization problem (15)-(17) can be expressed as the following optimization problem:

$$\min_{(v,d)} \max_{p_1^t} L(p_1^t, v, d) \tag{18}$$

$$(f, v, d) \in \Omega_{(f,v,d)} \tag{19}$$

where the Lagrangian function is formulated as follows:

$$L(p_1^t, v, d) = \sum_{a \in A} \left(\int_o^{v_a^t} c_a^t(\omega) d\omega \right) - \sum_{w \in W} \left(\int_o^{d_w^t} D^{-1}(\omega) d\omega \right) + p_1^t * \left(\sum_{a \in A} u_a^t v_a^t - \xi^t \right) \tag{20}$$

Given the single-period equilibrium credit price $\tilde{p}^t$, the optimization problem (18)-(19) can be reformulated as follows:

$$\min_{(v,d)} \sum_{a\in A}\left(\int_{o}^{v_a^t} c_a^t(\omega)d\omega\right) - \sum_{w\in W}\left(\int_{o}^{d_w^t} D^{-1}(\omega)d\omega\right) + \tilde{p}^t * \left(\sum_{a\in A} u_a^t v_a^t - \xi^t\right) \tag{21}$$

$$(f, v, d) \in \Omega_{(f,v,d)} \tag{22}$$

Now, let us consider the transfer of credits under the multi-period TCS to period t. If, under the equilibrium condition, travelers consume the transferred credits in a future period t at the net rate of $\Delta^t > 0$, the optimization problem for multi-period TCS in period t can be formulated as follows:

$$\min_{v^t,d^t} U_2(v,d) = \sum_{a\in A}\left(\int_{o}^{v_a^t} c_a^t(\omega)d\omega\right) - \sum_{w\in W}\left(\int_{o}^{d_w^t} D^{-1}(\omega)\, d\omega\right) \tag{23}$$

$$\sum_{a\in A} u_a^t v_a^t = \xi^t + \Delta^t \tag{24}$$

$$(f, v, d) \in \Omega_{(f,v,d)} \tag{25}$$

Constraint (24) states that credits are either consumed in period t or transferred to future periods under the multi-period TCS. Since the credit consumption rate is higher under the multi-period TCS than that of the single-period TCS in period t, the equilibrium link flows and traffic network O-D demand rates under the multi-period TCS $\left(\bar{v}^t, \bar{d}^t\right)$ are not equal to those under the single-period TCS $\left(\tilde{v}^t, \tilde{d}^t\right)$. Because the transferred credits to the last period, T, are either consumed or discarded, the net credit transfer rate of period t, expressed in constraint (9), can be re-written as follows:

$$\Delta^t = \sum_{j=1}^{t-1} z^{j,t} - \sum_{j=t+1}^{T-1} z^{t,j} - \zeta^{t,T} - \psi^{t,T} \qquad T > t > 1 \tag{26}$$

where $\zeta^{t,T}$ and $\psi^{t,T}$ denote the rates of consumption and discarding of the transferred credits from period t in period T. By combining with constraint (26), constraint (24) can then be re-written as follows:

$$\sum_{a\in A} u_a^t v_a^t = \xi^t + \sum_{j=1}^{t-1} z^{j,t} - \sum_{j=t+1}^{T-1} z^{t,j} - \zeta^{t,T} - \psi^{t,T} \tag{27}$$

Constraint (27) states that the credit consumption rate in period t is equal to the summation of the issued credit rate and the net credit transfer rate of that period.

Let $\sigma^t = \sum_{j=1}^{t-1} z^{j,t} - \sum_{j=t+1}^{T-1} z^{t,j} - \zeta^{t,T}$. From constraint (26), $\sigma^t > 0$. Using constraint (27), the optimization problem (23)-(25) for the multi-period TCS in period t can be reformulated as follows:

$$\min_{v^t,d^t} U_2(v,d) = \sum_{a\in A}(\int_o^{v_a^t} c_a^t(\omega)d\omega) - \sum_{w\in W}(\int_o^{d_w^t} D^{-1}(\omega)\, d\omega) \tag{28}$$

$$\sum_{a\in A} u_a^t v_a^t \le \xi^t + \sigma^t \tag{29}$$

$$(f,v,d) \in \Omega_{(f,v,d)} \tag{30}$$

As $\sigma^t > 0$ in constraint (29), the feasible region of the optimization problem (28)-(30) is larger than that of the optimization problem (15)-(17). Further, since $(\tilde{v}^t, \tilde{d}^t)$ is not the optimal solution to the optimization problem (28)-(30), it follows:

$$U_2^* < U_1^* \tag{31}$$

where inequality (31) states that the optimal objective value, U_2^*, of the optimization problem (28)-(30) for the multi-period TCS is less than the optimal objective value, U_1^*, of the optimization problem (15)-(17) for the single-period TCS. Let p_2^t denote the Lagrange multiplier of credit conservation constraint (29); it is the multi-period equilibrium credit price $\bar{p}^t$ under the equilibrium condition. Using the Lagrangian function, the optimization problem (28)-(30) can be expressed as the following optimization problem:

$$\min_{(v,d)} \max_{p_2^t} L(p_2^t, v, d) \tag{32}$$

$$(f,v,d) \in \Omega_{(f,v,d)} \tag{33}$$

where the Lagrangian function is formulated as follows:

$$L(p_2^t, v, d) = \sum_{a\in A}(\int_o^{v_a^t} c_a^t(\omega)d\omega) - \sum_{w\in W}(\int_o^{d_w^t} D^{-1}(\omega)d\omega\,) + p_2^t * (\sum_{a\in A} u_a^t v_a^t - \xi^t - \sigma^t) \tag{34}$$

Given the multi-period equilibrium credit price $\bar{p}^t$, the optimization problem (32)-(33) can be reformulated as follows:

$$\min_{(v,d)} \sum_{a\in A}(\int_o^{v_a^t} c_a^t(\omega)d\omega) - \sum_{w\in W}(\int_o^{d_w^t} D^{-1}(\omega)d\omega\,) + \bar{p}^t * (\sum_{a\in A} u_a^t v_a^t - \xi^t - \sigma^t) \tag{35}$$

$$(f,v,d) \in \Omega_{(f,v,d)} \tag{36}$$

There are four possible cases for the single-period and multi-period equilibrium credit prices in period t. In the first case, the multi-period and single-period equilibrium credit prices are strictly positive. In this case, constraints (16) and (29) are binding (that is, $\sum_{a\in A} u_a^t \tilde{v}_a^t = \xi^t$ and $\sum_{a\in A} u_a^t \bar{v}_a^t = \xi^t + \sigma^t$). The optimal value of the objective function (15), U_1^*, is the best lower bound for the value of objective function (21). Although the multi-period equilibrium link flows and traffic network O-D demand rates $(\bar{v}^t, \bar{d}^t)$ are a feasible solution for the optimization problem (21)-(22), they are not the optimal solution. Substituting $(\bar{v}^t, \bar{d}^t)$ into objective function (21), it follows:

$$U_1^* < \sum_{a\in A}(\int_o^{\bar{v}_a^t} c_a^t(\omega)d\omega) - \sum_{w\in W}(\int_o^{\bar{d}_w^t} D^{-1}(\omega)\, d\omega) + \tilde{p}^t * (\sum_{a\in A} u_a^t \bar{v}_a^t - \xi^t) \tag{37}$$

Then,

$$U_1^* < U_2^* + \tilde{p}^t * (\sum_{a\in A} u_a^t \bar{v}_a^t - \xi^t) \tag{38}$$

Because $\sum_{a\in A} u_a^t \bar{v}_a^t = \xi^t + \sigma^t$, inequality (38) is re-written as follows:

$$U_1^* - \tilde{p}^t \sigma^t < U_2^* \tag{39}$$

Further, the optimal value of the objective function (28), U_2^*, is the best lower bound for the value of the objective function (35). Although the single-period equilibrium link flows and traffic network O-D demand rates $(\tilde{v}^t, \tilde{d}^t)$ are a feasible solution for the optimization problem (35)-(36), they are not the optimal solution. Substituting $(\tilde{v}^t, \tilde{d}^t)$ into objective function (35), it follows:

$$U_2^* < \sum_{a\in A}(\int_o^{\tilde{v}_a^t} c_a^t(\omega)d\omega) - \sum_{w\in W}(\int_o^{\tilde{d}_w^t} D^{-1}(\omega)\, d\omega) + \bar{p}^t * (\sum_{a\in A} u_a^t \tilde{v}_a^t - \xi^t - \sigma^t) \tag{40}$$

Then,

$$U_2^* < U_1^* + \bar{p}^t * (\sum_{a\in A} u_a^t \tilde{v}_a^t - \xi^t - \sigma^t) \tag{41}$$

Since the credit consumption rate is equal to the issued credit rate ξ^t under the single-period TCS in period t, inequality (41) is re-written as follows:

$$U_2^* + \bar{p}^t \sigma^t < U_1^* \tag{42}$$

Inequality (42) can be re-written as follows:

$$U_2^* + \bar{p}^t \sigma^t - \tilde{p}^t\, \sigma^t < U_1^* - \tilde{p}^t \sigma^t \tag{43}$$

Since $\sigma^t > 0$, it follows from inequality (39) that $\bar{p}^t < \tilde{p}^t$. Hence, the multi-period equilibrium credit price is less than the single-period equilibrium credit price in period t.

