Dynamic and disequilibrium analysis of interdependent infrastructure systems

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ABSTRACT

There is a growing awareness in recent years that the interdependencies among the civil infrastructure systems have significant economic, security and engineering implications that may influence their resiliency, efficiency and effectiveness. To capture the various types of infrastructure interdependencies and incorporate them into decision-making processes in various application domains, Zhang and Peeta (2011) propose a generalized modeling framework that combines a multilayer infrastructure network (MIN) concept and a market-based economic approach using computable general equilibrium (CGE) theory and its spatial extension (SCGE) to formulate a static equilibrium infrastructure interdependencies problem. This paper extends the framework to address the dynamic and disequilibrium aspects of the infrastructure interdependencies problems. It briefly reviews the static model, and proposes an alternative formulation for it using the variational inequality (VI) technique. Based on this equivalent VI formulation, a within-period equilibrium-tending dynamic model is proposed to illustrate how these systems evolve towards an equilibrium state within a short duration after a perturbation. To address a longer time scale, a multi-period dynamic model is proposed. This model explicitly considers the evolution of infrastructure interdependencies over time and the temporal interactions among the various systems through dynamic parameters that link the different time periods. Using this model, numerical experiments are conducted for a special case with a single region to analyze the sensitivity of the model to the various parameters, and demonstrate the ability of the modeling framework to formulate and solve practical problems such as cascading failures, disaster recovery, and budget allocation in a dynamic setting.

Keywords: Infrastructure systems interdependencies, computable general equilibrium model, dynamic analysis, variational inequality, generalized transportation network, decision-making

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1. Introduction

The potential economic, security, and engineering implications of the interdependencies among infrastructure systems (such as transportation, telecommunication, energy, power and water systems) have been highlighted through several recent extreme events such as the 9/11 terror attacks, the 2003 Northeast power blackout, and the 2005 hurricanes in the United States. The terror attacks and the hurricanes demonstrate that a significant event can impact several systems simultaneously, and the linkages among these systems may further amplify the magnitude of the subsequent disruptions. The power blackout event represents a case where the disruption or failure in one system can propagate to other systems through cascading effects due to their functional linkages. In these examples, the infrastructure interdependencies manifest in a negative manner within a short period of time, while their long-term impacts could last for months or years. Infrastructure interdependencies also exist in the "business-as-usual" scenarios. They can influence the effectiveness and efficiency of the infrastructure systems, especially in the context of growing urbanization and the need for the renewal of aging infrastructure. The inefficiency in one system can build up over time, and may eventually propagate to other systems, as demonstrated by the California electricity crisis of 2000-2001. These examples suggest that a dynamic perspective of infrastructure interdependencies is necessary to analyze the performance and robustness of the individual systems in the real-world context.

Despite the importance of this issue, the dynamic aspects of the infrastructure interdependencies have not been sufficiently addressed in the literature, partly because the domain of infrastructure interdependency analysis is itself relatively new (Zhang and Peeta, 2011; Ouyang, 2014). Among the few theoretical approaches (such as Haimes and Jiang, 2001; Nagurney and Dong, 2002; Duenas-Osorio et al., 2004; Rose and Liao, 2005; and Zhang and Peeta, 2011) currently available for the infrastructure interdependencies analysis, most are static in the sense that: (i) a single time period is considered, precluding the capture of the temporal interactions among the infrastructure systems over time; (ii) only the final equilibrium state of the individual systems is modeled, while the process of how the systems adjust from the initial and intermediate states to the equilibrium state is ignored; and (iii) the model parameters are considered as fixed throughout the modeling period.

Though the static equilibrium analysis provides the fundamental building blocks (Zhang and Peeta, 2011) for more complex infrastructure interdependency problems, dynamic and/or disequilibrium models are needed for several reasons. First, due to the changes in factors such as network configuration, system capacity, socioeconomic environment, technology, public policy, as well as the disruptive and/or random events that commonly occur in infrastructure systems, a static equilibrium may never be reached, even in a single infrastructure system. Second, engineering projects for infrastructure systems usually take a long time to accomplish. The planning, design, and construction of many infrastructure facilities can last for years. Therefore, the transition phases could influence the overall benefits from a project and the return on investment. Third, the type and magnitude of interdependencies among infrastructure systems vary from time to time as the circumstances causing the interdependencies can themselves be different over time. Inefficiencies stemming from these interdependencies can build up over time, and eventually cause catastrophic consequences if not properly addressed. Fourth, the dynamic modeling is essential for the disequilibrium and disruption analysis of infrastructure systems, especially in the cascading failures context. Due to their physical and operational characteristics such as inherent complexity, large physical scale, openness to the public, exposure to natural

environment, and close interactions with other systems, these systems are vulnerable to natural hazards, component failures, human mistakes, and intentional attacks. Therefore, incidents and disruptions are common phenomena in these systems, and a dynamic approach enables a more realistic analysis of these phenomena.

The aforementioned needs entail the consideration of several key dimensions. The first dimension is the multi-period dynamics analysis, which focuses on the implications caused by the changes to the configurations of the systems, the ambient socioeconomic environment, and the system interdependencies over long periods of time. The term "period" is used here to denote a unit of future time during which these characteristics are not likely to change due to the time span that these changes require. However, between two periods, these characteristics may be updated, implying the need to consider the relevant dynamic factors. In our problem context, the length of a typical period is a year. The second dimension is the within-period equilibrium-tending dynamics, which is formulated to analyze how the system state variables adjust over time towards an equilibrium state assuming that system configurations such as the system capacity, population, and economic environment do not change within the period. Such analysis generally starts from an initial state, and the various decision-makers adjust their actions based on the ambient system conditions impacted by the actions of other decision-makers in both that system and other systems. The direction and the rate of the adjustment are normally determined by the behavior of and the information received by the various decision-makers when optimizing their objectives over time. The third dimension involves the disruption or disequilibrium analysis related to various events that cause sudden drops in the level of service, or even complete loss of capacity in short periods of time. This dimension aims to model disturbances in the dynamic and equilibrium models. In our problem context, its primary objectives are to understand how the disruption in one system can be transmitted to other systems through various interdependencies, and to capture the cascading phenomena across multiple systems. It also enables the evaluation of remedial strategies after a disruption in order to recover the overall system in a coordinated manner. In all these three dimensions, the way that the systems evolve is not only determined by each individual system itself, but also by its linkages to other systems.

To address these dimensions, this paper develops dynamic infrastructure interdependency models using a market-based approach that explicitly consider various interdependencies among multiple infrastructure systems. The approach proposed in this paper is general in the sense that it can consider all or a subset of the infrastructure systems in a single modeling platform, and it allows incorporating the spatial characteristics of the various infrastructure networks by including multiple geographic regions. The region here can be a city, county, state or country depending on the problem being addressed. Therefore, both the interdependencies among the different infrastructure systems and the interactions within the same system located in different regions can be addressed simultaneously using the approach. In the illustrative numerical experiments conducted in Section 4, a special case of the model with a single region is used because a multiple-region case requires system-specific expertise on factors such as network cost functions, which is beyond the scope of this paper. The paper proposes both within-period and multi-period dynamic models based on the platform of the static SCGE model of Zhang and Peeta (2011). Only the multi-period dynamic model is used in the experiments as our framework mainly focuses on addressing the interdependency issues in the long-term planning problems for the infrastructure systems, and the multi-period dynamic model fits this purpose better.

The paper is organized as follows. Section 2 introduces the theoretical background of the problem by describing the methodological approach used in this study. It briefly reviews a static

multilayer infrastructure network spatial computable general equilibrium (MINSCGE) model that serves as the basic theoretical platform. It also proposes the equivalent variational inequality (VI) formulation of the static MINSCGE model. Section 3 proposes a within-period dynamic model based on the VI formulation and a multi-period dynamic MINSCGE model based on the dynamic computable general equilibrium (CGE) theory. Section 4 summarizes numerical experiments using the multi-period dynamic MINSCGE model, and discusses their insights. Section 5 provides some concluding comments.

2. Theoretical background

This section provides theoretical background for the dynamic infrastructure interdependency analysis. It first discusses the market-based SCGE approach proposed by Zhang and Peeta (2011) for modeling infrastructure interdependencies, and then briefly reviews their static equilibrium model based on this approach that serves as the starting point for the dynamic models. It then proposes an equivalent VI formulation that will be used in Section 3.1 to illustrate that the steady state of the proposed within-period dynamic MINSCGE model is consistent with the solution of the static MINSCGE model. Issues related to transitioning from the static model to dynamic models will also be discussed.

2.1. Market-based SCGE approach

A key difficulty in modeling the infrastructure interdependencies is integrating the disparate physical, operational and scale characteristics of the various systems. In order to address these differences and provide a platform on which all or a subset of the systems can be modeled simultaneously based on a common interface, Zhang and Peeta (2011) propose a market-based approach to capture infrastructure interdependencies through supply-demand mechanisms that involve the production/consumption of commodities and associated production/utility substitution functions. The market-based interactions are captured using computable general equilibrium (CGE) theory, and its spatial extension (SCGE) is used to additionally capture the interactions within the same system among different geographical locations to address the spatial characteristics of the infrastructure networks. Thereby, interdependent infrastructure systems are treated as interacting sectors in an economy, and are represented as multilayer infrastructure networks (MINs).

While market mechanisms are used to capture interdependencies, their implications can be interpreted at the level of the individual infrastructure system using generalized cost functions. For example, in a transportation network these could imply link performance functions or link pricing functions influenced by capacity changes or pricing structure. In return, the output from the holistic consideration of the integrated systems can impact the decision-making in the individual systems to achieve more meaningful outcomes. Fig. 1 illustrates how the proposed infrastructure interdependency analysis manifests and impacts the real-world. Residing between the policy-making layer and the individual infrastructure system layer, the interdependency model integrates and analyzes data from the various individual systems through a generalized modeling framework. After performing decision-making and what-if analyses with a more global consideration of all systems, it provides feedback to both the policy makers and the individual system operators. Based on the feedback, the policy makers can adjust policy, investment

decisions and other forms of control vis-à-vis the individual systems. Responding to these adjustments and the outcome of the interdependency model, the individual infrastructure systems adapt their production and consumption behaviors, and these interactions continue recursively between the three levels.

This conceptual framework is generalized in the sense that: (i) it can model various types of interdependencies, (ii) it can be tailored to specific problems involving two or more infrastructure systems, (iii) it avoids the flow/physical characteristics and associated units for specific systems, (iv) it uses available real-world data sources, and (v) the implications of the interdependencies for a specific infrastructure system can be directly translated into characteristics addressed at the level of this infrastructure system.

2.2. The static MINSCGE model

Section 2.2 provides details of the static MINSCGE model of Zhang and Peeta (2011). It is included here for completeness as it provides the necessary platform for the dynamic models. It is important to note here that most of Section 2.2 follows directly from Zhang and Peeta (2011), and does not represent a contribution of this paper.

The model is primarily characterized by three types of players: (i) representative producer (also called firm in the SCGE context), who produces a certain type of commodity such as fuel, electric power, and transportation or telecommunication service by consuming commodities from other producers as well as primary factors such as labor and capital from households; (ii) household, who receives payment for selling the primary factors to producers, and determines its demand for each commodity using a utility function subject to cost constraints; and (iii) transport agent, who links the producers in different regions. In our problem context, an infrastructure system can be thought of as a sector in the economy in the SCGE model, and each node in an infrastructure network is modeled as a producer in the corresponding geographical region. The links in an infrastructure network represent the physical connections used by the transport agents to move the corresponding commodity with certain cost. Such SCGE models can capture various types of infrastructure interdependencies: (i) the input-output relationships involving the production and utility functions reflect the functional interdependency whose magnitude is captured by the parametric values of these functions, (ii) a common event may impact production levels and costs in different infrastructure systems simultaneously, thereby reflecting physical interdependencies, (iii) a common funding pool or resource may be available to multiple competing systems, indicating budgetary interdependencies, and (iv) all systems share the same households in the same market and region, implying market interdependencies.

Consider a closed spatial economy with N geographical regions, $n \in [1, ..., N]$, each hosting an economic system with K infrastructure systems, $k \in [1, ..., K]$ and one representative household. A transport agent is explicitly associated with each system, and denotes the operators/carriers that move goods/passengers, messages/data, water, electricity or energy in the corresponding network of that infrastructure system. Consistent with the MIN framework, the network associated with an infrastructure system k is represented by a directed graph $G^k(R, A^k)$, where R is the set of N regions and A^k is the set of links that represent the physical or logical/economic linkages between the nodes in that network.

The commodity movement between the producer and the commodity pool is performed at a certain cost by transport agents. A commodity has two prices in each region: (i) the supply price, denoting the payment that a producer receives for selling its output; and (ii) the demand

price, denoting the payment that the final or intermediate users pay for purchasing the commodity. As the transport agent incurs some transportation cost during the movement, typically the demand price of a commodity is higher than the supply price in a region. Thereby, a transport agent can also be viewed as a retailer in an infrastructure system.

