Clearance Time Estimation for Incorporating Evacuation Risk in Routing Strategies for Evacuation Operations

Yu-Ting Hsu · Srinivas Peeta

Abstract This study seeks to develop an approximate but efficient approach for estimating evacuation clearance time which is defined as the time required to evacuate the population of a location to areas of safety. Elsewhere, this estimate of the clearance time is used as input to infer evacuation risk for a location which reflects whether the population of that location can be safely evacuated before the disaster impacts it. As the computed evacuation risk is used in a real-time stage-based framework for evacuation operations, the approach for clearance time estimation needs to be computationally efficient while being capable of approximating traffic flow dynamics reasonably. To address these needs, a dynamic routing policy labeled the location-priority routing is proposed, and the associated clearance time estimation problem is formulated as a dynamic network flow problem. The location-priority is based on the lead time that a location has until it is impacted by the disaster, and designates that the population at a location with a shorter lead time has higher priority in using roadway capacity for evacuation. By combining the location-priority routing with the consideration of a super sink, the routing problem for the evacuation network is transformed into sequential single-origin single-destination dynamic routing problems. It avoids expensive iterative search processes, enabling computational efficiency for real-time evacuation operations. The solution approach approximately captures the evolution of traffic dynamics across the stages of the operational framework. Results from numerical experiments illustrate that the proposed approach can address the aforementioned needs related to clearance time estimation.

Keywords Evacuation risk · Clearance time estimation · Dynamic network flow problem · Location-priority routing · The quickest flow

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1 INTRODUCTION

The traffic routing for evacuation operation problems seeks to efficiently relocate the threatened/affected population to safe places using the available transportation network, thereby avoiding or mitigating potential loss of life due to disasters. Most current approaches to address evacuation operations focus on the traffic management aspects of the problem, where traffic-related network performance is of major concern (for example, total system travel time or network clearance time). However, an evacuation operation is also significantly affected by disaster characteristics and impacts. However, most current approaches do not explicitly factor the impacts of disaster characteristics in determining operational evacuation strategies, and may notionally address the role of a disaster by identifying the population to be evacuated in terms of evacuation zones, designated *a priori* through evacuation planning. The lack of explicit accounting for the role of disaster impact in an evacuation operation may raise two key issues. First, a pre-specified evacuation zone may not be identifiable for disasters with high randomness in spatial location of occurrence. Also, in the context of an evacuation operation, disaster impact may spread and/or vary with time. Hence the use of a pre-specified evacuation zone may preclude a capability to capture the dynamics of the disaster impact. Second, as indicated in the natural hazards management literature (Neumann 1987; Cutter et al. 2000; Odeh 2002) and some evacuation-related studies (Sorensen et al., 1992), individuals at different spatial locations even within an evacuation zone may sustain different risk levels of being impacted by the disaster. To more explicitly and systematically address these issues, a concept of an evacuation risk zone is proposed and introduced next.

1.1 Evacuation risk zone and stage-based operational framework

In current practice, the evacuation zone is determined in the planning context based on some simple principles pertaining to the expected disaster impact over the associated region; for example, locations within a 10-mile radius of a nuclear power plant (Sorensen et al., 1987). By contrast, Hsu and Peeta (2011) propose the concept of an evacuation risk zone (ERZ) which is determined based on the "evacuation risk" associated with each location in the region potentially affected by a disaster. Evacuation risk for a location reflects whether the population of that location can be safely evacuated before the disaster impacts it. Church and Cova (2000) use clearance time, the time required to evacuate the population of a location to areas of safety, as a surrogate for evacuation risk in wildfire evacuation planning. Further, they propose the ratio of the aggregate demand to the available exit capacity for a location as being a simplified estimate of the clearance time. However, in an operational context, the risk associated with safely evacuating the affected population of a location depends on the evolution of the disaster characteristics with time as well as the dynamics of the traffic flow conditions in the associated network. We acknowledge these operational considerations in Hsu and Peeta (2011), and define evacuation risk of location a, R_a, as:

$$R_a = -(LT_a - CT_a) \tag{1}$$

where LT_a is the lead time, the time to when an location is estimated to be impacted by the disaster, and CT_a is the clearance time for location a. $LT_a - CT_a$ implies the lead time for the last evacuee who exits from location a. It can also be interpreted as the time margin available to the population of a location to be safely evacuated while avoiding being impacted by the disaster. The negative sign indicates that evacuation risk is negatively valued. Based on this definition, our proposed evacuation risk measure can integrate the dynamics of both traffic and disaster evolution.

The concept of evacuation risk is further applied to develop an ERZ deployment paradigm for stage-based evacuation operations in which disaster response operators seek to prioritize resources for operational deployment to evacuate the population with the highest levels of evacuation risk first. In each stage, the ERZ is determined as a spatially bounded zone in the disaster-affected region that encompasses the locations which have the highest evacuation risks up to a threshold determined by resource constraints, and also satisfies the need for spatial contiguity. Hence, in the stage-based operational framework, one ERZ is determined for each stage of the evacuation operation sequentially based on the evacuation risks of various locations in that stage. In this context, the dynamics of the evacuation network are captured in evacuation risk computation through the stage-based updates of the evolving disaster characteristics and traffic conditions. The details of the model to determine an ERZ are provided in Hsu and Peeta (2011).

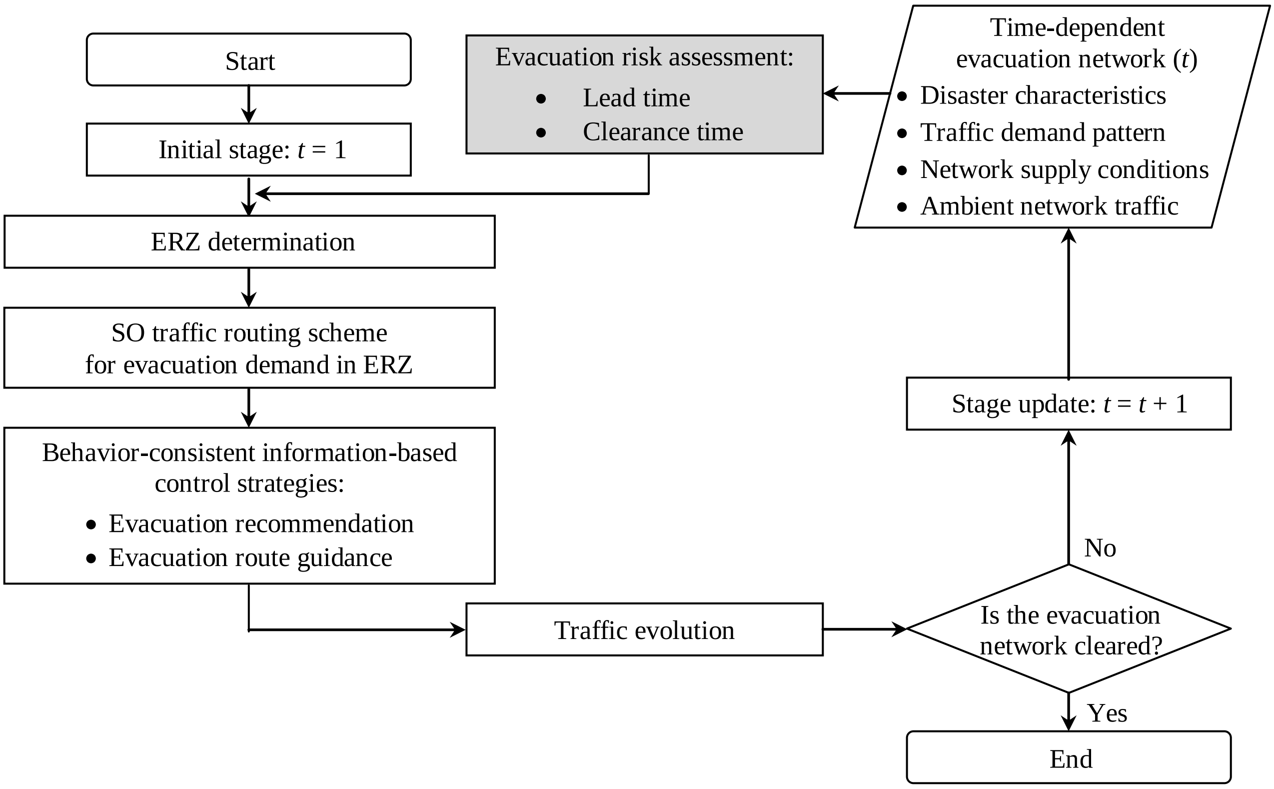


Fig. 1 Stage-based framework for evacuation operations with ERZ deployment paradigm

Fig. 1 conceptually illustrates the stage-based framework for evacuation operations. In it, information-based control strategies are used for managing evacuation traffic. For each stage of the operation, the ERZ is first determined based on the evacuation risk of each location in the disaster-affected region, which factors disaster and traffic dynamics through the lead and clearance times, respectively. Based on the determined ERZ, a system optimal dynamic traffic assignment (SO-DTA) model is applied to derive the optimal routing scheme for the evacuation demand in that ERZ. The SO traffic routing scheme is an idealized benchmark for traffic management as all individuals are assumed to fully comply with the SO routes provided. To capture the behavioral realism of evacuee responses to the information strategies, a behavior-consistent information-based control module is developed to better account for the likely responses of individuals in determining the information strategies. In the control module, evacuee behavior related to the decisions of whether to evacuate and which evacuation route to take under information strategies is performed using the corresponding behavior models developed at an aggregate level (Hsu and Peeta, 2013). Accordingly, the behavior-consistent information strategies, including evacuation recommendation and route guidance, are determined and disseminated to the population in the ERZ. The described process is repeated for each operational stage, with the updated disaster characteristics and network traffic conditions, until the evacuation network is cleared.