In the second case, the multi-period and single-period equilibrium credit prices in period t are both equal to zero. Then, inequality (39) implies $U_1^* < U_2^*$ which contradicts inequality (31) and hence, this case is infeasible. In the third case, the multi-period equilibrium credit price is strictly positive while the single-period equilibrium credit price in period t is equal to zero. Similar to the previous case, this case is also infeasible since inequality (39) contradicts inequality (31). In the fourth case, the single-period equilibrium credit price is strictly positive while the multi-period equilibrium credit price is zero in period t. Then, the single-period equilibrium credit price is higher than the multi-period equilibrium credit price in period t.

Similarly, it can be proved that the multi-period equilibrium credit price in period $t'$is higher than its single-period equilibrium credit price. Since the multi-period equilibrium credit prices of periods t and t' are equal, the single-period equilibrium credit price in period t' is less than the single-period equilibrium credit price in period t. In other words, travelers transfer credits from a period with a lower single-period equilibrium credit price to a period with a higher single-period equilibrium credit price. This completes the proof.■

The above result has an intuitive interpretation. In a multi-period TCS, travelers store unused credits in a period with a lower single-period equilibrium credit price which can be independent of the actual travel needs in future periods. Instead of using these unused credits in the current period, travelers store them to sell them in future periods with a higher single-period equilibrium credit price. Consequently, the multi-period credit price increases in the period with the lower single-period credit price due to the reduced credit supply to consume in that period. Also, the multi-period credit price reduces in the period with the higher single-period credit price because of the additional credits. This flow of credits continues until the multi-period credit prices are equal in both periods. Hence, the multi-period TCS dampens high single-period credit prices by allowing travelers to store and transfer credits to future periods.

Next, Proposition 2 generalizes the proposition by Yang and Wang (2011) that the single-period equilibrium credit price is unique if there exists at least one O-D pair whose equilibrium path set contains two (or more) paths with the same generalized travel costs and different credit charges.

Proposition 2. Given the TCS, the multi-period equilibrium credit price in period t is unique if one of the following conditions is satisfied:

1. If there exists at least one O-D pair in period t whose equilibrium path set contains two (or more) paths with different credit charges.

2. Travelers transfer credits from a previous period with unique multi-period equilibrium credit price to period t.

3. Travelers transfer credits from period t to a future period with unique multi-period equilibrium credit price.

Proof. Yang and Wang (2011) prove the uniqueness of equilibrium credit price in single-period TCS under condition 1 where the uniqueness of link flows is used to demonstrate the uniqueness condition of the single-period equilibrium credit price. They show that the single-period equilibrium credit price in period t is unique if there exist at least two equilibrium paths with the same generalized travel costs and different credit charges for travelers in period t. If this condition is satisfied under the multi-period TCS in period t, the multi-period equilibrium credit price $\bar{p}^t$ is unique since the equilibrium link flows are unique similar to the single-period TCS. For condition 2, we prove that the multi-period credit price is unique if travelers transfer credits from a previous period t' with a unique multi-period credit price to period t. Assume that the multi-period equilibrium credit price $\bar{p}^{t'}$ of period t' is unique. From constraint (13), the current and future multi-period credit prices are equal. In other words, if $z^{t',t}$ is positive, then:

$$\bar{p}^t = \bar{p}^{t'} \tag{44}$$

Since the multi-period equilibrium credit price of period t' is unique, the multi-period equilibrium credit price of period t is also unique. Hence, it follows that the equilibrium credit price in period t is unique if travelers transfer credits from a period t' with unique multi-period equilibrium credit price to a period t. Similarly, the uniqueness of the multi-period equilibrium credit price can be proved under condition 3. This completes the proof.■

Proposition 2 has an important implication. The multi-period equilibrium credit price during a specific period of the planning horizon is unique if the uniqueness condition proposed by Yang and Wang (2011) is satisfied in one of the periods in which travelers transfer credits from/to that period. Yang and Wang discuss that the non-uniqueness of credit price in a single-period TCS results in an unhealthy market where travelers have to purchase required credits at uncertain prices in the market. So, their travel choices reduce with an increase in the multi-period equilibrium credit price. In the multi-period TCS, the linkage of multi-period equilibrium credit prices across periods enhances the possibility of uniqueness of the multi-period equilibrium credit prices.

4. Multi-period tradable credit scheme with transfer fee and reservation credit price

In this section, a different tradable scheme is proposed for implementation over the planning horizon whereby travelers store credits and transfer them to future periods at a predetermined transfer fee that can vary by period. The cost to manage and operate this TCS can be a relevant issue. These costs stem from the need to establish the market and technology required for this scheme. Nie (2012) proposes that the central authority charge a transaction cost on the credits traded in the market. While a transaction cost can compensate a portion of the budget to implement this scheme, it can entail negative connotations due to it being viewed as a penalty (Williamson, 2010). In the scheme proposed in this study, there is an incentive for travelers to reduce the credit consumption in some periods. They can use/sell stored credits in future time periods if the multi-period credit price is higher compared to that in the current period. Reducing credit consumption can decrease traffic network demand and mitigate consequent negative effects related to emissions, energy wastage, etc., and thereby address the system-level goals of the central authority. Hence, encouraging travelers to sell back credits to the central authority at the end of planning horizon can represent another scheme to manage traffic network demand and reduce emissions, both of which are desirable goals for the central authority. Thereby, the central authority buys back and discards unused credits previously issued to travelers using a predetermined reservation credit price that can vary by period. The reservation price is a widely-used economic concept and denotes the minimum price that a seller is willing to pay for a specific good/commodity. For example, this regulatory instrument has been recommended to overcome the issue of price volatility in the market for European Union emissions TCS (Hepburn et al., 2011).

In this TCS, credits can be transferred from any time period to future time periods by incurring the transfer fee. Travelers pay a transfer fee $P_g^{t',t} \geq 0$ to transfer their unused credits from period t' to a future period t. Let P_b^t be the reservation credit price for selling back the unused credits issued in period t to the central authority. Let m^t denote the rate at which credits are sold back during time period t to the central authority. The other notations follow the ones in the previous formulation. The equilibrium problem with transfer fee and reservation credit price can be formulated as follows:

$$\min \sum_{t}\sum_{a}((\int_{o}^{v_a^t} c_a^t(\omega)d\omega - P_b^t m^t + \sum_{j=1}^{t-1} P_g^{j,t} z^{j,t}) - \sum_{w}(\int_{o}^{d_w^t} D^{-1}(\omega)d\omega)) \tag{45}$$

$$\sum_{j=2}^{T} z^{t,j} + \sum_{a} u_a^1 v_a^1 + m^1 = \xi^1 \tag{46}$$

$$\sum_{j=t+1}^{T} z^{t,j} + \sum_{a} u_a^t v_a^t + m^t = \xi^t + \sum_{j=1}^{t-1} z^{j,t} \qquad 1 < t \leq T \tag{47}$$

$$z^{t',t} \geq 0 \qquad \forall t' < T, \forall t > t' \tag{48}$$

$$(f, v, d) \in \Omega_{(f,v,d)} \tag{49}$$

Constraint (46) states that the issued rate of credits to travelers in the first period by the central authority is equal to the sum of the credit consumption rate for that period, the rate of selling back to the central authority and the transfer rate of credits to future periods. Constraint (47) states that the transfer rate of credits from previous periods and the issued credit rate in each period should sum up to the credit transfer rate from the current period to future periods, the rate of selling back to the central authority and the rate of credit consumption during the current period. Unlike the multi-period TCS without penalty, travelers can sell credits back to the central authority to gain benefit instead of discarding credits in the last period. Constraint set (48) ensures the non-negativity of transfer rate of credits. It also implies that travelers cannot transfer credits from the last period. Constraint set (49) ensures that the traffic network O-D demand, link flows and path flows satisfy the traffic network conditions; it is identical to constraint (8). It should be noted here that when $P_g^{j,t} \to \infty$ and $P_b^t = 0$, model (45)-(49) reduces to the multi single-period TCS formulation.