The fundamental mechanism of the MINSCGE model can be illustrated by the circulation of commodity and payment flows in each region, and then for the entire spatial system. Fig. 2 illustrates the flows for a two-region (*r* and *m*) case, which can be generalized to multiple regions. A producer sells the commodity to, and receives payment from, the transport agent. This income is spent on purchasing primary factors and intermediate inputs from the various local commodity pools. Based on the value conservation principle, the unit value of the produced commodity should equal the total value of the primary factors, the intermediate inputs, and the transportation costs. On the consumption side, the household receives payment by selling primary factors to the local producers. It spends this income on purchasing various types of commodities from the local commodity pool through the corresponding transport agents. A transport agent collects the commodity from the producer in a region, and then provides it to the local and remote (other regions) households and producers through the corresponding commodity pool, during which process the transportation costs are added to the value of the commodity.

We start from the household. We assume that the primary factors are only consumed by local producers. The demand for a primary factor should not exceed the total available endowment in each region:

$$V_n^f \geq \sum_{j=1}^{K} v_n^{fj}, \ \forall f \in [1,...,F], n \in [1,...,N] \tag{1}$$

where V_n^f and v_n^{fj} are the endowment of primary factor f for all F primary factors, and the amount of primary factor f required by system j in region n, respectively. Income balance should hold for each region:

$$\varpi_n^f V_n^f = \varpi_n^f \sum_{j=1}^{K} v_n^{fj}, \ \forall f \in [1,...,F], n \in [1,...,N] \tag{2}$$

where ϖ_n^f is the unit price of primary factor f in region n. The total income of the household is composed of the payments received from selling the primary factors to all local producers, and should not exceed the amount spent on purchasing the various types of commodities:

$$\sum_{f=1}^{F} \varpi_n^f V_n^f = \sum_{f=1}^{F} (\varpi_n^f \sum_{j=1}^{K} v_n^{fj}) \geq \sum_{k=1}^{K} \rho_n^k d_n^k, \ \forall n \in [1,...,N] \tag{3}$$

where d_n^k is the final consumption demand of commodity k by the household in region n, and ρ_n^k is the demand price of commodity k in region n. The behavior of the household in each region is represented by a utility maximization problem:

$$\text{Max } u_n(D_n) \tag{4}$$

subject to the income constraint (3), where D_n is a vector of final consumption levels for all commodities in region n.

On the supply side, the gross output of a producer should be no less than the total amount sent to the commodity pools in all regions; it can also be viewed as the total amount of the commodity collected by the associated transport agent:

$$s_r^i \geq \sum_{m=1}^{N} T_{rm}^i, \ \forall i \in [1,...,K], r \in [1,...,N] \tag{5}$$

where s_r^i is the production level of commodity *i* produced by the corresponding producer in region *r*, and T_{rm}^i is the amount of commodity *i* produced in region *r* and exported to region *m* for intermediate or final consumption. The commodity exported to another region is collected by the transport agent first and then moved to the destination region, and the commodity available in each region is not differentiated based on where it is produced. This is illustrated in Fig. 2 by the arrows linking the producers in regions *r* and *m* to the transport agent through the output.

In terms of the value flows, the produced commodity should conserve its value when transferred to the transport agents:

$$\pi_r^i s_r^i = \pi_r^i \sum_{m=1}^{N} T_{rm}^i,\ \forall i \in [1,...,K], r \in [1,...,N] \tag{6}$$

where π_r^i is the supply price of commodity *i* in region *r*. After moving the right hand side to the left hand side and combining the two terms, Equation (6) becomes:

$$\pi_r^i (s_r^i - \sum_{m=1}^{N} T_{rm}^i) = 0,\ \forall i \in [1,...,K], r \in [1,...,N] \tag{7}$$

This equation indicates that either π_r^i or $(s_r^i - \sum_{m=1}^{N} T_{rm}^i)$ has to be 0. Equation (5) can be re-written as $(s_r^i - \sum_{m=1}^{N} T_{rm}^i) \geq 0$. Therefore, when $s_r^i > \sum_{m=1}^{N} T_{rm}^i$, that is, when the total production level from a producer is higher than the amount of the commodity that the transport agent desires to purchase (in other words, there is excessive supply of commodity *i* in region *r*), then the supply price π_r^i has to be zero to keep the equality in Equation (7).

Treating the transport agent as the final "supplier" of the commodity in the demand region through the commodity pool, the following supply-demand condition holds under equilibrium:

$$\sum_{r=1}^{N} T_{rm}^i \geq \sum_{j=1}^{K} x_m^{ij} + d_m^i,\ \forall i \in [1,...,K], m \in [1,...,N] \tag{8}$$

where x_m^{ij} is the input of commodity *i* needed to produce commodity *j* in region *m*. The left hand side of the equation is the total amount of commodity *i* available in the commodity pool in region *m*. The right hand side is the total demand for commodity *i* in this region; the first term is the total intermediate demand from all producers and the second term is the final demand of the household in this region. In terms of value flow, the value supplied should always be balanced by the value consumed for each commodity in a region:

$$\rho_m^i \sum_{r=1}^{N} T_{rm}^i = \rho_m^i (\sum_{j=1}^{K} x_m^{ij} + d_m^i),\ \forall i \in [1,...,K], m \in [1,...,N] \tag{9}$$

This equation implies that when there is excess supply in a system in a region, the demand price of that commodity will be zero, according to Walras' law (Ginsburgh and Keyzer, 2002). Combining Equations (5) and (8), we obtain:

$$s_n^i + \sum_{r=1}^{N} T_{rn}^i \geq \sum_{m=1}^{N} T_{nm}^i + \sum_{j=1}^{K} x_n^{ij} + d_n^i,\ \forall i \in [1,...,K], n \in [1,...,N] \tag{10}$$

In Fig. 2, this condition implies that the commodity flows (represented by the solid lines) leading into the transport agent must be no less than the outbound flows (represented by the solid lines)

leading out of it. The physical meaning of this equation is that the local production level of a commodity plus the total imported amount of the commodity must be no less than the total local, intermediate, and final demands, plus the total amount of this commodity exported to other regions.

The zero profit condition in general equilibrium theory implies that the price of the output from a producer should be no greater than the value of all the inputs it takes, which is represented by the following equation:

$$\pi_n^j \le \sum_{i=1}^{K} \rho_n^i \alpha_n^{ij} + \sum_{f=1}^{F} \varpi_n^f \beta_n^{fj}, \ \forall j \in [1,...,K], n \in [1,...,N] \tag{11}$$

α_n^{ij} and β_n^{fj} are the region-specific input-output coefficients. In terms of the value flows, the value that a producer generates should be equal to the total value of intermediate inputs and primary factors it consumes:

$$\pi_n^j s_n^j = \sum_{i=1}^{K} \rho_n^i x_n^{ij} + \sum_{f=1}^{F} \varpi_n^f v_n^{fj} = \sum_{i=1}^{K} \rho_n^i \alpha_n^{ij} s_n^j + \sum_{f=1}^{F} \varpi_n^f \beta_n^{fj} s_n^j, \tag{12}$$
$$\forall j \in [1,...,K], n \in [1,...,N]$$

The problem that each regional producer faces is to maximize the profit by choosing levels of intermediate inputs and primary factors:

$$\text{Max } z_n^j = \pi_n^j - \sum_{i=1}^{K} \rho_n^i \alpha_n^{ij} - \sum_{f=1}^{F} \varpi_n^f \beta_n^{fj} \tag{13}$$

subject to its production function s_n^j.

The network and spatial issues are addressed in the process of determining commodity flows between the regions and the corresponding costs associated with transporting them. The commodity transportation activities and costs are explicitly modeled through generalized cost functions that represent the characteristics of the network structure. The generalized costs can be in the form of money, time, or risk of disruption. Typically, these costs are functions of the network capacity, transportation distance, and the amount of flow being carried.

Under equilibrium, a set of conditions related to the network flow and transportation costs should hold. Due to the value conservation principle, the commodity moved from one region to the other should keep the same value after adding the transportation cost:

$$(\pi_r^k + \lambda_{rm}^k) T_{rm}^k = \rho_m^k T_{rm}^k, \ \ \forall r \in [1,...,N], m \in [1,...,N], k \in [1,...,K] \tag{14}$$

where λ_{rm}^k is the equilibrium unit transportation cost between regions *r* and *m* for commodity *k*. The underlying implication of the above equation is that when the flow of commodity *k* between regions *r* and *m* is nonzero, then $\pi_r^k + \lambda_{rm}^k = \rho_m^k$; that is, the demand price of commodity *k* in region *m* should be the same as the supply price of this commodity in region *r* plus the equilibrium transportation costs between the two regions. On the other hand, if the price equality does not hold, then the flow between the two regions should be equal to zero, that is, there is no flow of this commodity from region *r* to region *m*.

In terms of commodity flow conservation, the transportation demand between an origin-destination (O-D) node pair should be the summation of the total amount of commodity moved between these two regions. There may be multiple paths connecting the same O-D pair. The total transportation demand has to be fully carried by the paths connecting the O-D regions:

$$T_{rm}^{k} = \sum_{p \in P_{rm}^{k}} h_{p}^{k}, \forall r \in [1,...,N], m \in [1,...,N], k \in [1,...,K] \tag{15}$$

where P_{rm}^{k} is a set of paths that can carry commodity k between regions r and m, and h_{p}^{k} is the flow of commodity k on path p. Generally, the minimum cost of transportation is a function of the transportation demand between two regions. The transportation demand is partially determined by the price difference of the same commodity in the two regions and the associated transportation cost. A larger price gap of the same commodity between two regions motivates a greater amount of this commodity to be transported from the region with lower price to the other region.

When there are several paths connecting the same O-D pair, only those whose path cost C_{p}^{k} equals the minimum cost λ_{rm}^{k} will be utilized, and those that have higher cost will carry no flow:

$$h_{p}^{k} \begin{cases} \geq 0, \ if \ C_{p}^{k} = \lambda_{rm}^{k} \\ = 0, \ if \ C_{p}^{k} > \lambda_{rm}^{k} \end{cases}, \ \forall r \in [1,...,N], m \in [1,...,N], k \in [1,...,K], p \in P_{rm}^{k} \tag{16}$$

Conditions (15) and (16) are equivalently the flow conservation and user equilibrium conditions in a standard network flow assignment problem, respectively.

In summary, the above SCGE conditions are used to formulate the multi-region infrastructure interdependencies problem as a multi-player game with one household and several producers in each region, and one transport agent connecting the different regions in each system. The household and producers in each region seek to solve their optimization problems (Equations (4) and (13), respectively). The transport agents concurrently aim to determine the network flow allocation for each system based on Equations (14) through (16).

The exact forms of production functions s and utility functions u have not been specified so far. While the proposed models are general enough that these functions can take any mathematical form, the choice of the specific form is important vis-à-vis the infrastructure interdependencies problem because these functions reflect the input-output and substitution relationships among the various infrastructure systems, define the underlying decision-making behavior of the various players, determine the mathematical complexity and properties of the model, and influence the models' capability of capturing the various types of interactions among different systems. We choose the nested constant elasticity of substitution (NCES) function, which is an extension of the standard CES function (McFadden, 1963) with a hierarchical structure of decision-making.

When there are more than two inputs in a production or utility function, as in our case, there may be different levels of elasticity of substitution (EOS) between different input pairs. This can be represented by the NCES function. It is labeled so because it allows a nested structure with multiple tiers. In the proposed MINSCGE model, when considering four infrastructure systems, energy (NRG), power (PWR), transportation (TRA) and telecommunication (TEL), the production functions of the producers take the following form:

$$s = A_1 [\phi_L L^{-\mu_1} + \phi_O O^{-\mu_1} + \phi_{NP} NP^{-\mu_1} + \phi_{TT} TT^{-\mu_1}]^{-1/\mu_1} \qquad 1^{st} \text{ Tier}$$

$$\left.\begin{aligned} NP &= A_2[\varphi_Y NRG^{-\mu_2} + \varphi_P PWR^{-\mu_2}]^{-1/\mu_2} \\ TT &= A_3[\varphi_R TRA^{-\mu_3} + \varphi_C TEL^{-\mu_3}]^{-1/\mu_3} \end{aligned}\right\} \qquad 2^{nd} \text{ Tier} \tag{17}$$

where:

A_i : Technology parameter for function $i \in [1,2,3]$; $A_i > 0$;
μ_i : Substitution parameter for function $i \in [1,2,3]$;
L, O : Labor and other primary factors, respectively;
NP: The virtual input that represents the combination of the NRG and PWR commodities;
TT: The virtual input that represents the combination of the TRA and TEL commodities;
NRG, *PWR*, *TRA*, *TEL*: Input of energy, power, transportation and telecommunication commodities, respectively;
Y, P, R, C : Subscripts for NRG, PWR, TRA and TEL, respectively;
ϕ_j : Distribution parameters for different commodities or virtual commodities $j \in [Y, P, R, C, TT, NP]$; $0 \le \phi_j \le 1$, and $\sum_j \phi_j = 1$ in the same function.

Without loss of generality, the above equation only shows the production function of one producer to illustrate the structure of the NCES production functions. When implemented, the parameters and variables in (17) should have superscripts for system $k = [1,...,K]$ and subscripts for region $r = [1,...,N]$. That is, the producer in each system has its specific NCES production function, and the values of the parameters vary from one producer to another depending on the type and magnitude of interdependencies between the various systems. These superscripts and subscripts are omitted for simplicity.