1.2 Clearance time estimation

To determine stage-based ERZs in the aforementioned operational framework, there is the need to first infer evacuation risk based on Eq. (1). This represents the focus of this study (as illustrated by the grey-shaded box in Fig. 1). We assume that the lead time of each location for each stage of the evacuation operation can be obtained from meteorological data or plume dispersion models from the associated emergency management agencies. Then, the major challenge for inferring evacuation risk lies in the estimation of clearance times, which represents the focus of the rest of this paper. As introduced in Section 1.1, Church and Cova (2000) use a simplified formula which conceptually expresses clearance time of a location as the ratio of the aggregate demand to the available exit capacity associated with it. While their approach requires negligible computational cost, it lacks realism related to traffic flow representation even in a static context. They also suggest using simulation-based approaches for traffic flow modeling, whereby the clearance times can be more robustly estimated based on the simulated traffic flow pattern. However, due to the substantial computational cost associated with the simulation-based approaches, they may not be feasible for real-time operations.

Based on the discussion heretofore, the approach for clearance time estimation in an operational context involves a trade-off between two major issues: (i) computational efficiency for real-time implementation, and (ii) capability of accounting for the realism of traffic flow dynamics. To factor the traffic flow pattern for evacuation-related problems, traffic assignment approaches have been considered in the literature. The early studies, motivated primarily by evacuation planning, model the problem in a static manner whereby static traffic assignment approaches are used to identify potential bottlenecks in evacuation networks (Sherali et al. 1991; Stern 1989; Hobeika and Kim 1998). These approaches assume that a traffic flow exists on all links of a path instantaneously upon the loading of the flow onto the path and are incapable of capturing the dynamics of network flows. More recent studies have sought to use dynamic traffic assignment (DTA) modeling techniques (Sbayti and Mahmassani 2006; Chiu et al. 2007; Shen et al. 2007; Yao et al. 2009) to capture the traffic flow dynamics in the evacuation context. In these studies, SO-DTA models are developed to derive the routing scheme to minimize system-wide travel time. However, the computational complexity associated with the commonly-used iterative DTA solution approaches represents a key application barrier of these models for real-time evacuation operations.

To address the needs related to clearance time estimation in the real-time context, this study explores another thread of research on evacuation-related problems, which operates in the domain of dynamic network flow problems (DNFPs), especially for building evacuation (Chalmet et al. 1982; Hoppe and Tardos 1994; Baumann and Skutella 2009). The classical DNFPs emphasize sending flows from designated sources to sinks over a given network, characterized by link travel times and link capacities. Herein, the link travel time, defined as the amount of time for a flow to traverse the link, is generally assumed time-invariant and flow-independent. This assumption simplistically represents the mechanisms involved in the movement of a traffic flow across a link, and hence some traffic flow phenomena cannot be fully captured; for example, link congestion and backward wave propagation. However, clearance time estimation is only one of several components of the operational framework proposed in Fig. 1, where the dynamics of the evacuation network are updated along the stages of the operation, and the traffic realism is still addressed in the overall framework. In such a context, DNFPs can potentially aid the development of intermediate methods that straddle between static and dynamic approaches to leverage the trade-offs between traffic realism approximation and computational efficiency for clearance time estimation. We do so by combining a DFNP with the concept of location-priority, as discussed hereafter.

Traffic in an evacuation network under disaster impact is different from that of a normal traffic network in two key characteristics: (i) evacuees mostly move in outbound directions from the disaster-affected locations to safe locations; and (ii) evacuees in the network are subject to different levels of disaster-induced danger, as the locations closer to the disaster will be impacted earlier. To reflect these two characteristics in modeling traffic management for an evacuation network, this study proposes a routing strategy for evacuation operations by applying the concept of location-priority. Location-priority refers to the priority that the population of a location has in using roadway capacity for evacuation. It is defined based on the time-dependent disaster-induced danger associated with a location. We use the lead time for a location as a proxy for the disaster-induced danger, as a shorter lead time implies temporal nearness of danger manifesting through disaster impact. The proposed dynamic routing strategy is implemented by processing the evacuation demand at an origin with higher location-priority (shorter lead time) earlier. In such a routing strategy, the routing of the demand at an origin is subject to the residual capacity resulting from the dynamic traffic flows of earlier-processed demand. In this manner, the dynamic traffic routing problem for an evacuation network can be transformed into a series of single-origin dynamic routing problems. This sequential deployment scheme avoids iterative processes for the solution search, thereby enabling computational efficiency for real-time applications. Additionally, by applying the concept of a super sink, we can further transform the problem into sequential single-origin single-destination dynamic routing problems (*s-t* DRPs). Thereby, in the context of clearance time estimation in this paper, the real-time traffic flow routing focuses on developing a solution method to determine the quickest flow for an *s-t* DRP in the framework of dynamic network flow problems. Most existing solution algorithms for DFNPs employ time-expanded networks to capture flow dynamics in discrete time stages, which can be computationally expensive. By contrast, this study proposes a solution approach which is developed as a continuous-time model without using a time-expanded network whose size can grow exponentially (Baumann and Skutella, 2009). The proposed approach directly searches for the solution over continuous-time functions, and is hence computationally more efficient. It is important to note here that the approach for routing evacuation traffic is for approximately estimating clearance time in this study to infer evacuation risk, and not for determining the SO traffic routing scheme in the stage-based operation framework which is addressed elsewhere.

The remainder of the paper is organized as follows. As the starting point for the proposed clearance time estimation approach, Section 2 introduces DNFPs in the evacuation context and specifies the corresponding problem characteristics in terms of several relevant objectives. Section 3 describes the concept of location-priority in evacuation operations and its implications for dynamic routing problems. Next, to find the quickest flow for an *s-t* DRP, in Section 4, a proposition regarding a property of the quickest flow is proved, and a solution method for the clearance time estimation is developed. This is followed by the complexity analysis of the method. Section 5 presents numerical experiments for illustrating the computational efficiency of the proposed approach. Section 6 provides some concluding comments and insights.

2 Dynamic network flow problems in the context of clearance time estimation

A dynamic network flow problem (DNFP) is typically addressed for a directed network $\mathbf{G} = (\mathbf{N}, \mathbf{A})$, which represents the roadway system used for evacuation in the current problem context. Each link $(i, j) \in \mathbf{A}$ is characterized by a link travel time $\tau_{(i,j)}$ and link capacity $c_{(i,j)}$, where $0 \leq \tau_{(i,j)} < \infty$ and $0 \leq c_{(i,j)} < \infty$. The link travel time defines the time required for an evacuee to traverse the link, which is generally assumed time-invariant and flow-independent in DNFPs. The link capacity limits the number of evacuees who can enter the link in a unit of time. That is, the link flow $f_{(i,j)}$ is constrained as $0 \le f_{(i,j)} \le c_{(i,j)}$. The network contains a set of source nodes $\mathbf{S} = \{s\}$ as the origins with evacuation demand, and a set of sink nodes $\mathbf{T} = \{t\}$ as the safe destinations for evacuation operations; $\mathbf{S} \subseteq \mathbf{N}$ and $\mathbf{T} \subseteq \mathbf{N}$. Each origin $s \in \mathbf{S}$ is associated with a number of evacuees to be evacuated, $v(s) \ge 0$. The task of the DNFP is to send $v(s)$ from $\mathbf{S}$ to $\mathbf{T}$ to achieve certain desired objectives.

It is important to point out two key aspects in the use of time-invariant link travel times in a DNFP for clearance time estimation. First, as illustrated by Fig. 1, the stage-based evacuation operations framework captures the dynamics of traffic flow evolution by updating the evacuation network traffic conditions at the beginning of each stage. Hence, the link travel times are assumed constant in the estimation of the clearance time for only the small time interval reflecting the roll period of a stage, as the stage shifts after the roll period, at which point the network traffic conditions are updated. Second, as discussed earlier, the clearance time estimation is only one component in the proposed stage-based evacuation operations framework. Hence, while the use of constant link travel times for estimating clearance time for the current stage is an approximation, it is not particularly restrictive in the context of the motivation for the use of a DNFP in this study which seeks to trade off computational efficiency and level of approximation in estimating the clearance times.

From the perspective of modeling DNFPs, there are three frequently used objectives which are relevant to evacuation operation problems:

(i) The maximum flow: to send the maximal possible amount of demand from $\mathbf{S}$ to a $\mathbf{T}$ within a given time Q.
(ii) The quickest flow: to send the given amount of demand from $\mathbf{S}$ to $\mathbf{T}$ in the shortest possible time.
(iii) The earliest arrival flow: to send a specific flow from each $\mathbf{S}$ to a $\mathbf{T}$ that maximizes the amount of arrivals at $\mathbf{T}$ by some time θ, for all $\theta \in (0, Q]$.