Similar to the multi-period TCS model (2)-(8), the model (45)-(49) has a unique solution for $\boldsymbol{v}$. Introducing $\boldsymbol{p}$ as the Lagrange multiplier for constraints (46) and (47), the first order (KKT) conditions for this convex nonlinear program are as follows:

$$0 \leq \sum_{a}(((c_a^t(v_a^t) + p^t u_a^t)\delta_{a,r,w}) - \mu_w^t) \perp f_{r,w}^t \geq 0 \qquad \forall r, w, t \tag{50}$$

$$0 \leq (\mu_w^t - D_w^{-1}(d_w^t)) \perp d_w^t \geq 0 \qquad \forall w, t \tag{51}$$

$$0 \leq \left(p^{t'} + P_g^{t',t} - p^t\right) \perp z^{t,i} \geq 0 \qquad \forall t' < T, \forall t > t' \tag{52}$$

$$0 \leq (p^t - P_b^t) \perp m^t \geq 0 \qquad \forall t \tag{53}$$

$$(f, v, d) \in \Omega_{(f,v,d)} \tag{54}$$

(46)-(47)

The Lagrange multipliers of constraints (46) and (47) are the multi-period credit prices for trading in the market. Constraints (50) and (51) state the traffic equilibrium condition with elastic traffic network O-D demand. Constraint (52) states that if travelers transfer stored credits to a future period, the transfer fee compensates the difference between the lower multi-period equilibrium credit price in the current period and the higher multi-period equilibrium credit price in the future period. Constraint (53) states that the multi-period equilibrium credit price should be equal to or higher than the reservation credit price in a period. If the multi-period equilibrium credit price is higher than the reservation credit price, travelers sell their unused credits in the market or store them for transfer to future periods. If the multi-period credit price in a period is less than its reservation credit price, travelers sell credits back to central authority in that period. Then, the multi-period equilibrium credit price becomes equal to the reservation credit price in that period, as explained hereafter. Due to the sale of credits to the central authority, a reduction in credit supply occurs which leads to an increase in the multi-period credit price in that period. Due to the increase in the multi-period credit price, the generalized travel cost of each O-D pair increases, and hence the traffic network O-D demand decreases in that period. As a consequence, travelers sell their credits to the central authority until the multi-period credit price rises to become equal to the reservation credit price. Hence, credit price volatility of the planning horizon reduces as the multi-period credit price reaches the reservation credit price. Also, a reduction in the traffic network demand aids the central authority to achieve system-level goals. Constraint (54) is identical to constraint (49).

Next, in proposition 3, it is proved that under the equilibrium condition, travelers pay a transfer fee for using credits issued in a period with a lower multi-period equilibrium credit price in a period with higher multi-period equilibrium credit price.

Proposition 3. Under the equilibrium condition, travelers pay a transfer fee to transfer credits from a period with a lower multi-period equilibrium credit price to a period with a higher multi-period equilibrium credit price.

Proof. Constraint (52) implies that the equilibrium condition for credits issued in period t' to be transferred to period t is:

$$\bar{p}^t - \bar{p}^{t'} = P_g^{t',t} \tag{55}$$

As the right hand side of equation (55) is greater than or equal to zero because $P_g^{t',t} \geq 0$, it can be concluded that credits are transferred from the period with a lower multi-period equilibrium credit price to the period with a higher multi-period equilibrium credit price. This completes the proof.■

Proposition 3 has an intuitive interpretation. Travelers sell or consume credits in a period with a higher multi-period equilibrium credit price and purchase required credits in a future period with a lower multi-period equilibrium credit price rather than paying the transfer fee to transfer credits. As shown in proposition 1, travelers transfer credits from a period to a future period if the

future single-period equilibrium credit price is higher than the current single-period equilibrium credit price. When travelers have to pay a transfer fee, the flow of credits from a period to a future period continues until the transfer fee compensates the difference between the current and future multi-period equilibrium credit prices. As discussed earlier, if the central authority regulates a very high transfer fee, travelers do not transfer credits to future periods. The maximum acceptable transfer fee for travelers to transfer credits can be determined using the following proposition:

Proposition 4. Suppose that the single-period equilibrium credit price in period t' is lesser than the single-period equilibrium credit price in a future period t. Travelers transfer issued credits from period t' to consume in the future period t if the transfer fee is less than the difference between the single-period equilibrium credit prices in periods t and t'.

Proof. Suppose travelers transfer issued credits from a period t' to consume in a period t. Constraint (52) implies that if travelers transfer credits from period t' to period t, then (as in constraint (55)):

$$\bar{p}^{t} - \bar{p}^{t'} = P_g^{t',t} \tag{56}$$

Let $\tilde{P}_g^{t',t}$ denote the difference between the single-period equilibrium credit prices in period t', $\tilde{p}^{t'}$, and period $t, \tilde{p}^{t}$. It is proved in proposition 1 that if credits are transferred from period t', then the multi-period equilibrium credit price in that period is higher than its single-period equilibrium credit price. So,

$$\bar{p}^{t'} > \tilde{p}^{t'} \tag{57}$$

Also, it is proved in proposition 1 that the multi-period equilibrium credit price in period t is lower than its single-period equilibrium credit price. Then,

$$\tilde{p}^{t} > \bar{p}^{t} \tag{58}$$

From inequalities (57) and (58), we have:

$$\bar{p}^{t} - \bar{p}^{t'} < \tilde{p}^{t} - \tilde{p}^{t'} \tag{59}$$

The left hand side of inequality (59) denotes the transfer fee between periods t and t'. So, travelers transfer issued credits from a period t' to consume in a future period t if the transfer fee is less than the difference between the single-period equilibrium credit prices in periods t and t'. In other words, the difference between the single-period equilibrium credit prices in periods t and t' is the maximum acceptable transfer fee for travelers. This completes the proof.■

To understand proposition 4, let travelers pay a transfer fee $P_g^{t',t}$ to transfer issued credits from a period t' with a lower single-period equilibrium credit price to consume in a future period t with a higher single-period equilibrium credit price. Assume that this transfer fee is exactly equal to the difference between the single-period equilibrium credit prices of these periods. Since travelers transfer credits to use in a period with a higher single-period credit price, the multi-period credit price in period t' increases while that of period t decreases. Then, the value of each transferred credit, which includes the multi-period equilibrium credit price and the transfer fee, would become higher than the multi-period equilibrium credit price for period t. Hence, travelers will not transfer the credits to period t. So, if the central authority regulates the transfer fee to be between zero and $\tilde{P}_g^{t',t}$, travelers transfer credits from period t' to period t.

5. Numerical results

Numerical experiments are conducted to analyze the evolution of the multi-period equilibrium credit price under the two tradable credit schemes. The six-node network of Shirmohammadi et al. (2013), shown in Fig. 3, is used for the analysis. It has one O-D pair between nodes 1 and 6. The traffic network O-D demand function and corresponding link performance function parameters are presented in equation (60) and Table 1, respectively. The network supply parameters such as link capacities and free flow travel times are assumed to be unchanged over the planning horizon in this example. The link travel times are assumed to follow BPR functions. The traffic network O-D demand function between origin 1 and destination 6 is assumed to be:

$$D_{16}(\mu_{16}^t) = q_{16}^t exp(-0.05\mu_{16}^t) \qquad \forall t \tag{60}$$

where μ_{16}^t is the minimum generalized travel cost between nodes 1 and 6 in period t.

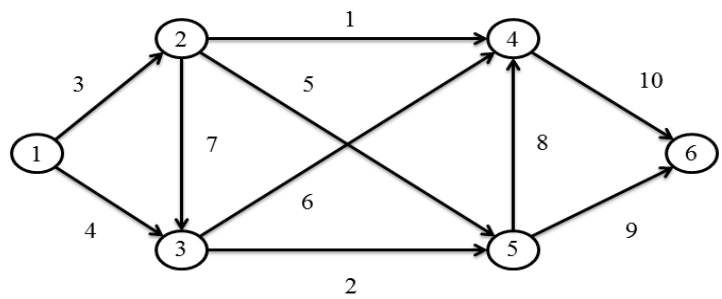


Fig. 3. The network.

Table 1

Link parameters for the six-node network.