The utility functions of the households take the following form:

$$u = B_1[\varphi_{NP} NP^{-\upsilon_1} + \varphi_{TT} TT^{-\upsilon_1}]^{-1/\upsilon_1} \qquad 1^{\text{st}} \text{ Tier}$$

$$\left.\begin{aligned} NP &= B_2[\phi_Y NRG^{-\upsilon_2} + \phi_P PWR^{-\upsilon_2}]^{-1/\upsilon_2} \\ TT &= B_3[\phi_R TRA^{-\upsilon_3} + \phi_C TEL^{-\upsilon_3}]^{-1/\upsilon_3} \end{aligned}\right\} \qquad 2^{\text{nd}} \text{ Tier} \qquad (18)$$

where:
B_i : Technology parameters for function $i \in [1,2,3]$; $B_i > 0$;
υ_i : Substitution parameters for function $i \in [1,2,3]$;
φ_j : Distribution parameters for different commodities or virtual commodities $j \in [Y, P, R, C, TT, NP]$; $0 \le \varphi_j \le 1$, and $\sum_j \varphi_j = 1$ in the same function.

Similar to the production function case, the parameters and variables in (18) should have subscripts $r \in [1,...,N]$ as the household in each region may have its own utility preferences.

The above model can be solved using the GAMS software. The objective functions and constraints described above can be written using the GAMS programming language and the MPSGE library available from the GAMS package. The model is first automatically converted to an MCP by GAMS, and is then solved using a NLP solver available in GAMS, as the model is typically nonlinear. The values of the variables from the solution output are used to determine the various performance measures such as the household utility (the value of the utility function for each household), the social welfare (the summation of the utility values for all households, when multiple households are considered), and the system activity level (the production level of a system). These performance measures are then used to assess the status of the systems and to characterize the various effects of the infrastructure interdependencies. To address the additional network flow aspects that represent horizontal links in the MIN framework, Equations (14)

through (16) can be directly added as network flow equilibrium conditions to the GAMS code in addition to the equations that represent the optimization problems for the household and producers. The transportation cost C_p^k requires system-specific knowledge for the associated functions and data, at which point they can be incorporated into the solution procedure in a straightforward manner.

2.3. VI formulation of the MINSCGE model

This subsection proposes an equivalent VI formulation of the MINSCGE model introduced in Section 2.2. Nagurney (1999) provides a comprehensive introduction of the VI approach and its application to network economics problems. The formulation proposed in this section follows a similar model structure.

Suppose for each sector $i \in [1,...,K]$ in each region $r \in [1,...,N]$, the production level $s_r^i(\pi)$ is a continuous (and monotonically increasing) function of the production price of the associated commodity, demand level $d_s^i(\rho)$ of the household is a continuous (and monotonically decreasing) function of the supply price of the commodity, and the minimum transportation cost $\lambda_{rm}^i(T)$ between two regions *r* and *m* is a continuous (and monotonically increasing) function of the transportation demand. To reiterate our problem context, each "sector" represents an infrastructure system, with one representative producer producing a commodity which may either be a physical product or a service. The "region" here is broadly used to refer to geographical locations of the scale of a city, county/district, state, or country.

First, we rewrite the equilibrium conditions of Section 2.2. For the producer's problem, Equation (6) can be rewritten as:

$$s_r^i(\pi^*) \begin{cases} = \sum_{m=1}^{N} T_{rm}^{i*}, \; if \;\; \pi_r^{i*} > 0 \\ > \sum_{m=1}^{N} T_{rm}^{i*}, \; if \;\; \pi_r^{i*} = 0 \end{cases}, \qquad \forall r \in [1,...,N], i \in [1,...,K] \tag{19}$$

The * superscript denotes the equilibrium value of the corresponding variable. The above equation implies that from the production perspective, when there is excess supply of a commodity *i* in a region *r*, its production price will be zero in this region.

On the demand side, Equation (9) can be rewritten as:

$$d_m^i(\rho^*) \begin{cases} = \sum_{r=1}^{N} T_{rm}^{i*} - \sum_{j=1}^{K} x_m^{ij}, \; if \;\; \rho_m^{i*} > 0 \\ < \sum_{r=1}^{N} T_{rm}^{i*} - \sum_{j=1}^{K} x_m^{ij}, \; if \;\; \rho_m^{i*} = 0 \end{cases}, \quad \forall m \in [1,...,N], i \in [1,...,K] \tag{20}$$

where the term $\sum_{r=1}^{N} T_{rm}^{i*}$ represents the total supply of commodity *i* in region *m*, d_m^i is the final demand of commodity *i* by the household in region *m*, and $\sum_{j=1}^{K} x_m^{ij}$ represents the total intermediate demand of commodity *i* by all other producers in region *m*. Equation (20) implies that from the demand perspective, when there is excessive supply of a commodity in a region, its

supply price will be zero in this region. Since x_s^{ij} is typically a function of the production activity of producer *j*, the above equation can be written as:

$$d_m^i(\rho^*)\begin{cases} = \sum_{r=1}^{N} T_{rm}^{i*} - \sum_{j=1}^{K} x_m^{ij}(s_m^j(\pi^*)),\ if\ \ \rho_m^{i*} > 0 \\ < \sum_{r=1}^{N} T_{rm}^{i*} - \sum_{j=1}^{K} x_m^{ij}(s_m^j(\pi^*)),\ if\ \ \rho_m^{i*} = 0 \end{cases}, \tag{21}$$

$\forall m \in [1,...,N], i \in [1,...,K]$

For the transport agent, Equation (14) can be re-written as:

$$\pi_r^{i*} + \lambda_{rm}^i \begin{cases} = \rho_m^{i*},\ if\ T_{rm}^{i*} > 0 \\ > \rho_m^{i*},\ if\ T_{rm}^{i*} = 0 \end{cases},\quad \forall r, m \in [1,...,N], i \in [1,...,K] \tag{22}$$

The above equation implies that for a commodity, if the production price in region *r* plus the transportation cost between *r* and *m* is higher than the supply price of the commodity in region *m*, then the transport agent will not move the commodity from *r* to *m* because its overall cost ($\pi_r^{i*} + \lambda_{rm}^i$) is higher than the revenue (ρ_m^{i*}) it may earn between that O-D pair.

Based on the above equilibrium conditions, the SCGE model described in Section 2.2 can be converted into a VI formulation as stated in the following theorem:

Theorem 1 VI Formulation of the MINSCGE Model

In a spatial economic system with *K* commodities and *N* regions, the production price, transportation demand, and supply price vector $\chi^* = (\rho^*, T^*, \pi^*)$ is in equilibrium *iff* the following variational inequality holds for all $\chi \equiv (\rho, T, \pi)$:

$$\sum_{i=1}^{K}\sum_{r=1}^{N}[s_r^i(\pi^*) - \sum_{m=1}^{N} T_{rm}^{i*}](\pi_r^i - \pi_r^{i*}) + \sum_{i=1}^{K}\sum_{r=1}^{N}\sum_{m=1}^{N}[\pi_r^{i*} + \lambda_{rm}^i(T^*) - \rho_m^{i*}](T_{rm}^i - T_{rm}^{i*})$$
$$+\sum_{i=1}^{K}\sum_{m=1}^{N}[\sum_{r=1}^{N} T_{rm}^{i*} - d_m^i(\rho^*) - \sum_{j=1}^{K} x_m^{ij}(s_m^j(\pi^*))](\rho_m^i - \rho_m^{i*}) \geq 0 \tag{23}$$

Proof:

$\Rightarrow$: We look at the second term in the above inequality first. Based on equilibrium condition (22), when equilibrium is reached, for each commodity *i* and region pair (*r, m*) one must have:

$$[\pi_r^{i*} + \lambda_{rm}^i(T^*) - \rho_m^{i*}](T_{rm}^i - T_{rm}^{i*}) \geq 0 \tag{24}$$

for any nonnegative transportation demand T_{rm}^i. This is because that if $T_{rm}^i > 0$, then based on (22), $\pi_r^{i*} + \lambda_{rm}^i(T^*) - \rho_m^{i*} = 0$ and (24) holds for the equality. If $T_{rm}^i = 0$, then again based on (22), $\pi_r^{i*} + \lambda_{rm}^i(T^*) - \rho_m^{i*} > 0$ while $T_{rm}^i - T_{rm}^{i*} \geq 0$, and (24) still holds.

Using the same logic, condition (19) implies that:

$$[s_r^i(\pi^*) - \sum_{m=1}^{N} T_{rm}^{i*}](\pi_r^i - \pi_r^{i*}) \geq 0 \tag{25}$$

and condition (21) implies that:

$$[\sum_{r=1}^{N} T_{rm}^{i*} - d_m^i(\rho^*) - \sum_{j=1}^{K} x_m^{ij}(s_m^j(\pi^*))](\rho_m^i - \rho_m^{i*}) \geq 0 \tag{26}$$

Since Equations (24)-(26) are true for all commodities (systems) and all regions, summing them over all commodities and regions will generate the variational inequality (23).

$\Leftarrow$: Suppose (23) holds for all commodities and regions. This implies that VI (23) holds for any value of $\chi \equiv (\pi, T, \rho)$. Let $T_{rm}^{i} = T_{rm}^{i*}$ for all commodities and region pairs except one randomly chosen pair (*y, z*) and one randomly chosen commodity *l*. Further, let $\pi_r^i = \pi_r^{i*}$ and $\rho_m^i = \rho_m^{i*}$ for all commodities and regions. Then, from (23), we obtain:

$$(\pi_y^{l*} + \lambda_{yz}^{l}(T^*) - \rho_z^{l*}) \times (T_{yz}^{l} - T_{yz}^{l*}) \geq 0 \tag{27}$$

Since *l*, *y*, *z* are randomly chosen, Equation (27) must hold for any χ at equilibrium, which is equivalent to Equation (22).

Let $\pi_r^i = \pi_r^{i*}$ for all commodities and regions except one randomly chosen region *y* and one randomly chosen commodity *l*. Further, let $T_{rm}^{i} = T_{rm}^{i*}$ and $\rho_m^i = \rho_m^{i*}$ for all commodities and regions. Then, from (23):

$$[s_y^l(\pi^*) - \sum_{m=1}^{N} T_{ym}^{l*}](\pi_y^l - \pi_y^{l*}) \geq 0 \tag{28}$$

Since *l* and *y* are randomly chosen, Equation (28) must hold for all χ at equilibrium, which is equivalent to Equation (19).

Let $\rho_m^i = \rho_m^{i*}$ for all commodities and regions except one randomly chosen region *z* and one randomly chosen commodity *l*. Also, let $\pi_r^i = \pi_r^{i*}$ and $T_{rm}^{i} = T_{rm}^{i*}$ for all commodities and regions. Then, from (23):

$$[\sum_{r=1}^{N} T_{rz}^{l*} - d_z^l(\rho^*) - \sum_{j=1}^{K} x_z^{lj}(s_z^l(\pi^*))](\rho_z^l - \rho_z^{l*}) \geq 0 \tag{29}$$

Since *l* and *z* are randomly chosen, Equation (29) must hold for all χ at equilibrium, which is equivalent to Equation (21). This completes the proof. □

The VI representation of the MINSCGE model has some advantages compared to its formulation in Section 2.2. First, the VI form can be embedded as an equilibrium constraint in optimization formulations such as network optimization, system design, and policy analysis problems. Without the VI technique, these problems would have to be formulated as sequential bi-level optimization problems, which lack behavioral intuition and are relatively more difficult to analyze and solve. Second, the VI formulation provides a convenient way to analyze mathematical properties such as the existence and uniqueness of the solution, which is addressed elsewhere for the static MINSCGE model (Zhang and Peeta, 2013). Third, analytical solution methods available to VI can be used to solve the corresponding equilibrium problems.

2.4. From static to dynamic

A natural approach to develop dynamic infrastructure interdependencies models is to add a temporal dimension to existing static equilibrium models in the literature. In the context of the current study, it can also be viewed as the process of extending single-system dynamic models to address multiple infrastructure systems. Various types of single-system dynamic models are available for different infrastructure system domains. For example, in the transportation domain, dynamic models are available in different contexts such as dynamic traffic assignment, traffic

network equilibrium, interregional freight flow modeling, and dynamic traveler behavior modeling, to name just a few problem contexts. However, when considered at the usual level of detail, the formulation of these single-system models is typically determined by the system-specific physical and operational characteristics of the corresponding infrastructure system. Therefore, these models differ from system to system in terms of temporal and spatial scales, network flow characteristics, user behavior assumptions, data needs, and formulation constraints and objectives. For example, compared to those for the transportation system, the telecommunication models typically have shorter time scales and larger spatial range. In terms of cost, the travel time or monetary expenses of end users are normally considered in transportation models, while the operational costs encountered by the operators are the key consideration in telecommunication models. Due to these disparate characteristics, it is difficult to develop a common microscopic dynamic model that can incorporate all infrastructure systems.