In the context of evacuation problems, the maximum flow is equivalent to the flow maximizing network throughput, and the quickest flow is equivalent to the flow of the minimum network clearance time. Ford and Fulkerson (1958) construct a DNFP based on the framework of a time-expanded network and propose a maximum flow algorithm. Jarvis and Ratliff (1982) prove that applying the Ford and Fulkerson maximum flow algorithm can determine a dynamic flow which simultaneously satisfies the aforementioned three objectives. Zheng and Chiu (2011) prove that the earliest arrival flow is equivalent to the SO-DTA solution in the framework of cell transmission models (CTMs). It should be noted here that the Ford and Fulkerson algorithm is developed for a single-source single-sink problem (by applying the concept of super source and super sink), and so is the Zheng and Chiu (2011) proof. However, the use of this algorithm for evacuation problems is restrictive as the concept of a single super source may not justifiable for addressing evacuation demand as different locations in the affected region are under different levels of disaster-induced danger.

In an evacuation problem, the population at each origin is required to be evacuated to a destination as soon as possible. It is reasonable to assume that evacuees attempt to reach the closest safe locations to exit the disaster-affected region as soon as possible and then advance to their final destinations later (for example, to home or to meet family members), especially under no-notice evacuations. Since each evacuee is not designated to reach a specific destination, the concept of a super sink can be applied. A super sink is a virtual node connecting all possible destinations with links of zero travel time and infinite capacity. However, as discussed earlier, the notion of a super source is not meaningful for this problem. Hence, the dynamic traffic routing problem for an evacuation operation in the context of clearance time estimation can be discussed in terms of a multiple-origins single-destination problem. The next section uses the concept of location-priority introduced earlier to further reduce the dynamic routing problem for evacuation to one of sequential single-origin single-destination problems.

3 Location-priority in the context of clearance time estimation

As discussed earlier, an evacuation network is characterized by traffic moving in outbound directions from the disaster-affected locations to safe areas. This implies that evacuees compete for the roadway capacities in these outbound directions. From this characteristic of evacuation networks, we note that evacuees at locations closer to the disaster are subject to a higher level of disaster-induced danger, as they may be impacted by the disaster earlier. From the perspective of disaster management, the evacuees who are under a higher level of disaster-induced danger should be removed from their current locations first, so as to avoid the impending disaster impact on them. This motivates an approach in which evacuees are prioritized in using roadway capacity for evacuation based on the lead times associated with their locations. That is, evacuees at a location with a shorter lead time are allotted a higher priority for evacuation. We label the associated routing strategy for evacuation operations as the location-priority routing, where the evacuation demand at the origin with higher priority is to be evacuated first.

Based on the proposed location-priority routing policy, the demand in the evacuation network is processed in the order of the lead times associated with the origins. That is, the routing is conducted origin by origin, starting from the one with the shortest lead time until all evacuees are processed. Thereby, the routing problem for an origin is subject to the residual capacity resulting from the dynamic traffic flows from earlier-processed origins. In conjunction with the concept of super sink introduced in the previous section, the problem is transformed into sequential single-origin single-destination dynamic routing problems or *s*-*t* DRPs. Then, the routing problem can be solved using the procedure shown in Fig. 2.

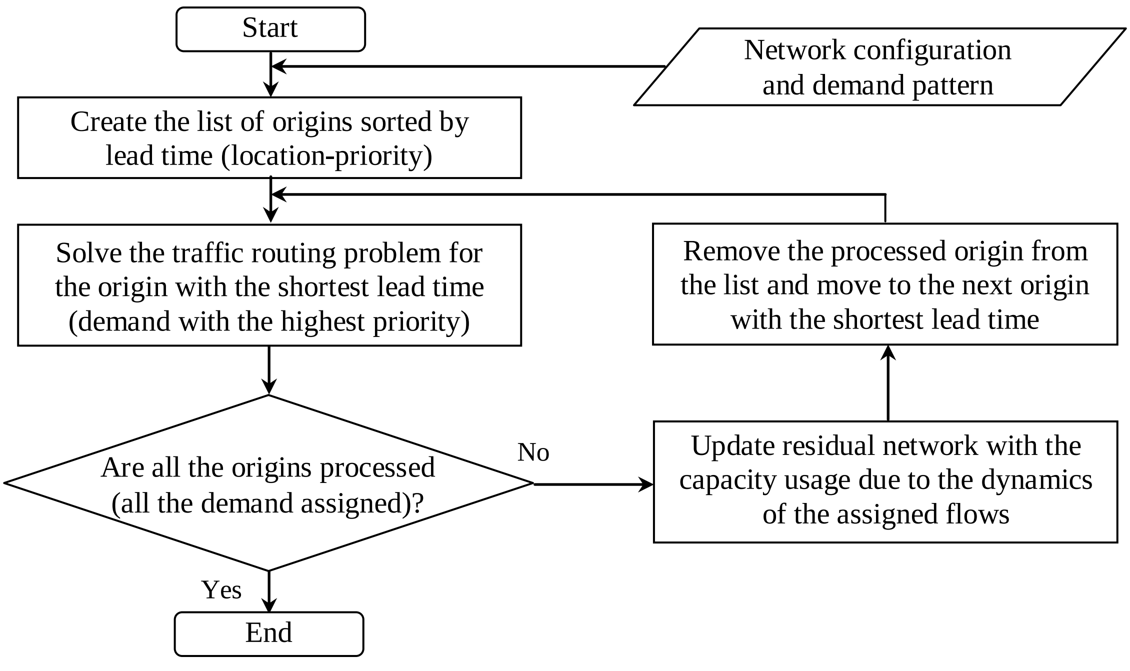


Fig. 2 Procedure to solve the dynamic evacuation routing problem with location-priority for clearance time estimation

4 Finding the quickest flow

As discussed heretofore, the dynamic routing problem for a multiple-origin multiple-destination evacuation network can be decomposed into sequential *s-t* DRPs, and solved in the order defined by the location-priority of the origins. In this section, we develop a solution method to address an *s-t* DRP in the framework of DNFPs. Most existing models for DNFPs are discrete-time models, and use time-expanded networks. A time-expanded network consists of the copies of the nodes of the original network along discrete time steps of the time horizon, so that it can capture the flow dynamics in the network over time. Unfortunately, it also inflates computation time, as the size of the expanded network grows linearly with the number of the discrete time steps, which depends heavily on the demand size and may grow exponentially. Time-expanded networks generally prevent efficient solution algorithms for real-time implementation. In this section, by contrast, we propose a continuous-time model to find the quickest flow. A proposition regarding a property of the quickest flow in an evacuation problem will be proven first, and then the solution method for finding the quickest flow will be developed thereupon.

4.1 Property of the Quickest Flow

In our evacuation problem context, determining the quickest flow is equivalent to minimizing the network clearance time, which is the time that the last evacuee exits the evacuation network to a safe location. Consider the problem of evacuating the demand from a single origin to a single destination (a super sink). To obtain the quickest flow, we model it as a standard DNFP where link travel times are assumed flow-independent and time-invariant. In an evacuation network, each evacuee is routed to an evacuation path to reach a safe destination. The evacuation time for an evacuee is defined as the time he/she has spent in the network, which consists of two parts: (i) path holding time: the time from when the evacuee starts at the origin and is waiting until he/she can enter the assigned path, and (ii) path travel time: the travel time from the origin to reach the destination using the assigned path. Path holding time is regulated by the flow rate entering the path, which is constrained by the capacity of the entrance link associated with the path. Additionally, it is also related to the sequence that an evacuee is loaded onto the path. Path travel time is the sum of link travel times along the path. Let *n* denote the loading sequence of an evacuee. For an evacuee *n* who is assigned to evacuation path *k*, his/her evacuation time is denoted as $(\lambda_{nk} + T_k)$, where λ_{nk} and T_k represent path holding time and path travel time, respectively.

Since the network clearance time is the time that the last evacuee exits the evacuation network, minimizing the network clearance time is equivalent to minimizing the evacuation time for the last evacuee. Accordingly, the objective function to minimize network clearance time can be written as:

$$\text{Min} \quad \underset{n}{\text{Max}}(\lambda_{nk} + T_k) \qquad \forall n, k \tag{2}$$

To solve this problem, we propose a solution utilizing an underlying characteristic of the quickest flow as stated in the following proposition.

Proposition 1 *For the quickest flow in a single-origin single-destination network, the last evacuee on each used path will arrive at the destination at the same time.*

Proof Consider *V* evacuees to be evacuated from an origin to the designated destination. Assume that a set $\mathbf{K}$ of independent paths are identified for evacuation. Based on the capacities of the links on path $k \in \mathbf{K}$, the residual capacity of path *k* is denoted as

$c_k = \underset{(i,j)}{\text{Min}}\, c_{(i,j)}, \forall(i,j) \in k$. Without loss of generality, it is assumed the flow's entrance to a path is constrained by its residual capacity instead of the capacity of the entrance link. That is, the waiting time to enter a link under the residual capacity is included in the path holding time. Thereby, by modeling the flow entering path *k*, v_k, as a continuous flow over time, the time required for the evacuees to enter path *k* can be calculated as v_k / c_k. Since link travel times are assumed time-invariant, T_k can be viewed as a parameter, and the evacuation time for the last evacuee on path *k* can be represented as $\underset{n}{\text{Max}}(\lambda_{nk} + T_k) = (v_k / c_k) + T_k$.