Link ID	Start node-end node	Free flow travel time (min)	Capacity	Toll (# of credits)
1	2-4	7	45	7
2	3-5	3	22.5	3
3	1-2	3	45	3
4	1-3	5	45	5
5	2-5	6	22.5	6
6	3-4	4	22.5	5
7	2-3	1	22.5	1
8	5-4	1	22.5	1
9	5-4	3	45	3
10	4-6	2	45	2

Table 2

Issued credit rate and potential O-D demand rate through the planning horizon.

Time period	q_w^t	ξ^t
1	567.175	550
2	634.327	450
3	605.038	300
4	580.114	400
5	579.221	700
6	333.608	350
7	352.475	400
8	428.441	500
9	310.067	250
10	375.032	650

The planning horizon of interest is divided into 10 periods. Table 2 shows the issued credit rate ξ^t and the potential O-D demand rate q_w^t in each period t. The models are solved using GAMS (Brooke et al., 2015) and the CONOPT solver (Drud, 1995). Section 5.1 analyzes the evolution of the multi-period equilibrium credit price and the O-D demand rate under the first multi-period TCS, and compares them to those under the multi single-period TCS. Section 5.2 performs sensitivity analysis of the multi-period equilibrium credit price with respect to the transfer fee and the reservation credit price in the context of the second TCS to understand the effect of these parameters on the TCS.

5.1.Multi-period tradable credit scheme without penalty

Fig. 4 illustrates the evolution of the equilibrium credit prices under the multi-period and the multi single-period tradable credit schemes through the planning horizon. The multi-period TCS reduces the fluctuations in the equilibrium credit prices in the market during the planning horizon compared to the multi single-period TCS. As the single-period equilibrium credit prices in the third and fourth periods are higher than the single-period equilibrium credit prices in the other periods, travelers do not transfer credits from these periods to future periods. However, they transfer credits from the first and second periods to the third and fourth periods, which results in the reduction of the multi-period credit prices in those periods. The flow of credits continues until the multi-period equilibrium credit prices of the first four periods are equal. The single-period equilibrium credit price is the lowest in the last period. So, travelers do not transfer credits from the other periods to the last period. This results in identical single-period and multi-period equilibrium credit prices for the last period. Fig. 4 also validates that a multi-period TCS can lead to a reduction in price volatility compared to the multi single-period TCS, which represents a key motivation for the design of multi-period tradable credit schemes by a central authority to foster

system-level goals. Fig. 5 shows the traffic network O-D demand rate during the planning horizon under the multi-period and multi single-period tradable credit schemes. As can be seen from Fig. 4, travelers store credits in periods 1, 2, 5 and 8 in the multi-period TCS due to the lower single-period equilibrium credit prices in these periods. Hence, the traffic network demand rates in these periods under the multi-period TCS are lower than those under the multi single-period TCS. By contrast, travelers transfer credits to periods 3, 6 and 9 in the multi-period TCS because of their higher single-period equilibrium credit prices. It leads to lower multi-period equilibrium credit prices in these periods under the multi-period TCS, and hence their traffic network O-D demand rates increase compared to those of the multi single-period TCS.

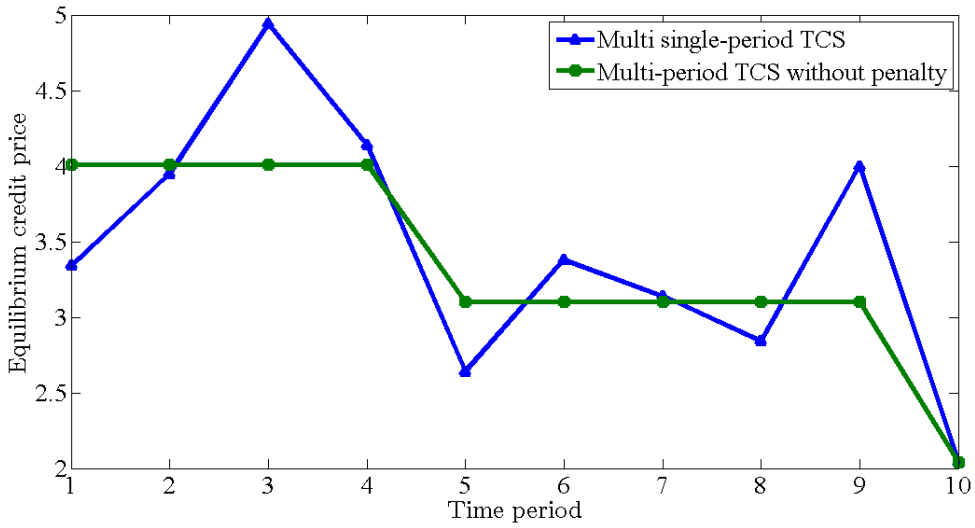


Fig. 4. Equilibrium credit prices under the multi single-period and multi-period tradable credit schemes.

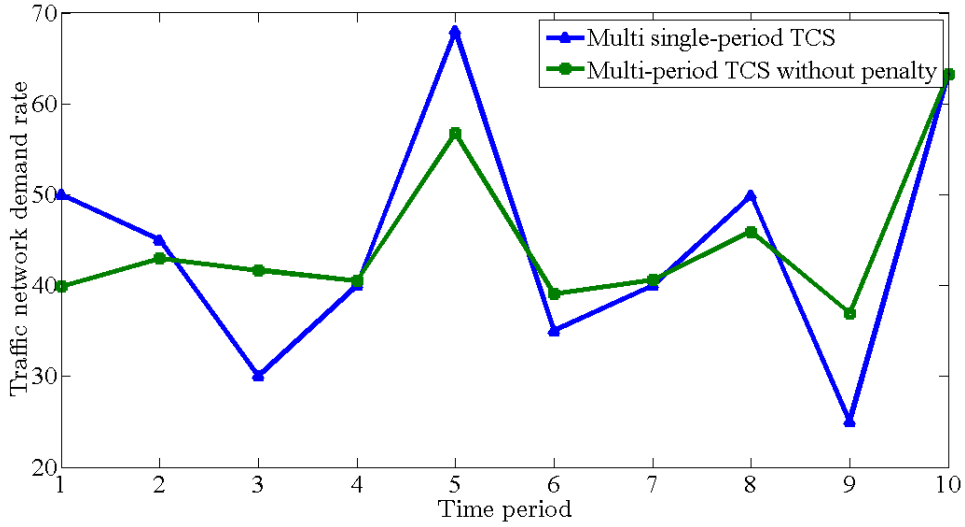
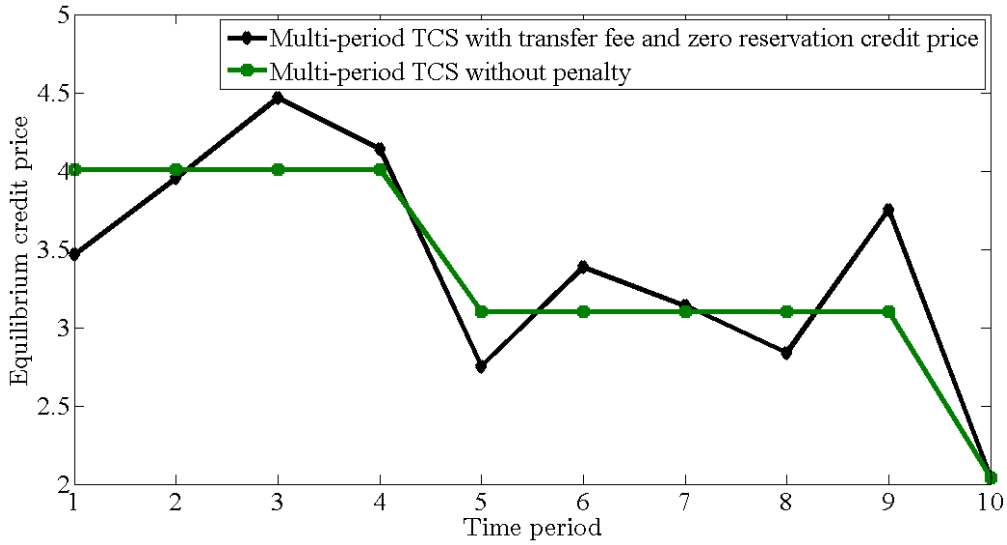


Fig. 5. Traffic network demand rates under the multi single-period and multi-period tradable credit schemes.