By contrast, macroscopic models, especially those associated with the economic and market perspectives, can provide a convenient platform to link the different infrastructure systems based on a common interface that enables circumventing the complexities caused by the disparate system characteristics. Macroscopic dynamic models for individual infrastructure systems are available, especially using the CGE approach to address the economic impacts for long-term planning problems. These models can be found in various domains, including the transportation (Giesecke and Madden, 2003; Kim et al. 2004), telecommunication (Hsiao and Lazar, 1991; Korilis and Lazar, 1995; Roller and Waverman, 2002; Altman and Wynter, 2004; Aravantinos and Harmantzis, 2005), energy (Jorgenson and Wilcoxen, 1993; McFarland et al. 2004a and 2004b), and water (Seung et al., 2000) systems. These models typically use a recursive dynamic approach to link a sequence of single-period equilibria through backward-looking and/or forward-looking equations to determine the activities of the involved entities in different periods, typically including the demand and supply sides. Some of them contain both within-period and cross-period sub-models at different time scales to formulate two types of adjustment behavior. First, the adjustment of commodity prices and production level occur over a shorter time scale. Then, the adjustments to the system configuration occur over multiple periods because of the lagged response. Among the aforementioned models, none explicitly focuses on formulating multiple infrastructure systems or their interdependencies, as they just analyze a single system.

In Section 3 we illustrate how the previously proposed market-based MINSCGE modeling framework can be modified by considering multiple time periods and linking the different time periods through economic or physical flows that represent the temporal interactions within or among the infrastructure systems to capture the dynamic aspects of infrastructure interdependencies problems. The approach preserves the modeling power of the static MINSCGE model in the interdependent infrastructure systems context, while being able to address both the dynamic and disequilibrium aspects.

3. Dynamic MINSCGE models

Based on the static MINSCGE model introduced in Section 2.2, we propose two classes of dynamic models for the infrastructure interdependencies problem: the within-period equilibrium-tending dynamic model and the multi-period dynamic MINSCGE model.

The two classes of dynamic models represent the dynamics of infrastructure interdependencies at different time scales and in different real-world scenarios. The within-period dynamic model represents a case with a shorter time scale, during which period the fundamental system configurations such as the system capacity, network connectivity, ambient socioeconomic characteristics, and the interdependencies among the systems are not likely to change due to the typically long life cycle of civil infrastructure projects/systems. However, the various parties, including the producers, households, and transport agents, can adjust their decisions reacting to the ambient system status such as the supply level, demand level, and prices of the commodities and the transportation demand/costs. Their decisions may subsequently influence these system state measures. This process typically unfolds over time in an iterative manner by the parties modifying their production, consumption and transportation decisions through the optimization behavior introduced in Section 2.2 based on the market signals such as the excessive supply/demand, or the difference between expected cost and the actual cost of a commodity or service. A typical example for the with-in period dynamic analysis is after a disruption, the original system equilibrium is broken, and the adjustment of the supply/demand levels by the involved parties enables the system to eventually reach a new equilibrium before the system recovers to its original capacity. As mentioned in Section 1, a typical length of a period is a year in our problem context.

For a multi-period dynamic model, a longer time scale is considered. Between different time periods, the aforementioned system configurations could change, naturally causing the adjustment of system state measures towards a new equilibrium. In this process, the dynamic factors such as the interest rate, population growth rate, depreciation rate of capital, system capacity, as well as the infrastructure interdependency parameters may change from period to period. Therefore, the multi-period dynamic model can be used to evaluate the impact of a new civil infrastructure project such as the construction of a new roadway or power plant. It may also be used to evaluate the long-term impact of a large-scale disruption involving multiple systems, as well as the remedial strategies after the disruption.

3.1. Within-period equilibrium-tending dynamic model

In this section, the static equilibrium model introduced in Section 2.2 is modified to incorporate the equilibrium-tending adjustment mechanism through which the commodity prices and interregional flows adjust over time following distinct market signals while the constraints ensuring balance conditions are not enforced prior to attaining an equilibrium state. The adjustment process is based on the assumption that no agent (household nor producer) is able to fully manipulate the prices or control the commodity flows between a pair of regions, and that the agents adjust their production and/or consumption levels and transportation demands based on the profit or utility maximization behavior described in Section 2.2. This assumption does not conflict with the setting in the static MINSCGE model that there is only one representative producer for each sector and one representative household in each region, because they can each be deemed as abstract agents with the aggregate behavior of a large number of individual entities. The proposed dynamic model has a structure similar to that of the projective dynamic system frameworks used by Dupuis and Nagurney (1993) and Friesz et al. (1994, 1996, 1998).

As in the static model, the producers, households and transport agents in the various infrastructure systems are modeled as players with self-interest goals and rules in a non-

cooperative game. The system starts from an initial state, and adjusts towards a Cournot-Nash equilibrium. During the adjustment process, the demand price of each commodity responds to the local excess demand, which is expressed as the difference between the total demand and supply in a region:

$$\begin{aligned} ED_s^i[\rho(t),\pi(t),h(t)] &= d_s^i[\rho(t)] + \sum_{j=1}^{K} x_s^{ij}(t) - \sum_{r\in N}\sum_{p\in P_{rs}^i} h_p^i(t) \\ &= d_s^i[\rho(t)] + \sum_{j=1}^{K} \alpha_s^{ij}\cdot s_s^j[\pi(t)] - \sum_{r\in N}\sum_{p\in P_{rs}^i} h_p^i(t), \\ &\forall i\in[1,\ldots,K], s\in[1,\ldots,N],\ t\in[0,\tau] \end{aligned} \tag{30}$$

where ED_s^i is the excess demand of commodity i in region s. The first term is the final demand level of the local household, the second term is the total intermediate demand by the local producers in other sectors, and the third term is the total supply of the commodity available from the region.

The market signal that each producer responds to is the excess production, which represents the difference between the total amount produced by the local producer and the total amount that can be absorbed by the transportation agent who will merge the output to the corresponding commodity pools of all regions including the local region. This can be expressed as:

$$\begin{aligned} &ES_r^i[\pi(t),h(t)] = s_r^i[\pi(t)] - \sum_{s\in N}\sum_{p\in P_{rs}^i} h_p^i(t), \\ &\forall i\in[1,\ldots,K], r\in[1,\ldots,N],\ t\in[0,\tau] \end{aligned} \tag{31}$$

The transportation demand between an O-D pair adjusts responding to the excess delivered price, which is the difference between the demand price at the destination region and the total cost of supply price plus the transportation cost. This can be written as:

$$\begin{aligned} &EC_{rs}^i[\rho(t),\pi(t),h(t)] = \rho_s^i(t) - \pi_r^i(t) - \lambda_{rs}^i[h(t)], \\ &\forall i\in[1,\ldots,K], r,s\in[1,\ldots,N],\ t\in[0,\tau] \end{aligned} \tag{32}$$

The sign and magnitude of excess quantities from functions (30)-(32) determine the fluctuations of production and consumption prices and the transportation activities perceived by producers, household and transportation agents, respectively. A commonly-used adjustment process in dynamical system analysis is used to model the way that the agents react to these signals. The adjustment process should guarantee that the trajectory of the intermediate states always stay in the feasible region at any time. Based on a property of our problem, we use a simple adjustment mechanism and then project the outcome onto the feasible region using the minimum norm projection. Particularly, when the only constraints are the nonnegativity of prices and transportation flows, the adjustment of demand price can be represented as:

$$\begin{aligned} &\rho_s^i(t+\Delta t) = \max(0, \rho_s^i(t) + \varepsilon 1_s^i ED_s^i(\rho(t),\pi(t),h(t))\Delta t), \\ &\forall i\in[1,\ldots,K], s\in[1,\ldots,N],\ t\in[0,\tau] \end{aligned} \tag{33}$$

where $\rho_s^i(t+\Delta t)$ is the future demand price that the household is willing to pay, and $\varepsilon 1_s^i > 0$ is the associated adjustment parameter. The projection guarantees that the price never becomes negative. Accordingly, the demand price dynamics follows:

$$\frac{d\rho_s^i}{dt} = \begin{cases} \max(0, \varepsilon 1_s^i ED_s^i(\rho(t), \pi(t), h(t))), \ if\ \rho_s^i(t) = 0 \\ \varepsilon 1_s^i ED_s^i(\rho(t), \pi(t), h(t)), \ if\ \rho_s^i(t) > 0 \end{cases} \tag{34}$$
$$\forall i \in [1,...,K], s \in [1,...,N], \ t \in [0, \tau]$$

This equation implies that if there is positive excess demand in a region, then the household is willing to pay higher price in the future, and vice versa. However, if the price is already zero, the price will not become negative even if there is negative excess demand. Similarly, the adjustment process of the supply price can be represented by:

$$\pi_r^i(t + \Delta t) = \max(0, \pi_r^i(t) - \varepsilon 2_r^i ES_r^i(\pi(t), h(t))\Delta t), \\ \forall i \in [1,...,K], r \in [1,...,N], \ t \in [0, \tau] \tag{35}$$

where $\pi_r^i(t + \Delta t)$ is the counterpart of $\rho_s^i(t + \Delta t)$ on the supply side, and $\varepsilon 2_r^i > 0$ is the adjustment parameter for production price. The supply price dynamics follows:

$$\frac{d\pi_r^i}{dt} = \begin{cases} \max(0, -\varepsilon 2_r^i ES_r^i(\pi(t), h(t))), \ if\ \pi_r^i(t) = 0 \\ -\varepsilon 2_r^i ES_r^i(\pi(t), h(t)), \ if\ \pi_r^i(t) > 0 \end{cases}, \tag{36}$$
$$\forall i \in [1,...,K], r \in [1,...,N], \ t \in [0, \tau]$$

The dynamics of transportation demands also follow a similar pattern. That is, the adjustment process of transportation demand follows:

$$T_{rs}^i(t + \Delta t) = \max(0, T_{rs}^i(t) + \varepsilon 3_{rs}^i EC_{rs}^i(\rho(t), \pi(t), h(t))\Delta t), \\ \forall i \in [1,...,K], r, s \in [1,...,N], \ t \in [0, \tau] \tag{37}$$

where $T_{rs}^i(t + \Delta t)$ is the future transportation demand of commodity *i* between regions *r* and *s*, and $\varepsilon 3_{rs}^i > 0$ is the adjustment parameter. The transportation demand dynamics follows:

$$\frac{dT_{rs}^i}{dt} = \begin{cases} \max(0, \varepsilon 3_{rs}^i EC_{rs}^i(\rho(t), \pi(t), h(t))), \ if\ T_{rs}^i(t) = 0 \\ \varepsilon 3_{rs}^i EC_{rs}^i(\rho(t), \pi(t), h(t)), \ if\ T_{rs}^i(t) > 0 \end{cases} \tag{38}$$
$$\forall i \in [1,...,K], r, s \in [1,...,N], \ t \in [0, \tau]$$

In summary, based on equations (33)-(38), the price and inter-regional commodity transportation demand dynamics in the MIN system can be readily stated, in vector form, as:

$$\begin{aligned} \frac{d\rho}{dt} &= \max(0, \varepsilon 1 ED(\rho(t), x(t), h(t))) \\ \frac{d\pi}{dt} &= \max(0, -\varepsilon 2 ES(\pi(t), h(t))) \\ \frac{dT}{dt} &= \max(0, \varepsilon 3 EC(\rho(t), \pi(t), h(t))) \\ \rho(0) &= \rho^0, \ \pi(0) = \pi^0, \ T(0) = T^0 \end{aligned} \tag{39}$$

The dynamical model (39) reflects how commodity prices and transportation demands adjust based on the market excess signals, and how fast this process is. The following theorem shows that the set of stationary points of the system are the set of solutions of the variational inequality (23) formulation of the SCGE model.

Theorem 2 Steady States of the MIN Dynamical System

A set of commodity demand prices, transportation flow demands, and commodity production

prices $z^* = (\rho^*, T^*, \pi^*)$ in the multilayer infrastructure systems is also the set of stationary points of dynamical system (39) if and only if it is an equilibrium solution of the variational inequality model (23).

Proof:

$\Rightarrow$: The stationary point condition implies that

$$\frac{d\rho_s^i}{dt} = 0 = \begin{cases} \max(0, \varepsilon 1_s^i ED_s^i(\rho(t), \pi(t), h(t))), if\ \rho_s^i(t) = 0 \\ \varepsilon 1_s^i ED_s^i(\rho(t), \pi(t), h(t)), if\ \rho_s^i(t) > 0 \end{cases} \tag{40}$$

$$\frac{d\pi_r^i}{dt} = 0 = \begin{cases} \max(0, -\varepsilon 2_r^i ES_r^i(\pi(t), h(t))), if\ \pi_r^i(t) = 0 \\ -\varepsilon 2_r^i ES_r^i(\pi(t), h(t)), if\ \pi_r^i(t) > 0 \end{cases} \tag{41}$$

$$\frac{dT_{rs}^i}{dt} = 0 = \begin{cases} \max(0, \varepsilon 3_{rs}^i EC_{rs}^i(\rho(t), \pi(t), h(t))), if\ T_{rs}^i(t) = 0 \\ \varepsilon 3_{rs}^i EC_{rs}^i(\rho(t), \pi(t), h(t)), if\ T_{rs}^i(t) > 0 \end{cases} \tag{42}$$

ρ^* satisfies Equation (40) iff it solves:

$$\rho_r^i ED_s^i(\rho(t), \pi(t), h(t)) = 0 \tag{43}$$

π^* satisfies Equation (41) iff it solves:

$$\pi_s^i ES_r^i(\pi(t), h(t)) = 0 \tag{44}$$

And, T^* satisfies (42) iff it solves:

$$T_{rs}^i EC_{rs}^i(\rho(t), \pi(t), h(t)) = 0 \tag{45}$$

Equations (43)-(45) form a set of nonlinear complementarity problems. Based on the relationship between NCP and VIP and the definition of excess variables (30)-(32), the set of equations is equivalent to VIP:

$$\sum_{i=1}^{K}\sum_{r=1}^{N}[s_r^i(\pi^*) - \sum_{s=1}^{N} T_{rs}^{i*}](\pi_r^i - \pi_r^{i*}) + \sum_{i=1}^{K}\sum_{r=1}^{N}\sum_{s=1}^{N}[\pi_r^{i*} + \lambda_{rs}^i(T^*) - \rho_s^{i*}](T_{rs}^i - T_{rs}^{i*})$$
$$+ \sum_{i=1}^{K}\sum_{s=1}^{N}[d_s^i(\rho^*) + \sum_{j=1}^{K} x_s^{ij}(s_s^j(\pi^*)) - \sum_{r=1}^{N} T_{rs}^{i*}](\rho_s^i - \rho_s^{i*}) \geq 0 \tag{46}$$

which is the same as (23).