Let F^* be the quickest flow, containing v_k^* evacuees to path $k \in \mathbf{K}$, $V = \sum_{k\in K} v_k^*$. If the last evacuee on each used path does not arrive at the destination at the same time, consider a case that the arrival of the last evacuee on path k^+ is later than the last arrivals on other paths. That is, the evacuation time (*ET*) of the last evacuee on path k^+:

$$ET_{k^+}^* = (v_{k^+}^* / c_{k^+}) + T_{k^+} > (v_k^* / c_k) + T_k = ET_k^* \qquad \forall k \in \mathbf{K} \setminus k^+ \tag{3}$$

Therefore the network clearance time (*NCT*) for this quickest flow $NCT^* = ET_{k^+}^*$. Let $\Delta ET_k = ET_{k^+}^* - ET_k^*$, $\forall k \in \mathbf{K} \setminus k^+$. Since T_k ($\forall k \in \mathbf{K}$) can be viewed as parameters, the flow on each path is the variable determining *NCT* and $V = \sum_{k\in K} v_k^*$. This implies that it is possible to obtain another flow F' by shifting some amount of the evacuees, $\Delta v'$, from path k^+ to another path $k' \in \mathbf{K} \setminus k^+$, so as to reduce $ET_{k^+}^*$ by $\Delta v'/c_{k^+} < \Delta ET_{k'}$, and increase $ET_{k'}^*$ by $\Delta v'/c_{k'} < \Delta ET_{k'}$. Then, the clearance time for F' will be:

$$NCT' = \text{Max}\left\{(ET_{k^+}^* - \Delta v'/c_{k^+}), (ET_{k'}^* + \Delta v'/c_{k'})\right\} < ET_{k^+}^* = NCT^* \tag{4}$$

However, this contradicts that F^* is the quickest flow leading to the minimum network clearance time, which concludes the proof.

While we prove Proposition 1 using the case where the identified paths for evacuation are independent, the proof can be extended to the case where paths are dependent. If the shared links across paths do not represent capacity bottlenecks, then the proof provided above can be directly applied to such cases. If one or more shared links represent capacity bottlenecks, it implies the existence of competition between the associated dependent paths for the capacity on these shared links. In such cases, if the arrival of the last vehicle to the destination for one path is later than that for another path, the v_k / c_k ratio can be adjusted to lower the longer evacuation time, thereby reducing the network clearance time. Hence, the underlying concept to obtain the quickest flow is to balance the flows over the available capacities of the used paths.

4.2 Solution Method

Based on Proposition 1, we develop a method to solve the quickest flow for an *s-t* DRP in a continuous-time framework, instead of modeling the problem over a time-expanded network. The underlying concept is to balance the flow on each used path such that the last evacuee on each path arrives at the destination at the same time. That is, the evacuation time (which includes the path holding time plus path travel time) for the last evacuee on each used path is identical, and equal to the network clearance time indeed. Based on this, the problem seeks to minimize the network clearance time by leveraging flows over the network configuration (topology, link capacities, and link travel times). To send *V* evacuees from the origin using the identified path set $\mathbf{K}=\{k\}$ to the destination in the quickest flow is equivalent to minimizing the evacuation time of the last evacuee. We can represent the problem as:

$$\text{Min} \quad NCT = \underset{n}{\text{Max}}(\lambda_{nk} + T_k) \qquad \forall n,k \tag{5}$$

subject to:

$$NCT = ET_{k'} = ET_{k''}$$

$$\Rightarrow \lambda_{k'} + T_{k'} = \lambda_{k''} + T_{k''} \qquad \forall k', k'' \in \mathbf{K},\ k' \neq k'' \tag{6}$$

$$V = \sum\nolimits_{k\in\mathbf{K}} v_k \tag{7}$$

where λ_k is path holding time for the last evacuee assigned to path *k*. Eq. (6) specifies the condition of Proposition 1, and Eq. (7) indicates that all evacuees have to be routed. In this study, we assume that *V*, the total number of evacuees at the associated origin to be evacuated, is obtained from pre-disaster planning processes or related demand estimation models.

As the problem focuses on the arrival of the last evacuee on each used path to his/her destination, λ_k can be interpreted as the time that the last evacuee enters path *k* (path entrance time), or the time required to load v_k evacuees onto the path. From another perspective, if the capacity of the path is fully used, the number of evacuees loaded to path *k* is equivalent to $\lambda_k c_k$, assuming c_k is constant. Consider the case that c_k is time-variant, the computation can be further generalized as $\int_0^{\lambda_k} c_k(\theta)d\theta$, where $c_k(\theta)$ now is a function of time, denoting the time-dependent residual capacity of the path. To satisfy Eq. (6), consider the flows on two used paths k' and k''. Assume $T_{k'} - T_{k''} = \Delta T > 0$. According to the constraint, $\lambda_{k'} + T_{k'} = \lambda_{k''} + T_{k''}$; we then obtain $\lambda_{k''} - \lambda_{k'} = \Delta T$. If we assume $\lambda_{k'}$ to be a variable, the number of evacuees assigned to paths k' and k'' can be represented as $\int_0^{\lambda_{k'}} c_{k'}(\theta)d\theta$ and $\int_0^{\lambda_{k'}+\Delta T} c_{k''}(\theta)d\theta$, respectively. By continuing the process by coupling path k' with other paths, we can represent the number of evacuees distributed to each path in the same manner. Summing up the evacuees distributed to all identified paths, we can re-write Eq.(7) as:

$$V = \sum\nolimits_{k\in\mathbf{K}} \left(\int_0^{\lambda_k} c_k(\theta)d\theta \right) \tag{8}$$

In this case, λ_k can be substituted by $\lambda_{k'} + (T_{k'} - T_k)$, $\forall k \in \mathbf{K}$. By solving Eq. (8), we can further determine λ_k, $\forall k \in \mathbf{K} \setminus k'$, and therefore obtain the routing scheme for the quickest flow. This represents the solution method for formulation (5)-(7). As stated previously, λ_k can be interpreted as both the path holding time for the last evacuee on path *k* and the time required to load all the associated evacuees onto path *k*. The equivalence of these two interpretations in the mathematical representation is one of key aspects allowing this solution method for the proposed optimization problem, which can be implemented in a continuous-time context.

To employ the proposed solution method, one important procedure is to identify the capacity functions of the paths in **K**. For the case that the paths in **K** are independent, the identification of c_k or $c_k(\theta)$ is straightforward, as previously discussed: $c_k = \underset{(i,j)\in k}{\text{Min}}\, c_{(i,j)}$, or $c_k(\theta) = \underset{(i,j)\in k}{\text{Min}}\, c_{(i,j)}(\theta)$, $\theta \in [0,\infty)$. On the other hand, if some paths share common links, it raises the issue of how to allocate the available capacities of the common links to these paths.

Here, we use the strategy that the path with shorter path travel time is prioritized in capacity usage, which is similar to the concept of the shortest path augmenting algorithm used to find the maximum flow in a static network (Ahuja et al. 1993).

4.2.1 Illustration of the solution method

A simple example is provided here to illustrate the implementation of the proposed solution method to find the quickest flow in the shared links case. Consider a network with two paths that share a common link, as shown in Fig. 3. The problem seeks to send the quickest flow of *V* evacuees from origin *s* to destination *t* using the two paths, which overlap on link 3. Each link is characterized with its initial capacity *c* (evacuees/minute) and travel time τ (minutes). We denote the path using link 1 as path A and that using link 2 as path B. As path B has a shorter path travel time (20 minutes), the evacuees are first distributed onto this path and consume the capacity up to the residual capacity of the path, $\mathrm{Min}\{c_2, c_3\} = 40$. The function of residual capacity for path B is $c_{\mathrm{B}}(\theta) = 40$, $\theta \in [0, \infty)$. Then, for path A, because 40 (evacuees/minute) of the capacity of link 3 is occupied from the 10th minute (the arrival of the first evacuee from path B), the residual capacity of link 3 is 20 for path A from thereon. The function of residual capacity for path A can be written as:

$$c_{\mathrm{A}}(\theta) = \begin{cases} 30 & 0 \le \theta < 10 \\ 20 & 10 \le \theta < \infty \end{cases} \tag{9}$$

However, as the first evacuee cannot arrive at node *j* by the 15th minute via link 1, the evacuees routed to path A at time θ are limited from entering link 3 by its residual capacity from time $\theta + 15$. Therefore, the function of the effective residual capacity for the flow on path A should be shifted by 15 minutes to the right and re-written as $c_{\mathrm{A}}(\theta) = 20$, $\theta \in [0, \infty)$, as illustrated in Fig. 4. Set λ as the time to load the last evacuee onto path A (or the entrance time for the last evacuee on path A). The time for loading evacuees onto path B is λ plus the difference of path travel time, $\lambda + 5$, so as to satisfy the condition of Proposition 1. In Fig. 4, the grey area implies the number of evacuees distributed onto each path. To obtain the routing scheme for the quickest flow, by applying Eq. (8), we can obtain a linear equation with one variable λ.

$$V = \int_0^{\lambda+5} c_{\mathrm{B}}(\theta) d\theta + \int_0^{\lambda} c_{\mathrm{A}}(\theta) d\theta = 40(\lambda + 5) + 20\lambda \tag{10}$$

As *V* is known, λ can be solved for to determine the number of evacuees distributed to each path.