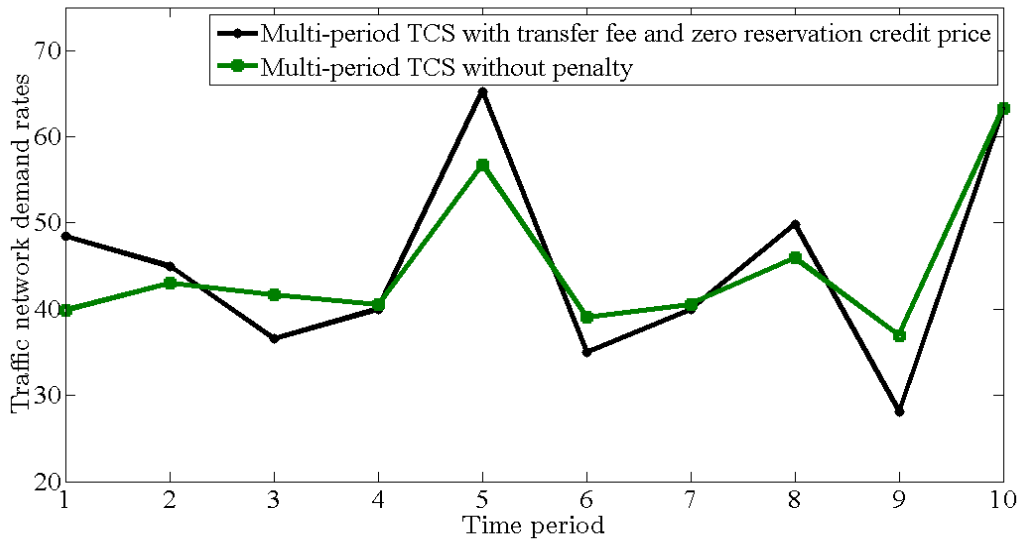
5.2.Multi-period TCS with transfer fee and reservation credit price

This section analyzes the multi-period TCS which incorporates transfer fees and reservation credit prices. To illustrate the effect of the transfer fee, the evolution of the equilibrium credit prices and the traffic network demand rates under the multi-period TCS without penalty and the multi-period TCS with transfer fee and zero reservation credit price are shown in Fig. 6. For this specific case, it is assumed that the transfer fee is constant through the planning horizon and equal to 1. As shown in Fig. 6(a), the multi-period TCS with transfer fee and zero reservation credit price leads to higher fluctuations in the equilibrium credit prices in the market during the planning horizon. This is because travelers transfer fewer credits to future periods under this multi-period TCS. As can be seen from Fig. 6(b), the traffic network demand of travelers and consequently, credit consumption are higher in periods 1, 2, 5 and 8 under the multi-period TCS with transfer fee and zero reservation credit price compared to the multi-period TCS without penalty. Fig. 4 demonstrates that the single-period credit prices are lower in aforementioned periods than other periods. So, travelers consume credits in periods with lower single-period equilibrium credit prices rather than transferring them to future periods with higher single-period equilibrium credit prices under this multi-period TCS. Under the multi-period TCS with no penalty, the total travel time of the traffic network through the planning horizon is equal to 5,958. If the central authority sets the transfer fee as constant (and equal to 1) through the planning horizon, the total travel time of the network increases to 6,043 through the planning horizon. One possible reason is because transfer fee increases the cost of using credits, and hence travelers choose paths with higher travel times and lesser credit consumption. It suggests that without the optimization of transfer fee, the total travel time can increase during the planning horizon.

Fig. 7 illustrates the effect of transfer fee in the multi-period TCS on the rate of storing credits by travelers in the periods 1, 5 and 8. Although hoarding behavior does not occur in this example, the figure indicates that if the central authority finds that travelers are hoarding credits, transfer fee can be used as an instrument to reduce the credit storing rate of travelers. It is assumed that transfer fee is constant during the planning horizon with zero reservation credit price. Although the single-period equilibrium credit prices of periods 5 and 8 are less than the single-period equilibrium credit price of period 1, travelers store higher number of credits in period 1 compared to periods 5 and 8. This is because travelers have higher opportunity to store credits in period 1 to transfer to future periods compared to periods 5 and 8. As transfer fee increases, the rate of storing credits reduces until it reaches the maximum acceptable transfer fee. The maximum acceptable transfer fee to transfer credits from the first period to future periods is the maximum difference between their single-period equilibrium credit prices (proposition 4). In this case, the maximum acceptable transfer fee is 1.78. If the central authority regulates the transfer fee to be higher than this value, then travelers sell or consume credits rather than store them to sell or consume in the future periods because it is cheaper to buy credits in future periods than transfer them. Hence, if the central authority identifies that travelers have hoarded credits in a period, this instrument enables the central authority to mitigate the hoarding effect by using a high transfer fee.



(a) Equilibrium credit prices under the multi-period TCS without penalty and the multi-period TCS with transfer fee and zero reservation credit price.



(b) Traffic network demand rates under the multi-period TCS without penalty and the multi-period TCS with transfer fee and zero reservation credit price.

Fig. 6. Equilibrium credit prices and traffic network demand rates under the multi-period TCS without penalty and the multi-period TCS with transfer fee and zero reservation credit price.

The reservation credit price is an important parameter in the second multi-period TCS, and denotes the least value of credits to trade in the market. It has a direct impact on the multi-period equilibrium credit price and the credit consumption. To understand the effect of the reservation credit price, the evolution of the equilibrium credit prices and traffic network demand rates under the multi-period TCS without penalty and the multi-period TCS with reservation credit price and zero transfer fee are shown in Fig. 8. It is assumed that the central authority sets the reservation credit price to 4 for all periods under the multi-period TCS with reservation credit price and zero transfer fee. As can be seen in Fig. 8(a), travelers sell their credits to the central authority in periods

5 through 10 until their multi-period equilibrium credit prices become equal to the reservation credit price. So, the regulation of the reservation credit price can reduce the credit price volatility in multi-period tradable credit schemes if the central authority chooses their values consistent with the objective of reducing volatility; a constant reservation credit price throughout the planning horizon can eliminate this volatility completely. As illustrated in Fig. 8(b), the total travel demand over the planning horizon is reduced by 21% under the multi-period TCS with a constant reservation credit price and zero transfer fee compared to the multi-period TCS without penalty. So, the reservation credit price can be used as a control parameter to reduce the network-level travel demand, which is a key motivation for the consideration of multi-period tradable credit schemes to address system-level goals.