3.2. Multi-period dynamic MINSCGE model

The multi-period dynamic MINSCGE model focuses on the impacts of the long-term dynamics of infrastructure interdependencies through the consideration of factors such as population growth rate (also simply referred to as "growth rate"), discount rate, and depreciation rate, as well as system capacity changes that occur over long periods of time (generally, over a year). In the modeling context, the future time is divided into multiple periods of uniform length (for example, a year). The impacts of the infrastructure interdependencies are captured through the multi-period dynamic model; it is assumed that the length of each period is long enough to enable the change in system parameters. This model provides a platform that enables more holistic policy, engineering and economic decisions by considering both the temporal and spatial interactions of the various systems.

In the static MINSCGE model introduced in Section 2.2, a key assumption is that the commodity and value flow conservation principles that reflect the equilibrium conditions require that the producers and households spend the entire income within a single period, and that the commodities produced by the producers will be completely consumed within that period. In this section, we extend it to a dynamic model by introducing multiple time periods within a finite horizon, and allowing the commodities to be potentially transferred to the next time period depending on the consumption levels. Similarly, the income that a household earns in one period can be saved and spent in the next period. Without loss of generality, we only consider the transfer of commodity and income between two consecutive time periods in the model proposed in this section. The case of allowing the transfer of commodity and income across multiple time periods can be modeled using a similar approach, but is not the focus in our problem context.

In addition to the future commodity supply and investment, two model parameters – population growth rate and depreciation rate – are also used to link the different time periods and address the temporal issues in the dynamic model. The population growth rate determines how fast the final demand side, which is represented by a single household in each region in our model, grows over time. The depreciation rate reflects the generic economic principle that the capital is weighted more for its current value, and its value is discounted at a certain rate when left for future expense. The sensitivity of the proposed dynamic model to these two parameters is examined through numerical experiments in Section 4.2.

The multi-period dynamic MINSCGE model follows a similar structure as the static MINSCGE model introduced in Section 2.2. That is, we consider N geographical regions, $n \in [1, ..., N]$, each hosting an economic system with K infrastructure systems, $k \in [1, ..., K]$ and one representative household, as described in Section 2.2. There is a transport agent in each system that collects the commodity produced by the corresponding producer and distributes it to the other producers and the household for intermediate and final consumptions, respectively, in each region. The equilibrium conditions, including the zero profit, market clearance, income balance, and network flow conservation conditions hold in the dynamic context as well, though their mathematical forms are based on the extension to the dynamic case.

Consider a planning horizon with $\mathcal{T}$ periods, $t \in [0,1,..., \mathcal{T}]$. The producers and households make production and demand decisions over the planning horizon. This leads to the following modeling differences compared to the static MINSCGE model: (i) The decision-making objective: The producers and households optimize their objectives over the entire planning horizon represented by the $\mathcal{T}$ periods, and the value of the objective function in a future period must be converted to the present value to ensure that the comparison is consistent over time. (ii) Commodity and value flows: In the dynamic model, these flows are not necessarily conserved within a single period. They can be transferred to the next period as future supply. (iii) System parameters: Parameters such as the production function, system capacity, consumption preference, and network structure can change from one period to the next.

Similar to the static MINSCGE model, the mechanism of the dynamic multi-period MINSCGE model can be illustrated by the circulation of commodity and payment flows among the regions, as demonstrated by Fig. 3. Without loss of generality, this figure shows a two-region (r to m) case, which can be generalized to multiple regions. Compared to the flow conservation conditions for the static MINSCGE model in Fig. 2, the key changes here are the addition of the "Future Supply" blocks leading out from the producers, implying that some commodities that the producers produce may be left for future consumption instead of being collected by the transport agent in the current period; and the "Investment" boxes leading out from the households,

representing the portion of income that is earned in the current period but is saved and invested in the next period. These changes serve as a key linkage that connects different time periods to reflect the temporal interactions caused by the infrastructure interdependencies.

We begin with the household in each region. In the factor market, an initial endowment of each factor may be available from the previous period:

$$V_{n,t}^{f} - \sum_{j=1}^{K} v_{n,t}^{fj} \geq 0, \forall f \in [1,...,F], n \in [1,...,N], t \in [0,...,\mathcal{T}] \tag{47}$$

The above equation is similar to Equation (1) in the static model, but with the time subscript, implying that the above primary factor supply condition must be satisfied in each period. The value conservation condition (2) can be rewritten as:

$$\varpi_{n,t}^{f}(V_{n,t}^{f} - \sum_{j=1}^{K} v_{n,t}^{fj}) = 0, \forall f \in [1,...,F], n \in [1,...,N], t \in [0,...,\mathcal{T}] \tag{48}$$

The dynamic MINSCGE can explicitly consider the impact of the growth rate of the population in the infrastructure interdependency context. The implications of the population growth are two-fold. First, a larger population implies more supply of primary factors, especially the labor. Second, it typically implies more consumption demand of each commodity from the various infrastructure systems. Depending on the weight that each commodity has in the utility function of the household, the interdependencies among the different systems and the activity level (production level) of each system may have varying impacts on the overall system welfare (the aggregated household utility across different regions) when the population growth rate is considered. In our problem context, the growth rate is modeled through the change in the primary factors over time, starting with the initial endowment at time 0:

$$V_{n,t}^{f} = (1+g)V_{n,t-1}^{f}, \forall f \in [1,...,F], n \in [1,...,N], t \in [1,...,\mathcal{T}] \tag{49}$$

where g is the growth rate of the population. In reality, this rate can be a variable that changes from period to period, and can take different values in different regions and for different primary factors. Without loss of generality, we use a single parameter here for simplicity to articulate the dynamic MINSCGE model.

The expense constraint of the household can be written as:

$$\begin{aligned} &\sum_{f}^{F} \varpi_{n,0}^{f} V_{n,0}^{f} - I_{n,0} - \sum_{k=1}^{K} \rho_{n,0}^{k} d_{n,0}^{k} \geq 0, n \in [1,...,N] \\ &\sum_{f}^{F} \varpi_{n,t}^{f} V_{n,t}^{f} + I_{n,t-1} - I_{n,t} - \sum_{k=1}^{K} \rho_{n,t}^{k} d_{n,0}^{k} \geq 0, n \in [1,...,N], t \in [1,...,\mathcal{T}] \end{aligned} \tag{50}$$

A key change relative to Equation (3) is that a portion of the household's income, $I_{n,t}$, is transferred to the next period for future investment. It corresponds to the "Investment" boxes leading out of the households through the dashed lines in Fig. 3. This future investment strategy represents a decision-making process that seeks to optimize the objective function across the multiple time periods in the planning horizon, and the investment is used link the different periods. Correspondingly, the objective function (4) for the household optimization problem can be converted to:

$$\text{Max} \sum_{t=0}^{\mathcal{T}} u_{n,t}(D_{n,t}) \tag{51}$$

Note that the objective function becomes the linear summation of the utility function over all time periods. That is, the value of the household utility is calculated for each time period, and these values are summed over all time periods. This implies that the utility in each period is treated equally and given the same weight. Similar to the net present value concept in economics, one can also give higher weight to the utility values in more recent periods, but this does not impact the overall structure of the model. The behavior of the household can be specified as the utilization maximization problem (51) subject to the expense constraint (50).

For the producers, due to the commodity flow principle, the supply should satisfy the demands from all regions. This corresponds to Equation (5):

$$\begin{aligned} & s_{r,0}^{i} - \sum_{m=1}^{N} T_{rm,0}^{i} - E_{r,0}^{i} \geq 0, \forall i \in [1,...,K], r \in [1,...,N] \\ & (1-e)E_{r,t-1}^{i} + s_{r,t}^{i} - \sum_{m=1}^{N} T_{rm,t}^{i} - E_{r,t}^{i} \geq 0, \forall i \in [1,...,K], r \in [1,...,N], t \in [1,...,\mathcal{T}] \end{aligned} \tag{52}$$

The variable $E_{r,t}^{i}$ represents the commodity produced in sector *i* of region *r* at time *t* that is kept by the producer as inventory for future supply, which is represented by the "Future Supply" boxes leading out from the various producers in Fig. 3. Due to depreciation on the value of the commodity over time, a depreciation rate *e* is applied when the commodity is produced in one period and supplied in the future. The depreciation can be caused by inflation over time. Another key factor that contributes to the depreciation is the inventory cost. In general, inventory cost can either be explicitly modeled as a monetary expense for each unit of commodity, or can be applied as a percentage discount over the value of the commodity over time. We take the latter approach and model it as a part of the depreciation rate. It should be noted that the inventory cost can vary significantly across different infrastructure systems. For example, the transportation capacity (product) cannot be saved for a future period. Therefore, the commodity from this system is generally 100% perishable. Electric power can be saved in some form with varying costs, while the inventory cost for water and other forms of energy such as fuel and coal is relatively low. Determining these rates requires domain knowledge readily available in each system; however, the exact value is not the focus of this paper. In Section 4.3 we test different values of the depreciation rate primarily to demonstrate the importance of this factor rather than seeking to determine its exact value. Like the growth rate *g*, we assume a single fixed depreciation rate *e* for all systems and time periods here, only for demonstration purposes. Accordingly, the following equation corresponds to the value conservation (similar to Equation (7) in the static context):

$$\begin{aligned} & \pi_{r,t}^{i}(s_{r,0}^{i} - \sum_{m=1}^{N} T_{rm,0}^{i} - E_{r,0}^{i}) = 0, \forall i \in [1,...,K], r \in [1,...,N] \\ & \pi_{r,t}^{i}((1-e)E_{r,t-1}^{i} + s_{r,t}^{i} - \sum_{m=1}^{N} T_{rm,t}^{i} - E_{r,t}^{i}) = 0, \\ & \forall i \in [1,...,K], r \in [1,...,N], t \in [1,...,\mathcal{T}] \end{aligned} \tag{53}$$

The excess demand function (8) can be rewritten as:

$$\sum_{r=1}^{N} T_{rm,t}^{i} - \sum_{j=1}^{K} x_{m,t}^{ij} - d_{m,t}^{i} \geq 0, \ \ \forall i \in [1,...,K], m \in [1,...,N], t \in [0,...,\mathcal{T}] \tag{54}$$

which implies that the total intermediate and final demand of a commodity should not exceed the supply capacity of the local commodity pool. Note that the future supply *E* does not appear in the

above excess demand functions because the supply of a commodity to the intermediate users (producers) and the final users (households) is solely provided by the transport agent. These functions do not differentiate whether the commodity they consume from the local commodity pool (and delivered by the corresponding transport agent) is produced in previous period or the current period. Thereby, the future supply influences only the supply side, and not the demand side.