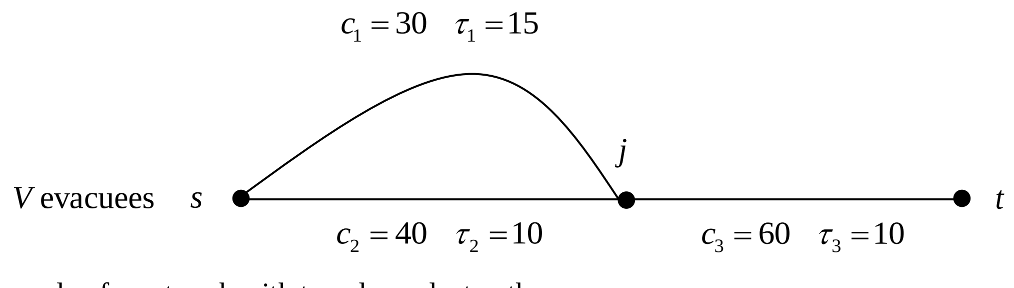


Fig. 3 Example of a network with two dependent paths

From this example, it can be observed that to determine the effective residual capacity of a path we need to consider the flow dynamics on the associated links, which can be designated by the movement of the first evacuee. The movement of evacuees on a path can be traced by the elapsed time in contrast to the travel times on the links along the path. In addition, after the routing scheme is determined, the temporal occupation/usage of link capacities along a path by the associated flows can be mapped. For example, given v_k evacuees routed to path *k* at the flow rate up to the residual capacity of the path, c_k, if the first evacuee arrives at link $(i, j) \in k$ at time θ_k^i, then the capacity to enter the link will be occupied by the flow c_k for a duration from θ_k^i to $\theta_k^i + v_k / c_k$. Assume the original capacity of link (*i*, *j*) is c_0 for $\theta \in [0, \infty)$, then the capacity of the link in the residual network for the following routing problems will be updated as a step function:

$$c_{(i,j)}(\theta) = \begin{cases} c_0 - c_k & \theta_k^i \le \theta < \theta_k^i + v_k / c_k \\ c_0 & \text{otherwise} \end{cases} \qquad \theta \in [0, \infty) \tag{11}$$

$c_0 - c_k \ge 0$ as c_k is defined as the residual capacity of the path *k* in the original network, and the function is in the form of a step function. Further, given $\tau_{(i,j)}$, the first evacuee will arrive at the next link at $\theta_k^i + \tau_{(i,j)}$. In this manner, we can capture the flow dynamics in terms of the capacity usage along the path, and update the residual network for the *s*-*t* DRPs for the origins with lower location-priority.

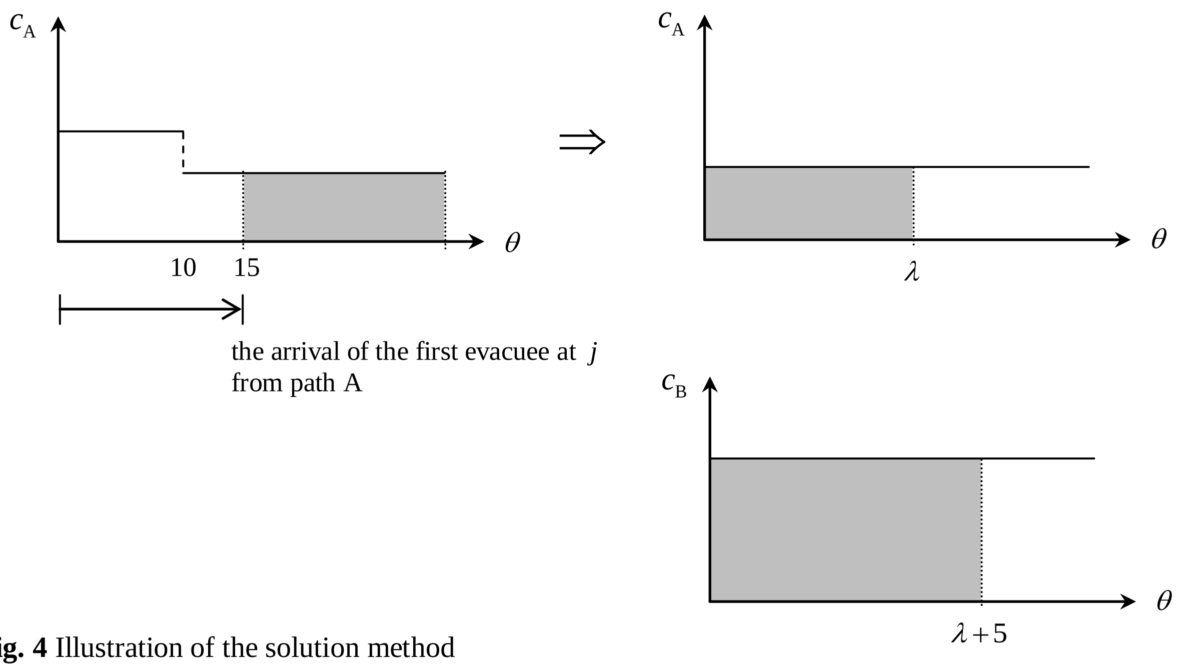


Fig. 4 Illustration of the solution method

The solution method was developed to solve one *s*-*t* DRP (one origin). To address the clearance time estimation for each origin within the disaster-affected region, the procedure introduced in Section 3 is applied to sequentially process demand assignment for each origin by solving the associated *s*-*t* DRP based on location-priority. Fig. 5 illustrates the flowchart for implementing the procedure, which is an expansion of Fig. 2 to include the details of the developed solution method. It is assumed that the lead time for each origin is obtained from the meteorological or disaster management agency, and therefore the location-priority of each origin is known. Starting from the origin with the highest location-priority, we identify the evacuation paths to safe locations for this origin and the associated path capacity functions and then solve the *s*-*t* DRP for this origin using the developed method for finding the quickest flow. Once the quickest flow for an origin is computed, the residual capacity of the network is updated according to the dynamics of the capacity usage by this flow. It should be noted that the capacity function of a link needs to be updated only if the link belongs to any evacuation paths for the derived quickest flow. In the residual network update component in Fig. 5,

$d_k(\theta)$ denotes the capacity usage of the quickest flow on path k along time; $\theta^i_{k(f)}$ and $\theta^i_{k(l)}$ are the arrival times of the first and last vehicles, respectively, at link (i, j) in the quickest flow on path k. That is, the capacity of the link is only consumed during the period between the arrival of the first and last vehicles of the associated flow. This procedure is repeated for the next origin with the highest location-priority, and the path capacity functions of the associated evacuation path for this origin are identified from the updated residual network. The procedure terminates when all the origins in the disaster-affected region are processed. At this point, the approximate clearance times for all origins are known.