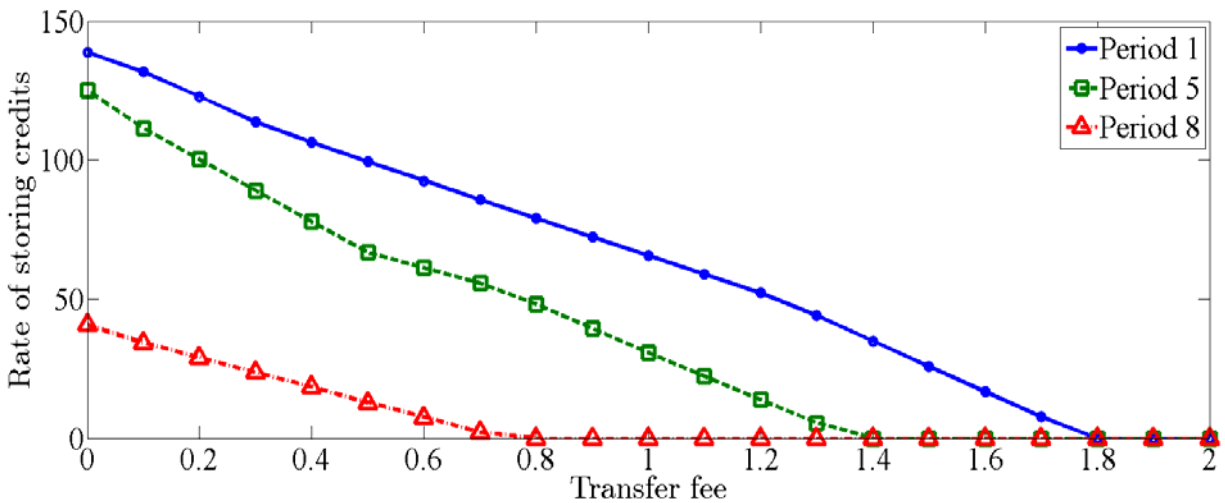


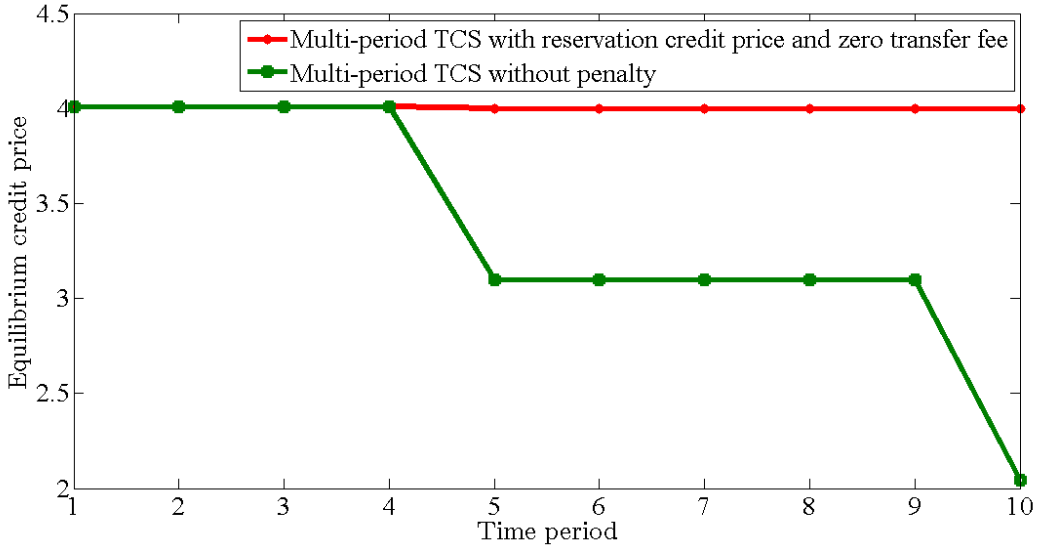
Fig. 7. Impact of transfer fee on rate of storing credits in different periods

As can be noted, the constant reservation credit price value of 4 considered in this experiment is equal to the highest multi-period equilibrium credit price under the no-penalty TCS in Fig. 4. It is relevant to note here that if the value chosen were about 2, which is slightly below the lowest equilibrium credit price under the no-penalty TCS in Fig. 4, the performance of the multi-period TCS with reservation credit price and zero transfer fee in Fig. 8 would be identical to that of the multi-period TCS without penalty. That is, the reservation credit price would have no role to play in this case, consistent with the insights from Section 4. It then follows that for constant reservation prices between 2 and 4, the performance of the multi-period TCS with reservation credit price and zero transfer fee would be between these two extremes.

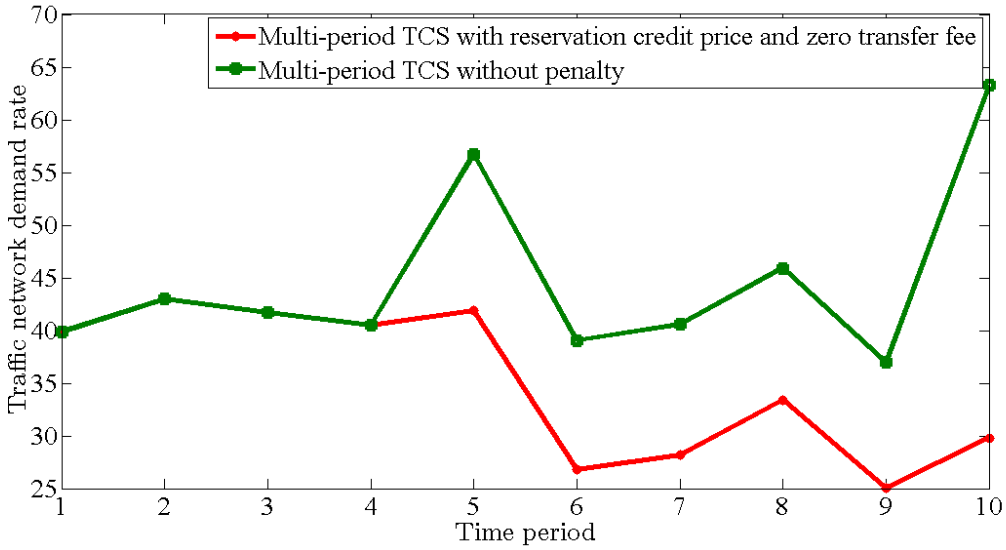
In summary, from a regulatory instrument standpoint, transfer fees play the role of dampening credit transfers across periods, thereby mitigating the possibility of hoarding. Reservation credit prices, depending on their time-dependent profile, can aid in the reduction of credit price volatility across the planning horizon. Hence, these two regulatory instruments can be used to reduce market manipulation and the artificial control of prices.

Fig. 9 illustrates the effect of the reservation credit price on the percentage of credits sold back to the central authority for a multi-period TCS in which the transfer fee is equal to zero. Travelers do not sell credits back to the central authority if the reservation credit prices are lower than the minimum single-period equilibrium credit price through the planning horizon. As

reservation prices increase, travelers have a greater incentive to sell credits back to the central authority instead of the market or to use it to travel. For example, as shown in Fig. 9, as the central authority increases the reservation credit price to 15, the percentage of sold back credits increases to 100%. It implies that the traffic network O-D demand rate is reduced to almost zero. This illustrates the benefit of introducing reservation credit price into tradable credit schemes so as to provide the central authority with a control parameter to meet system-level goals.



(a) Equilibrium credit prices under the multi-period TCS without penalty and the multi-period TCS with reservation credit price and zero transfer fee.



(b) Traffic network demand rates under the multi-period TCS without penalty and the multi-period TCS with reservation credit price and zero transfer fee.

Fig. 8. Equilibrium credit prices and traffic network demand rates under the multi-period TCS without penalty and the multi-period TCS with reservation credit price and zero transfer fee.

To analyze the effect of market performance under various combinations of transfer fee and reservation credit price, different combinations of P_g and P_b are tested. P_g values between 0 and 2 with a uniform 0.02 interval are considered. P_b values between 0 and 5 with a uniform 0.05 interval are considered. It is assumed that these P_g and P_b values are unchanged throughout the planning horizon for each combination scenario. Two-dimensional contour diagrams, in which the different shades of grayscale are used to denote the multi-period equilibrium credit prices corresponding to a combination of P_g and P_b, are illustrated for the third and ninth periods of the planning horizon in Fig. 10.

Fig. 10(a) illustrates the multi-period equilibrium credit price in the third period under different combinations of P_g and P_b. The minimum multi-period equilibrium credit price can be obtained under the multi-period TCS without transfer fee and reservation credit price; this is the multi-period TCS without penalty discussed in Fig. 4. If the central authority regulates the reservation credit price to be lower than the minimum of the single-period equilibrium credit prices of periods 1, 2 and 3, then travelers do not sell credits back in these periods to the central authority. Then, the single-period equilibrium credit prices of these periods under the multi-period TCS with transfer fee and reservation credit price are not affected by the regulation of the reservation credit price. Thereby, these single-period equilibrium credit prices are identical to those under the multi-period TCS without penalty. As discussed in proposition 4, the maximum acceptable transfer fee to transfer credits from periods 1 or 2 to period 3 is the difference between their single-period equilibrium credit prices. Since period 1 in Fig. 4 has a lower single-period equilibrium credit price compared to period 2, the maximum acceptable transfer fee to transfer credits from either periods 1 or 2 to period 3 is the difference between the single-period equilibrium credit prices of period 3 and period 1. In this case, the maximum acceptable transfer fee is 1.78 as illustrated on the y-axis in Fig. 10(a). Hence, if the central authority regulates the transfer fee to be higher than this value, travelers consume credits in periods 1 and 2 instead of transferring them to period 3. Then, the multi-period equilibrium credit price of period 3 is equal to its single-period equilibrium credit price under the multi-period TCS without penalty.