Based on Walras' law, a commodity in excess supply is free, and positive price implies the total supply meets the total demand. This can be written as the following equation:

$$\rho^i_{m,t}(\sum_{r=1}^{N} T^i_{rm,t} - \sum_{j=1}^{K} x^{ij}_{m,t} - d^i_{m,t}) = 0, \forall i \in [1,...,K], m \in [1,...,N], t \in [0,...,\mathcal{T}] \quad (55)$$

The non-positive profit condition is similar to Equation (11), but with a time subscript:

$$\sum_{i=1}^{K} \rho^i_{n,t}\alpha^{ij}_{n,t} + \sum_{f=1}^{F} \varpi^f_{n,t}\beta^{fj}_{n,t} - \pi^j_{n,t} \geq 0, \forall j \in [1,...,K], n \in [1,...,N], t \in [0,...,\mathcal{T}] \quad (56)$$

The value conservation condition corresponding to Equation (12) similarly becomes:

$$(\sum_{i=1}^{K} \rho^i_{n,t}\alpha^{ij}_{n,t} + \sum_{f=1}^{F} \varpi^f_{n,t}\beta^{fj}_{n,t} - \pi^j_{n,t})s^j_{n,t} = 0, \ \forall j \in [1,...,K], n \in [1,...,N], t \in [0,...,\mathcal{T}] \quad (57)$$

Like the household, the producers maximize their overall profit over time, in the form of net present value of the profit with discount rate q:

$$\text{Max } z^j_n = \sum_{t=0}^{\mathrm{T}} \frac{(\pi^j_{n,t} - \sum_{i=1}^{K} \rho^i_{n,t}\alpha^{ij}_{n,t} - \sum_{f=1}^{F} \varpi^f_{n,t}\beta^{fj}_{n,t})}{(1+q)^t} \quad (58)$$

For the transport agents, the transportation activities cannot be carried over to different time intervals. Therefore, the corresponding equations are similar to those for the static MINSCGE model of Section 2.2, but have a time subscript:

$$\pi^k_{r,t} + \lambda^k_{rm,t} - \rho^k_{m,t} \geq 0, \ \ \forall r,m \in [1,...,N], k \in [1,...,K], t \in [0,...,\mathcal{T}] \quad (59)$$

Similar to Equation (14), we have:

$$(\pi^k_{r,t} + \lambda^k_{rm,t} - \rho^k_{m,t})T^k_{rm,t} = 0, \ \ \forall r,m \in [1,...,N], k \in [1,...,K], t \in [0,...,\mathcal{T}] \quad (60)$$

Since $\lambda^k_{rm,t}$ is the minimum path cost, we have:

$$C^k_{p,t} - \lambda^k_{rm,t} \geq 0, \ \forall r,m \in [1,...,N], k \in [1,...,K], p \in P_{rm}, t \in [0,...,\mathcal{T}] \quad (61)$$

Similar to Equation (16), we obtain:

$$(C^k_{p,t} - \lambda^k_{rm,t})h^k_{p,t} = 0, \ \forall r,m \in [1,...,N], k \in [1,...,K], p \in P_{rm}, t \in [0,...,\mathcal{T}] \quad (62)$$

In summary, the above commodity flow and value flow conservation conditions are used to formulate the dynamic multi-region infrastructure interdependencies problem as a game that involves three types of players (the producers, the households, and the transport agents) that make their decisions simultaneously aiming to optimize their objective functions over the multiple time periods of the planning horizon. The key differences between this dynamic model and the static MINSCGE model introduced in Section 2.2 are that the household may keep some of its income from one time period to invest in the next period, and the producers may keep some amount of their commodities produced in one period to be sold in the next period. Thereby, they

use this future investment or inventory mechanism to adjust the future demand or supply levels, so as to optimize their respective objectives over the multiple time periods of the planning horizon.

When implementing and solving the dynamic MINSCGE model, we can use the same approach as used for the static model (as described in Section 2.2). The additional equations involving the future investment and supply need to be rewritten to reflect the temporal linkages of the corresponding variables, and the model is solved iteratively for all time periods. Similar to the static model, the dynamic model can be coded in the GAMS package, and solved using the non-linear programming solver available in GAMS.

In the numerical experiments illustrated in Section 4, only the multi-period dynamic model is used for the following reasons. First, the determination of the adjustment parameters such as $\varepsilon 1$, $\varepsilon 2$ and $\varepsilon 3$ that represent the adjusting behavior of the various players requires system-specific understanding of the production and consumption preferences in each system, which is beyond the scope of this paper. Second, since this paper mainly focuses on the analysis of infrastructure interdependencies and their impacts on the planning phase of these systems, as illustrated by Fig. 1, the multi-period dynamic MINSCGE model better fits the demonstration purpose due to its consideration of system configuration changes, as discussed in the beginning of this section. Third, most of the behavioral adjusting characteristics of the within-period dynamic model are similar to those of the single-system dynamic models, many of which are listed in Section 2.4. Extensive results and insights are available in the single-system contexts, therefore the value for including within-period experiments is not as significant compared to the multi-period examples. Of course, numerical examples involving the within-period model are nevertheless valuable in better understanding the implications of infrastructure interdependencies in a shorter time scale, and represent future work when appropriate data for the adjustment parameters is available.

4. Numerical experiments

This section presents the results of some numerical experiments based on the dynamic MINSCGE model proposed in Section 4.2. Real-world data is used to calibrate the model and illustrate that the proposed framework can be implemented to solve real-world application problems. The primary objectives of these experiments are to: (i) test the model's ability to capture infrastructure interdependencies over time in a dynamic setting, (ii) use the dynamic model to represent and analyze the impact of disruption scenarios in multiple infrastructure systems, and (iii) demonstrate the importance of holistic consideration of multiple systems over time in dynamic decision-making problems.

As discussed in Section 2.2, the implementation of a spatial model requires expertise on system-specific characteristics such as the transportation cost functions. To focus on the primary objective of this study, which is to analyze the impact of interdependencies among infrastructure systems in the dynamic context, it is sufficient to use a special case of the dynamic MINSCGE model that considers only one region. Without loss of generality, we present the single-region experiments here to highlight the key insights related to the dynamics of interdependencies among infrastructure systems. The extension to perform experiments involving multiple regions would use a similar approach, but with the additional network flow conservation constraints and additional data to calibrate the related parameters.

4.1. Experiment setup

Four infrastructure systems including transportation (TRA), telecommunication (TEL), energy (NRG) and power (PWR) are considered in the experiments. The production functions of the representative producers and the utility function of the representative household in the region take the NCES form introduced in Section 2.2. That is, these two-tier functions first consider the combination of PWR and NRG commodity demand to propose a "virtual commodity" *NP* from the consumption of these two commodities in the second tier. Similarly, the combination of TRA and TEL service demand is used to propose the virtual commodity *TT* in the second tier. Then, the overall production or utility levels are determined by the combination of these two combined virtual commodities in the first tier.

The dynamic MINSCGE model is calibrated using the national Social Accounting Matrix (SAM) data available from the Bureau of Economic Analysis of the U.S. Department of Commerce, as summarized in Table 1. Two types of primary factors are considered: labor and capital. The values of these two parameters are also calculated from the SAM data. The model is implemented in GAMS using the MPSGE (Mathematical Programming System for General Equilibrium) interface and the MCP (Mixed Complementarity Problem) solver (Rutherford, 1999).

When evaluating the results of the various experiments, the activity level in each system (that is, the production level of the associated producer) is measured as a proxy for the performance of that system, and in some experiments the household utility is used as a measure of the overall system performance.

4.2. Sensitivity analyses

The values of the model parameters determine the model realism and its ability to capture the dynamic impacts of infrastructure interdependencies. Therefore, we conduct a series of experiments to test the sensitivity of the model to two key parameters: depreciation rate e, and growth rate g. The discount rate q in the net present value calculation in Equation 58 is well-studied in economics, finance, capital budgeting and other problem domains. Hence, we will not focus on this parameter and it is set to 0 in all numerical experiments. Fig. 4 illustrates the sensitivity of the dynamic MINSCGE model to the depreciation rate e. As discussed in Section 3, this parameter represents the depreciation level of capital in the future compared to its present value. In this experiment, the capacity of the energy system is restricted to a percentage ranging from 60% to 100% of its original level, incrementally increased by 10% in each period from period 1 through period 6, while other systems preserve their original capacities. The trajectories of the activity levels in these systems are plotted in Fig. 4(a) through Fig. 4(d) with e values ranging from 0.01 to 0.04, respectively, for a population growth rate g of 0.02. The overall trend is that at the beginning of the time period, the production level of the electricity system increases beyond 1 due to the complementary substitution relationship between the NRG and PWR systems. As NRG is a key input, the TRA system production level drops dramatically in the beginning due to the shortage of NRG input, while the output of TEL system surges as it partly substitutes the TRA service through telecommuting. These phenomena are consistent with the insights for the static case (Zhang and Peeta, 2011). The new insights in these experiments are the trajectories by which the different systems converge to a stable state over time. As seen in

Fig. 4, the production level of all systems converges faster when the *e* value is lower. This is because when the depreciation rate is lower, the purchasing power of the capital accumulated from the previous time period is higher in each period, and more input can be obtained, which helps the faster recovery of the activity level. However, Fig. 4(b) through Fig. 4(d) suggest that the model is not very sensitive to this parameter once *e* is larger than a certain value (0.02 in this case). Fig. 5 shows the sensitivity of the model to the growth rate parameter *g*. As discussed in Section 3, this parameter represents how fast the population grows. The results indicate that the model is sensitive to the growth rate *g*. The impact of this parameter is two-fold: with higher population, the supply of labor is higher, and the demand for the various commodities is also higher. In this set of experiments, the depreciation rate *e* is fixed at 0.05, and the growth rate is varied from 0.01 to 0.04. Fig. 5(a) through Fig. 5(c) have similar trends as in Fig. 4, and the productivity levels of all systems eventually merge to a common value. However, as *g* increases beyond a certain value, this trend does not apply as illustrated by Fig. 5(d). This is because the increased population provides additional inputs to all systems, helping mitigate the lack of input from specific systems. Further, the growing population also stimulates higher utilization of production capacity in all systems to meet the demands from both households and other infrastructure systems. Therefore, Fig. 5(d) displays a nearly parallel increase of productivity level in all systems until the capacity limit of a system, TEL in this case, is reached.

4.3. Dynamic parameters for interdependency relationships

As discussed in Section 1, a key limitation of static equilibrium models is that the values of the model parameters are assumed to be fixed over the period of analysis. This issue can be addressed by the dynamic model. We illustrate this by varying the values of some system parameters related to the interdependency relationships (Equations 17 and 18) in different time periods. The depreciation rate *e* of 0.05 and population growth rate *g* of 0.02 are used in these experiments.

First, we test the impact of EOS on the performance of the various systems. In Fig. 6, we vary the values of the two first-tier EOS parameters, μ_1 in Equation 17 and υ_1 in Equation 18, from 0 to 1 with a 0.2 step size increment, and illustrate their influence on the activity levels of TRA, TEL and PWR systems over time while the capacity of NRG system changes from 0.5 to 1 with incremental improvement of 0.1 from period 1 to period 6, simulating that there is a disruption in this system in the initial stage and that the system gradually recovers over time. Fig. 6(a) indicates that the EOS value has a consistent impact on the productivity on the TRA system. That is, the higher the EOS value, the less the TRA output level is influenced by the disruption in the NRG system. This is because based on the SAM data, TRA relies more heavily on the NRG system compared to other sectors. Higher EOS implies that the loss in NRG supply can be replaced by other commodities more easily. However, this trend is less apparent for other systems. For example, Fig. 6(b) shows that in period 1, TEL has the highest activity level under the lowest EOS value. This is because the TRA system has the lowest activity level under this EOS value as explained previously, and more TEL supply is needed due to the substitution relationship between TRA and TEL. As time progresses, the supply of NRG and TRA systems eventually recovers, demanding less TEL service until period 4. After that, the demand for TEL rises again due to the growing population and increasing intermediate demands from other systems as their activity levels pick up. At the lower EOS value, the substitution relationship between the TRA and TEL systems is not strong enough to push the activity level of TEL system

up during the initial periods, so the TRA activity level increases consistently over time as the final and intermediate demands go up with the recovery of NRG. A similar conclusion can be obtained for the PWR activity level, as shown by Fig. 6(c).

In the above experiment, the EOS is still fixed over time, but different EOS values are used in the different cases. Additional experiments are conducted to illustrate that the proposed dynamic MINSCGE model also enables the use of dynamic EOS values that vary in different time periods. Fig. 7(a) represents a fixed-parameter benchmark case where the NRG capacity recovers from 0.5 to 1 over time after a disruption (which is similar to the previous experiment) and the supply levels of the other systems change in response to it. The two first-tier EOS parameters are fixed to a 0 value in this benchmark case. As illustrated by the figure, the TRA activity level first drops due to its dependency on the NRG system and rises over time with the recovery of NRG system. The activity level of PWR and TEL systems are higher in the initial stages (the first two periods) due to the substitution relation with the NRG and TRA systems, respectively. However, these activity levels drop over time (the middle two periods) as the NRG and TRA systems recover. Later, the PWR and TEL activity levels increase again (in the last two periods) due to the increased demand due to the overall recovery of all systems as well as the increase in population. By comparison, in Fig. 7(b) the EOS values change from 0 to 1 over time with an increment of 0.2 in each period. Compared with Fig. 7(a), the recovery of TRA activity level and the return of the PWR and TEL activity to their normal levels are faster, suggesting less impact of the NRG disruption on other systems. Adjusting the EOS value can be thought of as improving the production technology in the various systems leading to less dependency on the availability of the NRG commodity. Therefore, when the supply of NRG is limited, a higher EOS value implies that it is easier to substitute the lost NRG supply using other commodities, thereby limiting the impact of the reduction in the NRG commodity.

4.4. Recovery of system activity after disruption

Next, we illustrate the dynamic MINSCGE model's capability to capture infrastructure interdependencies under the dynamic setting by simulating the cascading effects initiated by the disruption in one system. Suppose a disruption occurs in the NRG system in the initial period, and the system is recovered eventually in the following periods. This can be simulated by reducing the production capacity of the NRG sector while keeping the capacity of other systems unconstrained. The depreciation rate e of 0.05 and population growth rate g of 0.02 are used in these experiments.