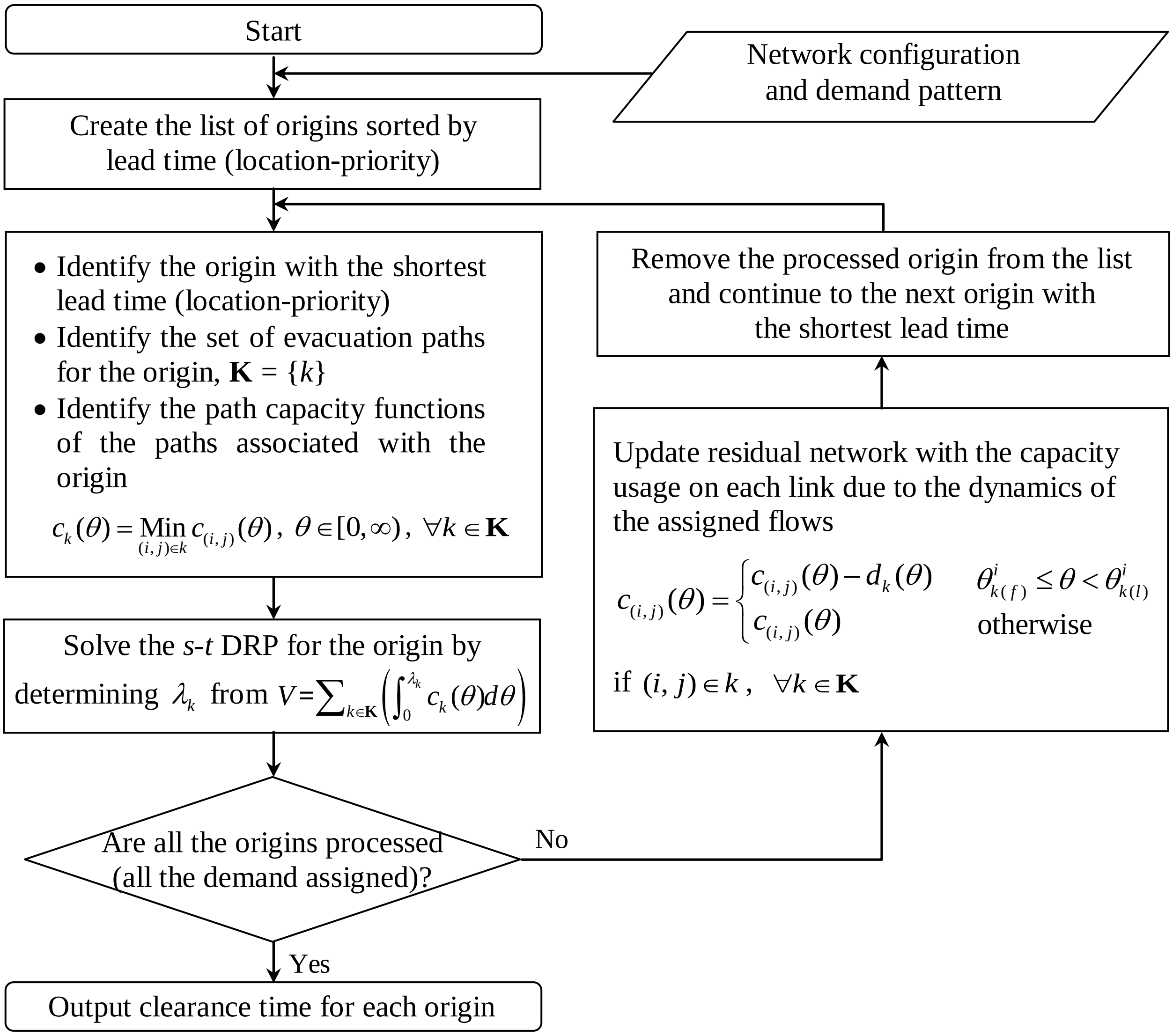


Fig. 5 Procedure for sequential *s-t* DRPs with location-priority for clearance time estimation

4.2.2 Path capacity functions

As link capacity functions have a step function form, the residual capacity function of a path, derived from the capacity functions of the links along the path, is also a step function. In this context, to solve Eq. (8), due to the property of integration along a piecewise function, the solution may not be as straightforward as solving the linear equation (Eq. (10)) in the presented example. Consider the case with two identified paths, k_1 and k_2, where both have two steps in their residual capacity functions (as shown in Fig. 6). Assume k_1 has shorter path travel time, and λ_{k_1} is the problem variable. To determine λ_{k_1}, we need to first identify the location of λ_{k_1} along θ from the sections $[0,\theta_1)$, $[\theta_1,\theta_2)$ and $[\theta_2,\infty)$. Then, we can construct the linear equation to determine its value. That is, in the worst case, we need to check three times to first roughly identify the location of λ_{k_1}. It can be further deduced that if the total number of steps in $c_k(\theta)$ $\forall k \in \mathbf{K}$ is Λ, the problem is solved using at most $(\Lambda - |\mathbf{K}| + 1)$ checks.

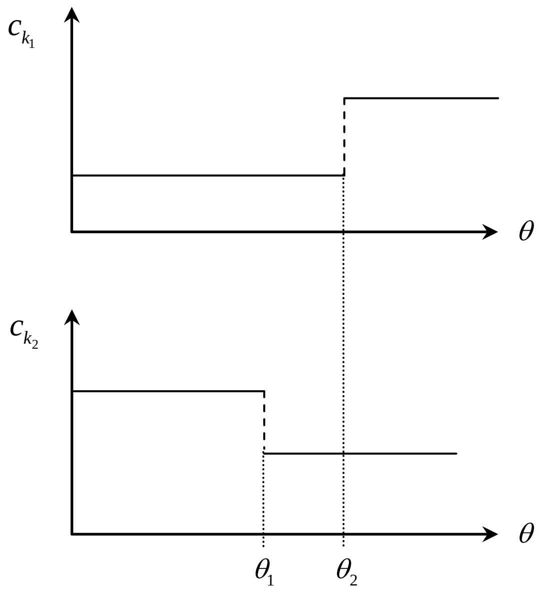
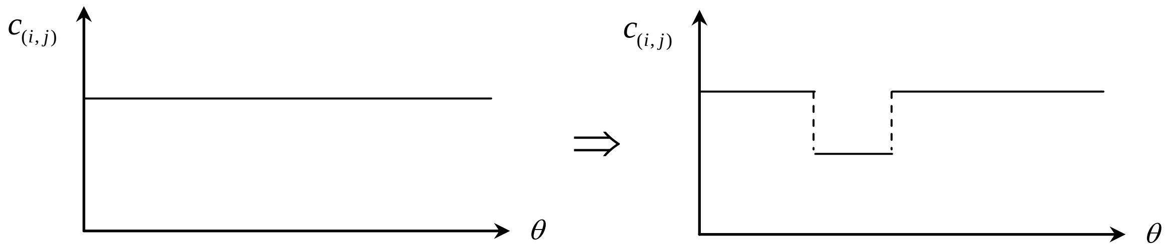
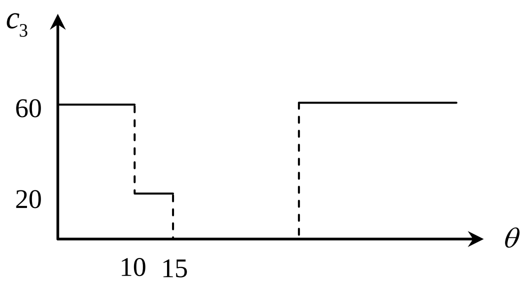


Fig. 6 Illustration of determining λ for step functions



(a) Steps created due to a flow traversing a link



(b) Steps created due to multiple flows from the same origin traversing a link

Fig. 7 Formation of “steps” in the residual capacity function of a link

“Steps” in residual capacity functions result from updating the residual network with capacity consumption by the previously-processed demand. Fig. 7(a) illustrates that given the initial capacity function of link (i, j), sending a flow though the link will turn its capacity function into a three-step piecewise function. That is, the number of steps increases from 1 to 3. Hence, every p flows sent through a link may add at most $2p$ steps to the original step function of the link. For the example illustrated in Fig. 3, if V is larger than c_{B}, the optimal routing scheme for V will result in the residual capacity function for link 3 as shown in Fig. 7(b). One more step is created due to the travel time difference between link 1 and link 2. Thereby, it can be deduced that every p flows from the same origin traversing a link will add (p+1) steps to the original capacity function of the link. This aspect is relevant to the complexity analysis of the proposed solution method, discussed hereafter.

4.3 Complexity Analysis

Introducing the location-priority concept to solve the DRP for an evacuation network is equivalent to solving $|\mathbf{S}|$ single-origin single-destination dynamic routing problems (s-t DRPs), where $\mathbf{S}$ is the set of origins. It includes the time for solving $|\mathbf{S}|$ s-t DRPs and ($|\mathbf{S}|$–1) times of updating the residual network. Since the update of link capacities can be performed in parallel for the relevant links, the major computational cost is attributed to the DRPs. As most current solution algorithms are developed for a single s-t DRP, in this section, we focus on the complexity analysis of our proposed method in the context of solving one s-t DRP for origin $s \in \mathbf{S}$. Assume s also denotes the sequence number ordered according to the location-priority, where a higher value of s indicates lower location-priority. That is, origin 1 has the highest location-priority. To solve the s-t DRP for origin s using the proposed solution method involves two major procedures, and the complexity of the problem is analyzed with respect to these two procedures hereafter.

In procedure 1, we need to first identify the shortest paths to be used for sending the quickest flow and their path capacity functions. Let $\mathbf{K}_s$ be the set of identified paths to be used for routing the evacuation demand at origin s. From a practical perspective, the number of paths in $\mathbf{K}_s$ should be within a reasonable range that can be managed by the disaster response operators given their resource constraints. Here, we do not specify the number of paths in $\mathbf{K}_s$, but suggest that it may be obtained in the following manner: We first identify the shortest path with path travel time τ^*, and then repeat the process of searching for the next shortest path until the path travel of the identified path exceeds $\alpha\tau^*$, where α is a positive problem parameter to be determined by the disaster response operators. This criterion can also ensure the set of the identified shortest paths is acceptable to the evacuees as well. For a network $\mathbf{G}$ with n nodes and m links, identifying the shortest path for an origin-destination (O-D) pair is of the order $O(nm)$ (Ahuja et al. 1993). When the shortest path is identified, we can simultaneously retrieve the residual capacity function of each involved link. There may be the flows from the previously processed (s–1) origins traversing through these links. If only one flow exists from each previously processed origin, there will be at most [1+2(s–1)] steps in their residual capacity functions. For the case with multiple flows from the same origin passing through a link, the total number of steps may exceed this number. However, it is not common to observe a situation in a single-origin routing problem that a large number of paths diverge at the origin but share common links at some downstream segments. Also, one more such flow just adds one more step to the step function. Without loss of generality in the complexity analysis, we do not further include such situations here. Then, to further obtain the capacity function of the identified path, we need to find the minimum capacity from the m links of the path along [1+2(s–1)] steps in the link capacity function, which requires $O(m(2s–1))$ time. Hence, to obtain the residual capacity function of one shortest path to be used leads to the complexity $O(nm+m(2s–1))$ = $O(m(n+s))$. Collectively, procedure 1 runs in $O(|\mathbf{K}_s|m(n+s))$.