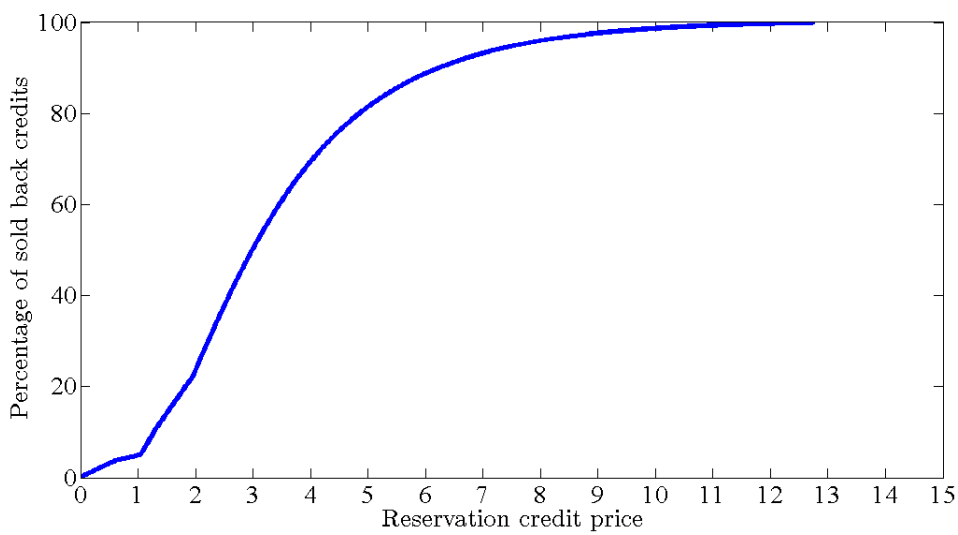
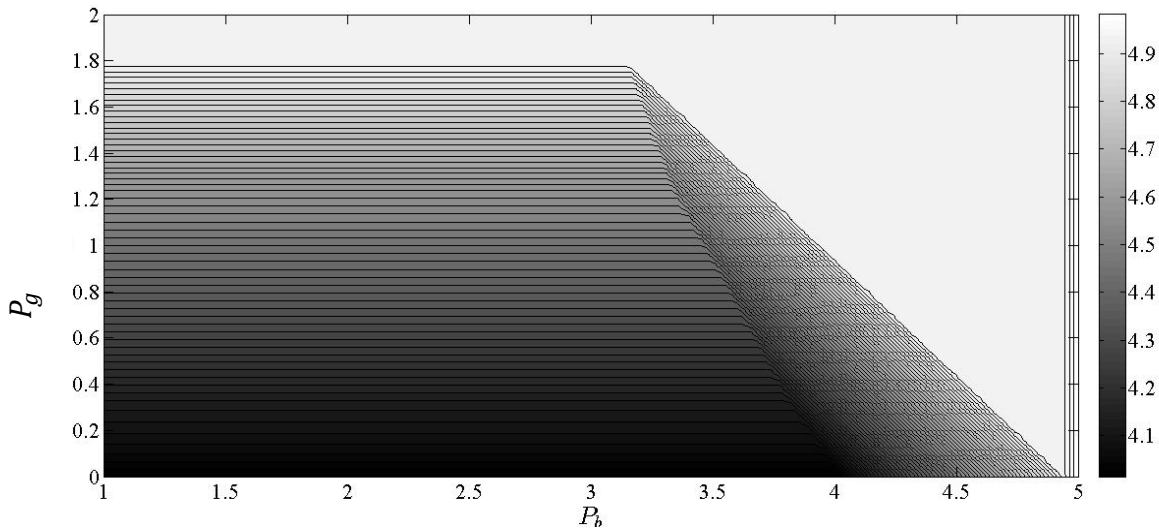


Fig. 9. Impact of reservation credit price on the percentage of credits sold back to the central authority.

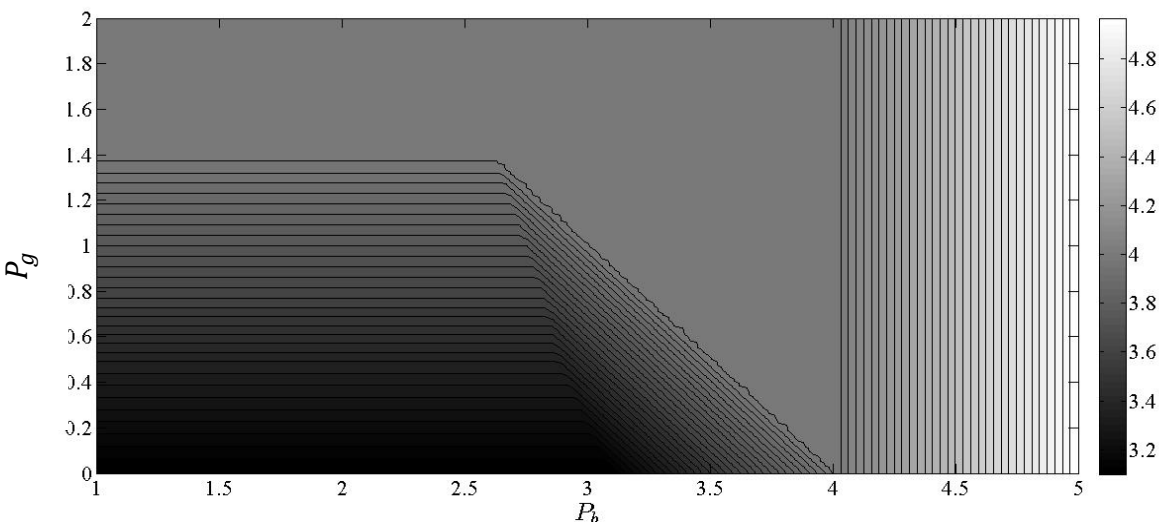
As the reservation credit price increases to a value higher than the single-period equilibrium credit prices of periods 1 and 2 under the multi-period TCS without penalty, travelers sell credits

back to the central authority in these periods. Then, the single-period equilibrium credit prices of periods 1 and 2 increase, and consequently the maximum acceptable transfer fee reduces in the context of the transfer of credits to period 3. If the reservation credit price becomes equal to the single-period equilibrium credit price of the third period, which is the highest among the first three periods under the multi-period TCS without penalty, then credits are either consumed or sold back to the central authority in the first three periods. In other words, travelers do not consider the option of transferring credits from period 1 and 2 to period 3 in this case.

A similar trend can be observed in the ninth period as shown in Fig. 10(b). Since travelers have more opportunities to store credits before the ninth period compared to the third period, the multi-period equilibrium credit price can be reduced to 3 in the ninth period though it can be reduced to only 4 in the third period.



(a) Multi-period equilibrium credit price in the third period.



(b) Multi-period equilibrium credit price in the ninth period.

Fig. 10. Multi-period equilibrium credit prices in different periods under different combinations of transfer fee and reservation credit price.

6. Concluding comments

This paper proposes the concept of multi-period tradable credit schemes. It focuses on a TCS-based multi-period equilibrium modeling framework from the long-term planning perspective of a central authority to meet some system-level goals, such as traffic-related emissions reduction over a long time horizon. The central authority divides the planning horizon into multiple time periods where the TCS parameters (credit supply and the credit charging scheme) are predetermined in advance for the planning horizon. While the central authority monitors the progress toward the system-level goals, the stability of the equilibrium credit prices through the planning horizon is a primary factor in the public acceptance of the TCS. It is because travelers can hedge against the potential monetary losses in storing and transferring credits. Hence, given these exogenous TCS parameters, this study seeks to understand the evolution of the multi-period equilibrium credit prices over the planning horizon. These planning-based equilibrium credit prices can be provided as forecasted future credit prices to travelers who would then make decisions related to credit usage and credit transfers to future periods.

Two multi-period tradable credit schemes are analyzed in this study. In the first multi-period TCS, travelers can transfer credits at no penalty to future periods. In this planning problem, it is shown that the multi-period equilibrium credit price is monotonically decreasing. Hence, a multi-period TCS is shown to reduce credit price fluctuations due to the market linkage across periods. However, in an operational context, the realized credit price can be different from the projected multi-period equilibrium credit prices because of potential inaccuracies in the forecasts of future traffic network demand and supply, as well as the heterogeneity in traveler behavior. That is, the monotonically decreasing characteristic may not hold in practice. Hence, the multi-period equilibrium credit prices solved for in this study represent desirable benchmarks from the perspective of the central authority in a deployment context, as well as enable a long-term operational plan to achieve the system-level goals. The study also demonstrates that the linkage of the multi-period equilibrium credit prices across periods enhances the possibility of uniqueness of the multi-period equilibrium credit prices, thereby mitigating the likelihood of unhealthy markets.

To further mitigate the possibility of market manipulation, a second multi-period TCS is analyzed in which travelers pay a transfer fee to transfer stored credits from previous periods to a future period. Also, in this TCS, the central authority predetermines reservation credit prices to buy credits back from travelers. It is shown that the reservation credit price is the least value to trade credits in the market. Also, the transfer fee is shown to compensate for the difference between the higher multi-period equilibrium credit price in a future period and the lower multi-period equilibrium credit price in the current period. It is proved that the maximum acceptable transfer fee between the current and a future period is the difference between their single-period equilibrium credit prices. Numerical results demonstrate that the regulation of the reservation credit price can further reduce credit price volatility during the planning horizon, while also aiding the central authority to meet system-level goals by reducing traffic network demand.