The experiment simulates the case when the capacity of NRG drops to 60% of the original equilibrium output level in the first time period, and then eventually recovers to 110% of the pre-disruption output level, improving 10% of its original capacity in each time step. Therefore, the capacity of NRG is 0.6 in period 1, 0.7 in period 2, 0.8 in period 3, and so on. Fig. 8 illustrates the propagation of the disruption from the NRG system to the other systems. The dashed line (labeled as "FIX") connected by the solid black circles represents the benchmark capacity of all systems under normal conditions with an annual population growth rate of 0.02. When the capacity of the NRG system drops from 1 to 0.6 in period 1, the activity level in the TRA system drops significantly. This is because the TRA system uses a large amount of the NRG commodity as intermediate input due to the functional and economic interdependencies. The activity level in PWR sector rises because there is a substitution relationship between the PWR and NRG systems. When the capacity of the NRG system reduces, the demand for the

PWR commodity increases in order to replace the lost NRG supply, and there is no capacity constraint on PWR at this point. Therefore, the supply of PWR commodity goes up. The supply level of TEL system also rises because there is a substitution relationship between TEL and TRA. When the capacity of TRA is reduced, more TEL supply is needed to replace the lost TRA supply. In the dynamic context, as the capacity of the NRG system is being recovered over time, the activity levels of the various systems converge to their normal levels.

Fig. 9 shows the cascading effects when multiple systems are impacted by an event. In many events such as earthquakes, hurricanes, and terror attacks, the capacities of several systems can be affected simultaneously. This may potentially be due a physical interdependency as the systems may be co-located together. Again, the dashed "FIX" line is plotted as a benchmark to show how each system should progress under normal circumstances compared to the changes in all the systems with the tested disruption. In this example, the capacities of the TRA and NRG systems are reduced at the same time. The capacity of TRA stays at 0.7 across all periods, while the capacity of NRG recovers from 0.6 to 1.1 over time as in the previous case. In the beginning the activity level of TRA drops below 0.4. This means that the capacity reduction of a system is not only determined by its own capacity constraint, but also is influenced by the availability of other interdependent systems. The activity level of TEL initially drops as TRA improves, because the two systems have substitution relationship. It then rises as TRA reaches its fixed limit 0.7, and the supply of TEL keeps advancing thereafter as a result of both the population growth and the increasing demand due to the improving NRG system, requiring more TEL commodity as its input.

Infrastructure interdependencies also have significant impact on the long-term planning of the various systems. Fig. 10 demonstrates that the lag of one system may restrict the capacity improvement of other systems. Assume that investment and population growth allows a 2% improvement year over year for all systems except for the TRA system, which has a constant activity level of 1 over time. Two scenarios are tested and correspond to high and low EOS values, respectively. As discussed in Section 2.2, the level of EOS determines how easily a system can be replaced by other systems that have a substitution relationship with that commodity. Under the high EOS scenario, illustrated in Fig. 10(a), an EOS value of 1 is assumed for the first tier. The result indicates that though the investment allows a 2% annual improvement (as represented by the "FIX" line), both NRG and PWR systems actually have a lower capacity improvement due to the capacity restriction in the TRA system. The only exception is the TEL system, which has a substitution relationship with TRA and grows at a slightly higher rate. Similar to Fig. 8, more TEL supply is needed to supplement the loss of TRA supply. In the low EOS scenario, as illustrated in Fig. 10(b), an EOS of 0.1 is used. Here, all systems including TEL develop at rates lower than 2%. This is because the substitution relationship between TRA and TEL is too weak to enable the TEL system to grow at a higher rate.

4.5. Dynamic decision-making

The experiments discussed so far demonstrate that the proposed dynamic MINSCGE is able to capture the interdependencies among infrastructure systems in a dynamic setting. Though these descriptive capabilities are important in understanding the nature and implications of these interdependencies, a normative/prescriptive perspective is necessary to provide more insights under both disruption and business-as-usual scenarios for effective, robust, and holistic decision-

making. The depreciation rate e of 0.05 and population growth rate g of 0.02 are used in the experiments below.

Fig. 11 represents a scenario when a recovery strategy needs to be determined to recover the overall system capacity after a disaster that reduces the capacity of each infrastructure system to 60% of the original level. In the figure, Strategy 1 shows the improvement of the overall household utility level when each system is improved by 10% of its original capacity in each time period. This represents a balanced recovery strategy. Strategy 2 represents the case where only one system is fully recovered to its original capacity in each period while other systems are left unchanged as in the previous period. That is, the TRA system is recovered to its full capacity while other systems are left to 60% in period 2; then TEL is recovered to full capacity while PWR and NRG systems are still constrained to 60% of their capacity (TRA has already been recovered in the previous period; therefore it retains its full capacity) in period 3; then all systems except NRG are recovered in period 4; and finally all systems are recovered to full capacity in the last period. Strategy 3 shows the household utility level when the PWR, NRG, TRA and TEL systems are recovered in that sequence in the different periods, one at a time. The latter two cases represent unbalanced or uncoordinated recovery of the different infrastructure systems. Though all three strategies imply the same amount of recovery level ($4\times0.1\times4=1.6$ in Strategy 1; and $1\times0.4\times4=1.6$ in Strategies 2 and 3) over all periods, the figure clearly illustrates that the balanced recovery strategy generates higher level of household utility. It represents a faster and more efficient strategy as it avoids the bottlenecks created by some systems that are not restored on time while other systems are fully restored.

5. Concluding comments

The dynamic analysis of infrastructure interdependencies is necessary to understand how the interdependencies influence the system status over time. Such an understanding provides insights that enable more holistic, robust and efficient decision-making for long-term planning and disruption recovery problems involving multiple infrastructure systems. Using an economic market-based approach (the SCGE theory) and a multilayer infrastructure network (MIN) concept, this paper proposes a VI form of the static MINSCGE model developed by Zhang and Peeta (2011), which serves as the basis for a within-period equilibrium-tending dynamic model to capture how the system status adjusts over time when the system configurations do not change within a relatively short period of time. It also proposes a multi-period dynamic MINSCGE model that explicitly considers the temporal interactions of the various infrastructure systems and key real-world parameters such as the depreciation and population growth rates. The use of a market-based approach and principles from economics circumvents the modeling complexities caused by the disparate physical, operational and institutional characteristics of the individual infrastructure systems and enables the modeling of all systems on the same platform.

Numerical experiments are conducted using the multi-period dynamic MINSCGE model to illustrate the ability of the proposed model in capturing infrastructure interdependencies in the dynamic and disequilibrium contexts, and to demonstrate the potential application of such models for practical decision-making problems. Sensitivity analysis is also conducted to test the impact of key parameters on the performance of the model. The numerical results illustrate that the model is sensitive to the depreciation and population growth rates, both of which are important when addressing the infrastructure interdependencies problem in a long-term dynamic

setting. It should be noted here that the trends from the numerical results are primarily to illustrate the ability of the model to provide qualitative practical insights. The different combinations of the depreciation/growth parameters as well as other parameters such as the EOS and the substitution structure of the models may also influence the actual outcomes for a specific real-world problem scenario.

In general, the numerical results from the experiments suggest that: (i) the dynamic MINSCGE model can be used to capture the infrastructure interdependencies under a dynamic setting; (ii) disruption in a single system may demand more supply from other systems over time due to complementary interdependency, but may also reduce the production capability of other systems due to input-output interdependency; which effect dominates the final impact on each individual system activity level depends on the specific relationships between each pair of infrastructure systems, and such relations can be conveniently captured using the SCGE structure and the NCES functions proposed in this study; (iii) holistic consideration of multiple systems is critical in decision-making under both disruption and business-as-usual scenarios; and (iv) a dynamic perspective considering the impacts of various dynamic factors such as population growth rate, interest rate and the change of system configurations enable a more holistic view not only over multiple systems, but also over time.

Additional insights on the dynamics of infrastructure interdependencies and the capabilities of the proposed dynamic model entail more elaborate experiments; the authors do so elsewhere (Zhang and Peeta, 2013). Also, since this paper primarily focuses on illustrating the interdependencies among different infrastructure systems in a dynamic context, the numerical experiments do not address the interactions among the different components of the same system in different regions, though this capability is provided by the dynamic MINSCGE model. In addition, the numerical experiments provided in the paper only involve the multi-period dynamic model. Future research efforts include exploring the spatial effects through numerical experiments by using domain-specific data (such as transport cost functions) for the individual infrastructure systems, as well as in the within-period scenario.

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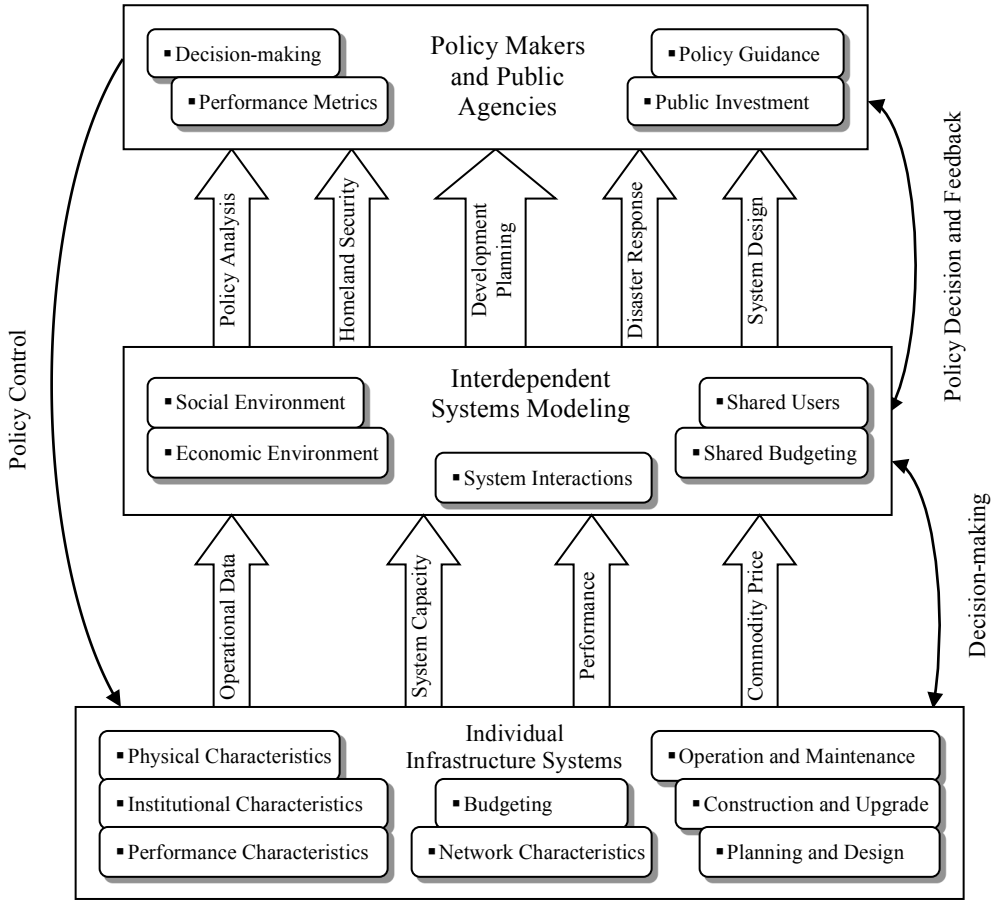

Fig. 1. Infrastructure interdependency analysis in the real-world context.

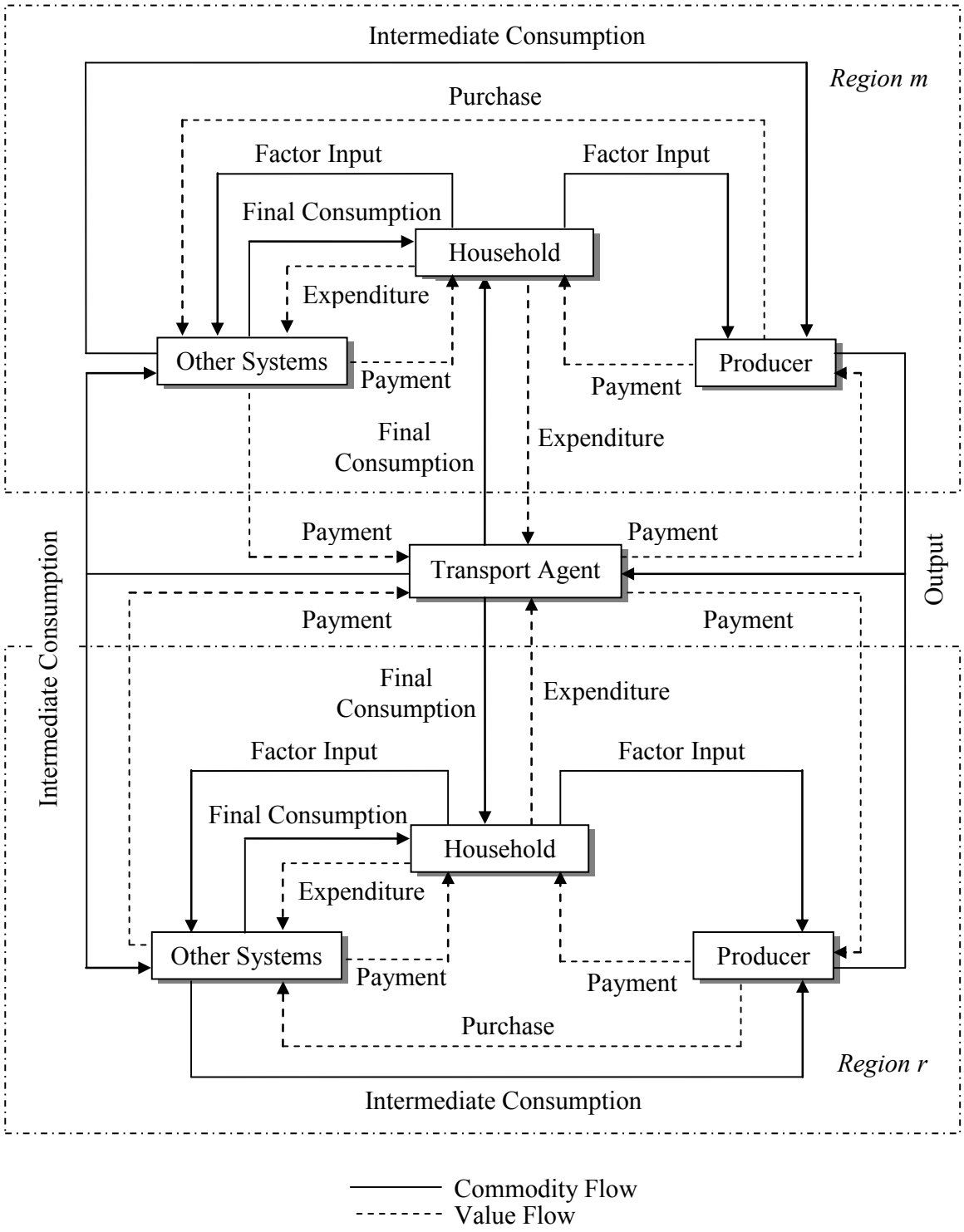

Fig. 2. Circulation of commodity and value flows in the static SCGE model (shown here for a two-region case).