Collecting the information obtained from procedure 1, in procedure 2, we employ the proposed the solution method to find the quickest flow. As discussed in the previous section, the complexity of the problem depends on the total number of "steps" in the residual capacity functions of all identified paths, Λ. $\Lambda = |\mathbf{K}_s|(2s–1)$ based on the discussion pertaining to procedure 1. Then, it needs at most $[|\mathbf{K}_s|(2s–1)–|\mathbf{K}_s|+1]$ checks to identify the location of λ

along the steps in the residual capacity functions. That is, this procedure runs in $O(2|\mathbf{K}_s|(s–1))$ = $O(|\mathbf{K}_s|s)$. Combining the two procedures, the overall complexity of the problem is $O(|\mathbf{K}_s|m(n+s)+|\mathbf{K}_s|s) = O(|\mathbf{K}_s|m(n+s))$.

To the best of our knowledge, the current fastest algorithm for an *s*-*t* DRP is proposed by Burkard et al. (Burkard et al. 1993). It is a strongly polynomial algorithm, running in the time complexity of $O(m^2 \log^3 n(m + n\log n))$. Our method may be preferable in the sense that the number of links m is normally larger than the number of nodes n in most traffic networks. Due to the difference in the problem contexts, the values of $|\mathbf{K}_s|$ and s are involved in representing the complexity of the proposed solution method. As previously defined that a larger value of s indicates a lower location-priority, it may also imply that the spatial location of the origin is further away from the disaster but closer to the safe destinations. As the distance for an O-D pair becomes smaller, there is more likely a simple network topology between the origin and the destination, and thereby a smaller number of shortest paths exist. By contrast, if the value of s is smaller, it may imply that the origin is further away from the safe destinations, and a larger number of shortest paths can be identified for routing. In such situations, the number of potential paths may be exponential in the size of the network. However, in the implementation of evacuation operations, it may be more practical for network operators to identify a set of evacuation paths which use major arterials and avoid complicated routing. In this manner, the value of $|\mathbf{K}_s|$ may largely depend on the practical aspects of the disaster response, but is generally limited to within a reasonable range (much smaller than m). Overall, the value of $|\mathbf{K}_s|$ can be managed to be at a scalable level in practice, and therefore the solution computation can be addressed efficiently.

5 Numerical experiments

Numerical experiments are conducted using the Borman Expressway Network in northwest Indiana, which has 197 nodes and 460 links as shown in Fig. 8. It also illustrates the location of the disaster occurrence. Evacuation demand is specified for 40 origins, where the associated location-priorities are determined based on the distances between these origins and the location of disaster occurrence. $\alpha = 1.5$ is used in the numerical experiments.

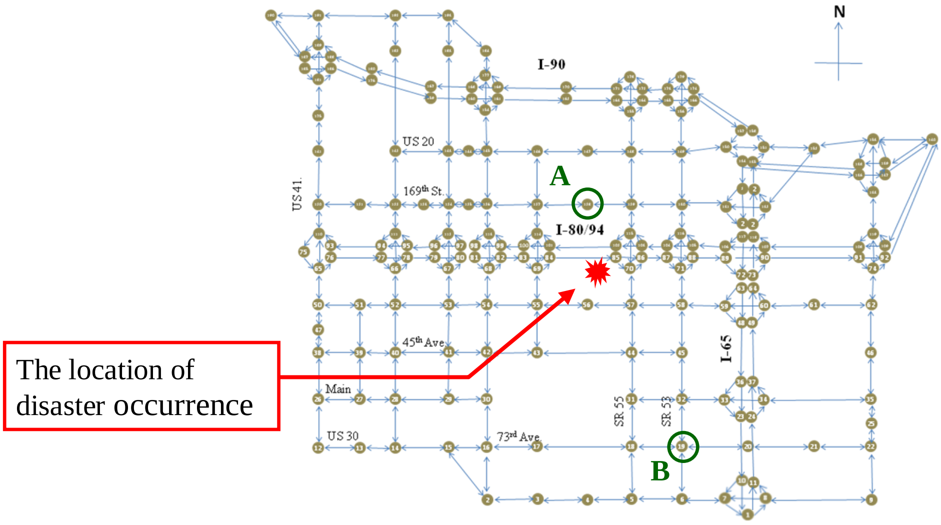


Fig. 8 Study network for numerical experiments (the Borman Expressway Network)

The DRPs of each scenario are solved by using the proposed solution method for finding the quickest flows. Additionally, for comparative purposes, the SO-DTA solutions are computed for the same scenarios using a mesoscopic traffic assignment-simulator, DYNASMART-P. DYNASMART-P uses a modified Greenshields model to describe traffic propagation across a link (Jayakrishnan et al., 1994) and therefore is capable of more accurately accounting for traffic realism and flow interactions in the evacuation network, in contrast to the assumption of flow-independent link travel time used in DNFPs. Also, the objectives of the SO-DTA problem and the proposed location-priority based problem are inherently different. The SO-DTA problem aims to determine the flow pattern minimizing overall network travel time using iterative search processes, which can incur substantial computational costs. The SO-DTA solution benchmarks the best case scenario, assuming that evacuees have full knowledge of the network conditions and fully comply with the routes suggested (which may not be behaviorally realistic as some SO paths can be significantly longer). By contrast, the proposed problem seeks the quickest flow for each origin, which is solved sequentially according to an order defined based on the lead times of the origins. This sequential problem structure circumvents the iterative search processes, which fundamentally differentiates the objectives of these two problems.

5.1 Experimental Results

The computational costs of solving each experimental scenario using these two approaches are summarized in Table 1. The clearance time estimates based on the traffic patterns derived from the associated solutions are also compared. The average estimated clearance times across all origins differ by 13–24%, and can be explained based on differences between the two approaches in terms of the objectives and the assumption of flow-independent link travel times used in the DNFP. Hence, under higher demand levels, which imply higher levels of link congestion, the difference in the estimated clearance times increases. As articulated above, this difference does not necessarily imply error in estimation, as the two approaches have different objectives. The SO-DTA can be viewed as a benchmark for the best traffic flow pattern, but does not, by definition, reflect behavioral realism. Nevertheless, these results assure that the clearance time estimates using the proposed approach are reasonable for the purpose of inferring evacuation risk. The computational cost of using the simulation-based approach can preclude its implementation in the operational context. By contrast, the proposed approach has a computational cost of less than 1% of that of the simulation-based approach. Thereby, the proposed approach for clearance time estimation is significantly more efficient in the real-time operational context.

Table 1 Computational results of the experiments for the Borman Expressway Network

Demand level	Computational cost		Average difference in clearance time estimation
	The quickest flow by the proposed approach	SO-DTA by DYNASMART-P	
25,000	2.50 sec	46 min 25 sec	13.27 %
50,000	2.55 sec	70 min 51 sec	15.76 %
75,000	2.52 sec	83 min 29 sec	16.85 %
100,000	2.46 sec	145 min 04 sec	19.10 %
125,000	2.62 sec	194 min 58 sec	23.48 %

As discussed in Section 1, Church and Cova's (2000) approach for estimating clearance time does not account for evacuation traffic flows even in the static context. By contrast, the approach proposed in this paper factors traffic flow dynamics in an approximate manner, while its computational efficiency, as illustrated in Table 1, enables its application for real-time operations. Further, while there is an estimation difference compared to the simulation-based SO-DTA approach, it can be partly explained by the assumption of constant link flow travel time in the proposed approach. However, since the proposed evacuation operation is implemented in a stage-based framework (Fig. 1), the network traffic conditions are updated across the stages of the operation. Hence, the realism of traffic flow dynamics is still addressed to a certain level. In addition, it should be emphasized that clearance time is only an input to one component of the proposed operational framework, used for inferring evacuation risk. Hence, computationally intensive approaches to estimate it may not represent meaningful tradeoffs given the broader objective of the operational framework in Fig.1. Instead, a reasonable approximation which can account for the realism of traffic flow dynamics at an adequate level and an approach which is computationally viable for real-time evacuation operations may be desirable.

5.2 Properties of the Proposed Approach Next, the properties of the proposed approach are investigated. Instead of deriving path set $\mathbf{K}_s$ for each origin using parameter α, experiments are conducted for scenarios where specific numbers of evacuation paths in $\mathbf{K}_s$ are pre-defined for both approaches. The computational cost of the proposed approach is still only a small fraction (under 10 seconds) of the cost under the simulation-based approach. Fig. 9 illustrates the average difference in clearance time estimates between the proposed and the simulation-based approaches. This difference ranges from 13% to 24% and increases with the demand level, which is similar to the results in Table 1. The number of evacuation paths in $\mathbf{K}_s$ does not perceptibly affect the clearance time estimate difference between the two approaches. Fig. 9 also indicates that the use of more evacuation paths may result in slightly smaller estimation differences. While there is no direct mapping between the number of evacuation paths and the value of α, adopting a higher value of α is more likely to generate more evacuation paths for assignment. However, if a larger number of evacuation paths are specified, some of them may entail significantly longer travel times and may not be meaningful alternatives in the real world.