The key findings of this study are as follows: (1) Multi-period TCS dampens the credit price volatility during the planning horizon. (2) This study demonstrates that the linkage of multi-period equilibrium credit prices across the periods of the planning horizon of interest enhances the possibility of uniqueness of the multi-period equilibrium credit prices. Past studies on single-period TCS illustrate uniqueness only if specific conditions are satisfied (one O-D pair with two used paths and positive credit charges). In our study, uniqueness in any one period enhances the

likelihood of uniqueness in other periods (Proposition 2). (3) Transfer fee as an instrument enables the central authority to mitigate the risk of credit hoarding by travelers. Reservation credit price can dampen credit price volatility while reducing credit consumption, which can have positive implications such as reduced emissions.

The study insights suggest that the central authority should design a multi-period TCS by adequately factoring traveler behavior in terms of the interactions between the credit and travel markets to enable the smooth progress of the system towards the system-level goals. Further, the effects of market manipulation and uncertainty of future traffic network demand under a long planning horizon need to be considered in the design of a TCS. Hence, through the strategic design of a multi-period TCS, the central authority can exert greater control in terms of achieving system-level goals rather than just myopically reacting to credit price fluctuations or system state deviations in specific time periods. Further, multi-period TCS enables the central authority to develop TCS with stable credit prices. It can increase public acceptance of TCS since travelers can hedge against potential monetary losses. Finally, the central authority can mitigate the risk of credit hoarding by regulating transfer fee. The reservation credit price can reduce traffic network demand and its consequent negative impacts.

Due to the planning context of the framework proposed in this study, it is assumed that perfect information is available to travelers on future credit prices, and travelers value time homogeneously. A future research direction is to relax these assumptions to address this problem in a real-world context. Another future research direction is to factor the uncertainties in traffic network demand and credit supply to analyze the evolution of the credit prices over the horizon of interest. A third future research direction is to determine the optimal design of multi-period TCS in terms of credit allocation scheme, credit charging scheme, transfer fee and reservation credit price to enable the central authority to achieve system-level goals while considering the discount rate over a long-planning horizon. When the discount rate is considered, travelers would consider selling credits and use this cash to purchase credits in future periods while collecting interest on the deposited cash from the bank. Thereby, the multi-period equilibrium credit prices should reflect the effect of this behavior.

The proposed planning-focused multi-period TCS framework is a building block to enable the central authority to evaluate the progress towards system-level goals. In an implementation context, the central authority will need to leverage the available data unfolding in each period. A fourth research direction is to develop a deployment approach using this framework that incorporates the real-world data available in each period. For example, this can be a rolling horizon approach in which data collected in the current period on the realized market credit price, the traffic network demand and the status of the system-level goals, is used to: (i) calibrate the elastic traffic network demand function, (ii) update the TCS parameters for the rest of the planning horizon, and (iii) determine the updated equilibrium credit prices for the planning horizon.

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References

Bao, Y., Gao, Z., Xu, M., Yang, H., 2014. Tradable credit scheme for mobility management considering travelers' loss aversion. Transp. Res. Part E 68, 138–154. doi:10.1016/j.tre.2014.05.007

Brooke, A., Kendrick, D., Meeraus, A., Raman, R., 2015. GAMS — A User's Guide. Washington, DC.

Dafermos, S., Sparrow, F.T., 1971. Optimal resource allocation and toll Patterns in user-optimized transport networks. J. Transp. Econ. Policy 5, 184–200.

Dafermos, S.C., 1973. Toll patterns for multiclass-user transportation networks. Transp. Sci. 7, 211–223. doi:10.1287/trsc.7.3.211

Drud, A.A., 1995. A System for Large Scale Nonlinear Optimization. Tutorial for CONOPT Subroutine Library. ARKI Consulting and Development A/S, Bagsvaerd, Denmark.

Ellerman, A.D., Joskow, P.L., 2008. The European Union's emissions trading system in perspective. Pew Center on Global Climate Change, Arlington, VA. doi:10.5755/j01.em.17.3.2113

He, F., Yin, Y., Shirmohammadi, N., Nie, Y., 2013. Tradable credit schemes on networks with mixed equilibrium behaviors. Transp. Res. Part B 57, 47–65. doi:10.1016/j.trb.2013.08.016

Hepburn, C., Grubb, M., Neuhoff, K., Matthes, F., Tse, M., 2011. Auctioning of EU ETS phase II allowances: how and why? Clim. Policy 6, 137–160.

Lawphongpanich, S., Yin, Y., 2010. Solving the Pareto-improving toll problem via manifold suboptimization. Transp. Res. Part C 18, 234–246. doi:10.1016/j.trc.2009.08.006

Leggett, J.A., Elias, B., Shedd, D.T., 2012. Aviation and the European Union ' s Emission Trading Scheme. Congressional Research Service, Washington, DC.

Nagurney, A., 2000a. Alternative pollution permit systems for transportation networks based on origin/destination pairs and paths. Transp. Res. Part D Transp. Environ. 5, 37–58. doi:10.1016/S1361-9209(99)00023-1

Nagurney, A., 2000b. Sustainable Transportation Networks. Edward Elgar Publishing, Cheltenham, UK.

Nagurney, A., Ramanujam, P., Dhanda, K.K., 1998. A multimodal traffic network equilibrium model with emission pollution permits: compliance vs noncompliance. Transp. Res. Part D Transp. Environ. 3, 349–374. doi:10.1016/S1361-9209(98)00016-9

Nash, C., Menaz, B., Matthews, B., 2008. "Inter-Urban Road Goods Vehicle Pricing in Europe," in: Road Congestion Pricing in Europe. Edward Elgar Publishing, Cheltenham, UK. doi:10.4337/9781848441453.00020

Nie, Y., 2012. Transaction costs and tradable mobility credits. Transp. Res. Part B 46, 189–203. doi:10.1016/j.trb.2011.10.002

Nie, Y., Yin, Y., 2013. Managing rush hour travel choices with tradable credit scheme. Transp. Res. Part B 50, 1–19. doi:10.1016/j.trb.2013.01.004

OECD, 1997. Towards sustainable fisheries: economic aspects of the management of living marine resources. Organisation for Economic Co-operation and Development, Paris, France.

Pigou, A.C., 1920. The economics of welfare, 1st ed. Macmillan and Company, London.

Schrank, D., Eisele, B., Lomax, T., 2012. 2012 urban mobility report. Texas Transportation Institute.

Shirmohammadi, N., Zangui, M., Yin, Y., Nie, Y., 2013. Analysis and design of tradable credit schemes under uncertainty. Transp. Res. Rec. 2333, 27–36. doi:10.3141/2333-04

Verhoef, E., Nijkamp, P., Rietveld, P., 1997. Tradeable permits: their potential in the regulation of road transport externalities. Environ. Plan. B 24, 527–548. doi:10.1068/b240527

Viegas, J.M., 2001. Making urban road pricing acceptable and effective: searching for quality and equity in urban mobility. Transp. Policy 8, 289–294. doi:10.1016/S0967-070X(01)00024-5

Wang, X., Yang, H., Zhu, D., Li, C., 2012. Tradable travel credits for congestion management with heterogeneous users. Transp. Res. Part E 48, 426–437. doi:10.1016/j.tre.2011.10.007

Williamson, O.E., 2010. Transaction cost economics: the natural progression. Am. Econ. Rev. 100, 673–690. doi:10.1257/aer.100.3.673

Xu, M., Grant-Muller, S., 2016. Trip mode and travel pattern impacts of a Tradable Credits Scheme: A case study of Beijing. Transp. Policy 47, 72–83. doi:10.1016/j.tranpol.2015.12.007

Yang, H., Huang, H.-J., 1998. Principle of marginal-cost pricing: how does it work in a general road network? Transp. Res. Part A Policy Pract. 32, 45–54. doi:10.1016/S0965-8564(97)00018-9

Yang, H., Wang, X., 2011. Managing network mobility with tradable credits. Transp. Res. Part B 45, 580–594. doi:10.1016/j.trb.2010.10.002

Ye, H., Yang, H., 2013. Continuous price and flow dynamics of tradable mobility credits. Transp. Res. Part B 57, 436–450. doi:10.1016/j.trb.2013.05.007

Zhu, D.-L., Yang, H., Li, C.-M., Wang, X.-L., 2015. Properties of the Multiclass Traffic Network Equilibria Under a Tradable Credit Scheme. Transp. Sci. 49, 519–534. doi:10.1287/trsc.2013.0508