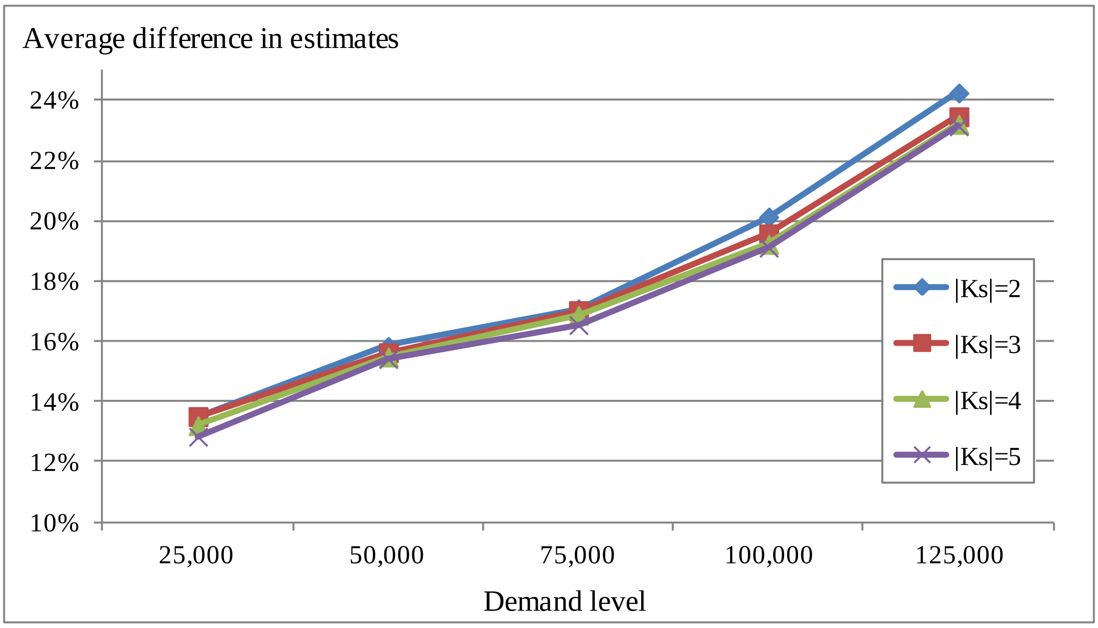
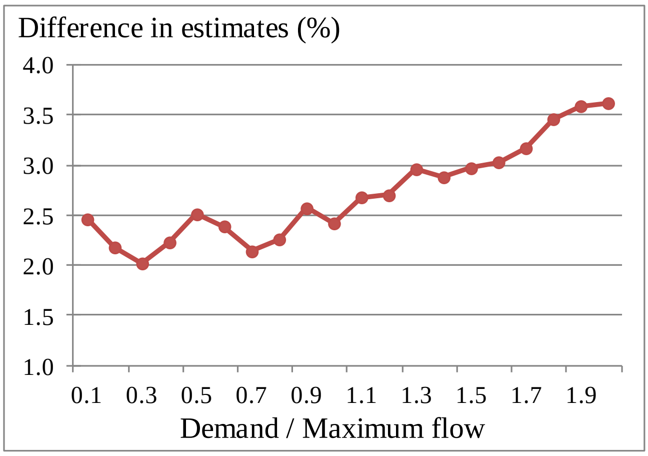
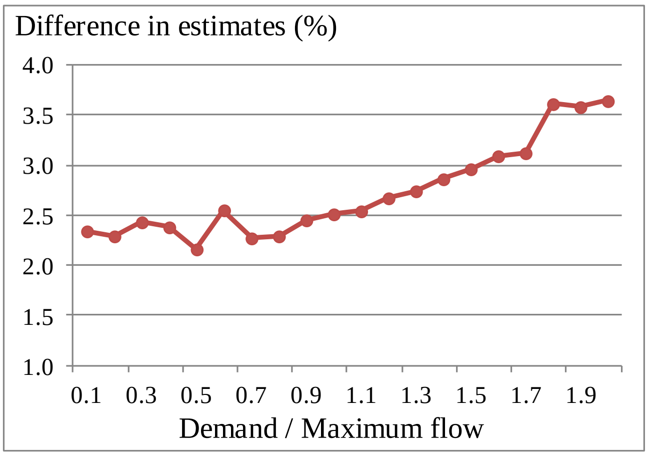


Fig. 9 Scenarios with different numbers of evacuation paths specified

The final set of experiments seeks to exclude the effect of location-priority and focus on the analysis of the DNFP and the proposed approach to find the quickest flow. To do so, the proposed and the simulation-based approaches are implemented for only one origin. With the concept of the super sink applied, the problem to evacuate the population from one origin to safe areas can be viewed as an *s-t* DRP in the DNFP context, and a single-origin single-destination SO-DTA problem in the simulation-based approach. Two origins are selected for these experiments, denoted as Nodes A and B in Fig. 8. Node A is further from the safe areas and Node B is closer to them. In these experiments, four evacuation paths each are identified in both approaches for evacuating the demand at Nodes A and B to the destinations associated with these paths, which are linked to a super sink. Next, the maximum flow for an origin in the DNFP context is determined as the maximum flow rate that can be achieved to send the demand of that origin to the associated super sink without exceeding the capacity of any link in the used evacuation paths (Ahuja et al., 1993). Fig. 10 illustrates the results of the difference in clearance time estimates between the two approaches as demand for an origin is increased, represented in terms of the ratio between the demand and the maximum flow for that origin in the figure. The estimation difference between the proposed and the simulation-based approaches, primarily due to the assumption of flow-independent link travel time in the proposed approach, is under 3% when the ratio of demand to the maximum flow is smaller than 1.5. Even when the demand is twice that of the maximum flow, the estimation difference is only about 3.6%. It may reflect that the SO-DTA solution seeks to minimize total system travel time and therefore may hold traffic at the origin to avoid severe congestion over the used evacuation paths when the total demand exceeds the available capacity of the four used evacuation paths. Hence, the difference in the estimated clearance times does not increase significantly with the ratio of the demand to the maximum flow. Further, by comparing Fig. 10(a) and Fig. 10(b), the distances from the origins to safe locations do not significantly affect the difference between the estimates using the two approaches. This suggests that the estimation difference does not accumulate across links when a single origin (or O-D pair) is considered.



(a) Results for Node A



(b) Results for Node B

Fig. 10 Scenarios considering one origin

6 Summary and concluding comments

This paper develops an approach for estimating network clearance time in the context of incorporating evacuation risk into an operational evacuation framework. The proposed approach is computationally efficient while being capable of adequately accounting for the realism related to representing traffic flow dynamics. To do so, it uses the framework of dynamic network flow problems while leveraging the directional characteristics of the evacuating traffic and factoring disaster–induced danger using the lead times of locations as surrogates.

The study makes two major contributions. First, it proposes the concept of location-priority which reflects a disaster management aspect that evacuees who are closer to the disaster are subject to a higher level of disaster-induced danger, as they may be impacted by the disaster earlier. Hence, location-priority, determined based on the lead times of locations, suggests the routing policy that evacuees with higher location-priority should be prioritized for using roadway capacity. This concept, combined with the notion of a super sink, is used to transform the routing problem for an evacuation network into a series of single-origin single-destination dynamic routing problems (*s*-*t* DRPs). Thereby, the location-priority routing strategy not only accounts for the disaster management perspective but also circumvents the iterative search process to determine the optimal flow conditions, thereby enabling computational efficiency for real-time applications. While this strategy is used in the framework of DNFPs in this paper to estimate the network clearance time, it can generally be applied to modeling frameworks (for example, simulation-based models) in the broader evacuation operations context itself.

Second, the study proposes a solution method to determine the quickest flow in an *s*-*t* DRP in the framework of DNFPs. In contrast to most existing solution algorithms for DNFPs which use time-expanded networks, the proposed method is developed as a continuous-time model and is more computationally efficient. The complexity analysis indicates that the proposed method solves the quickest flow in an *s*-*t* DRP with the complexity of $O(|\mathbf{K}_s| m(n+s))$, which is preferable to the current fastest method having the complexity of $O(m^2 \log^3 n(m + n \log n))$. In addition, the method is synergistic with computational tractability in estimating clearance times for real-time evacuation operations. The numerical experiments using a real-size network illustrate that the computational cost of the proposed method scales efficiently for real-time operations.

The current study focuses on clearance time estimation related to evacuation risk assessment under disasters for evacuation operations. Next, we will combine these estimated clearance times with the disaster lead times to determine the dynamic evacuation risk levels for the population in the disaster-affected region. As illustrated in Fig. 1, then the effect of evacuation risk will be incorporated into an operational framework that factors evacuee behavior in the determination of the evacuation advisories and associated route recommendations.

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