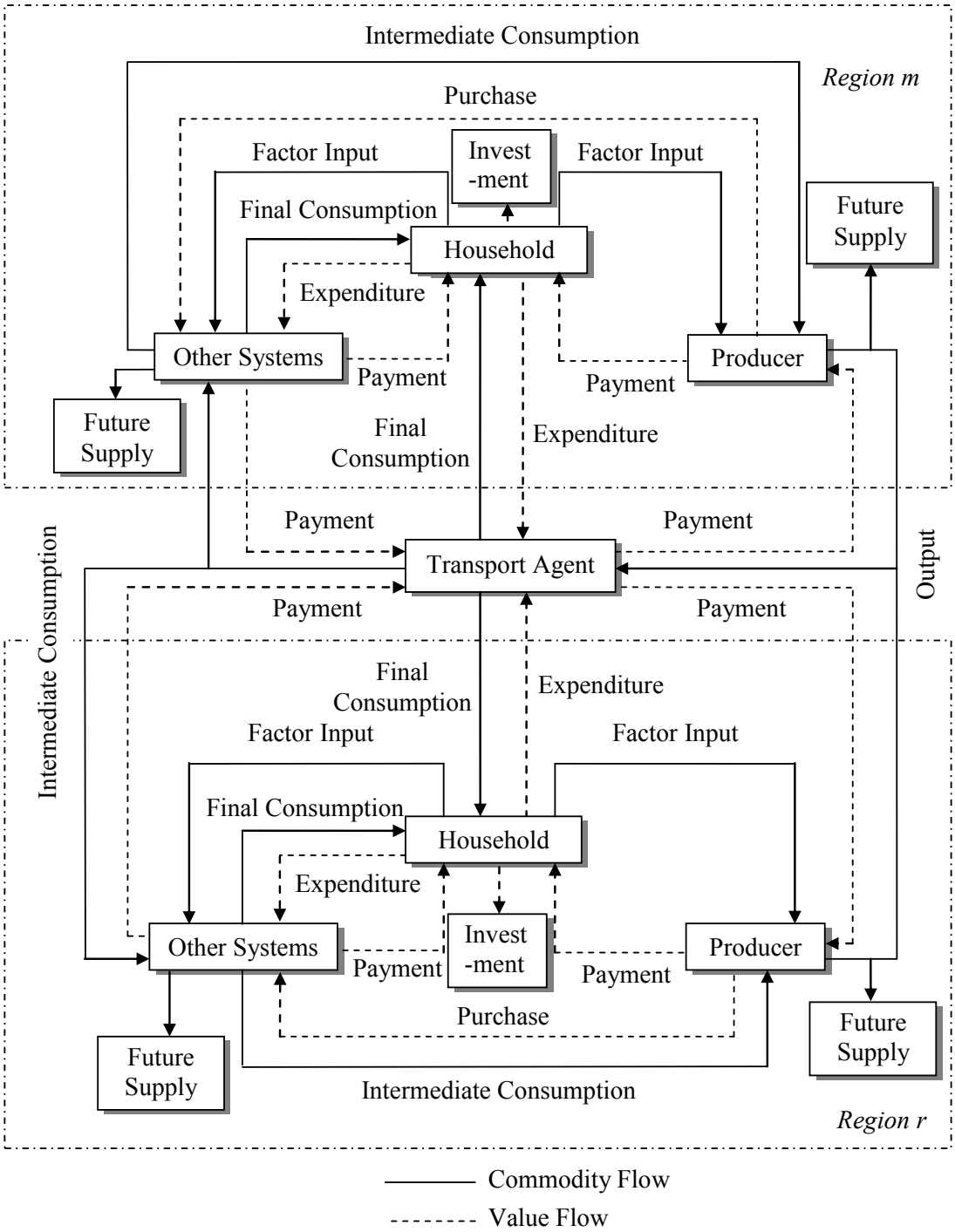

Fig. 3. Circulation of commodity and value flows in the dynamic SCGE model (shown here for a two-region case).

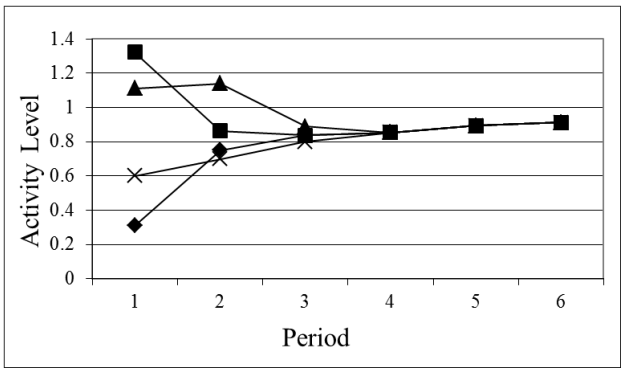

Fig. 4(a). $e = 0.01$.

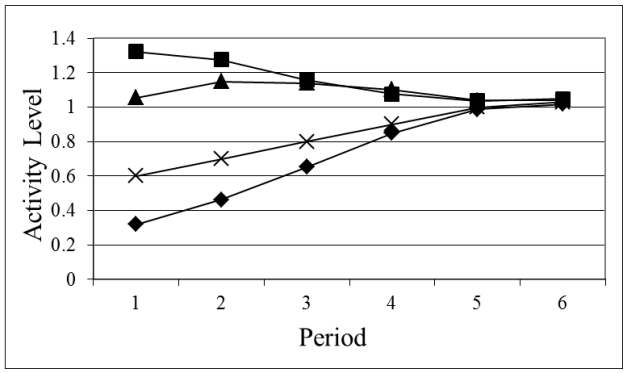

Fig. 4(b). $e = 0.02$.

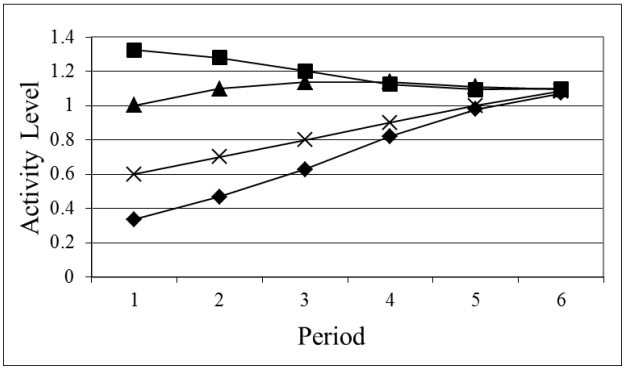

Fig. 4(c). $e = 0.03$.

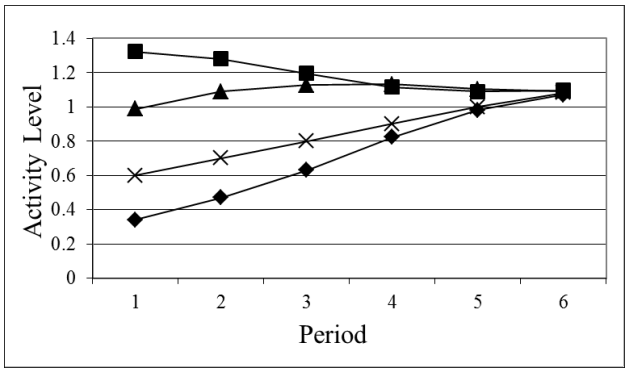

Fig. 4(d). $e = 0.04$.

Fig. 4. Sensitivity analysis for depreciation rate *e*.

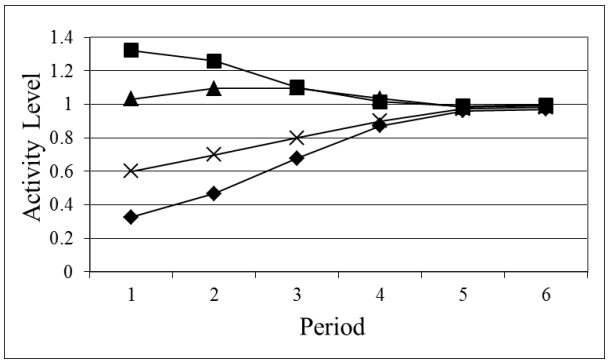

Fig. 5(a). $g = 0.01$.

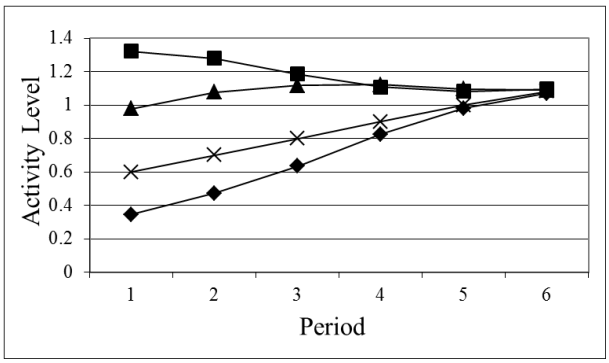

Fig. 5(b). $g = 0.02$.

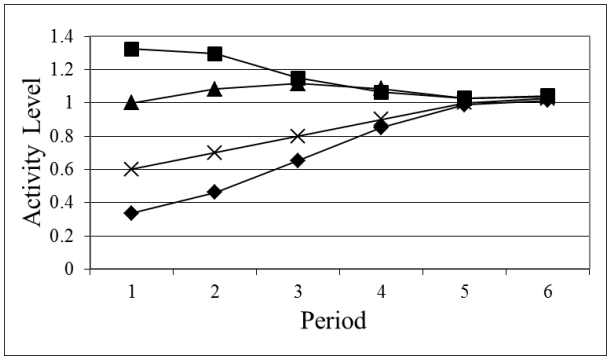

Fig. 5(c). $g = 0.03$.

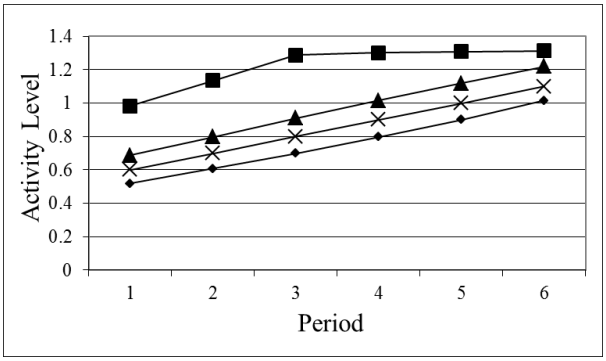

Fig. 5(d). $g = 0.04$.

Fig. 5. Sensitivity analysis for growth rate g.

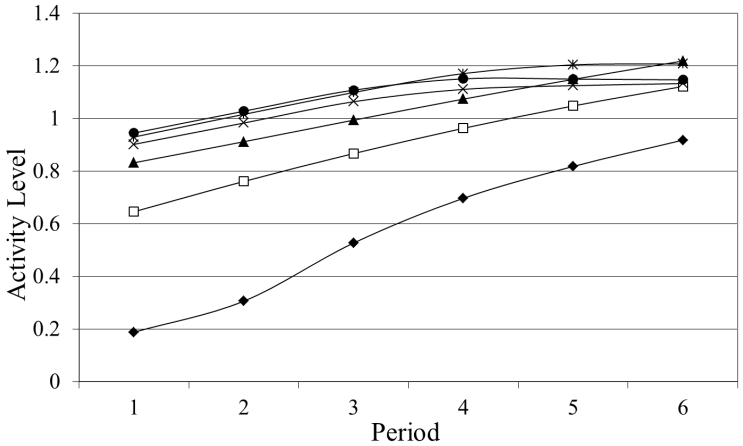

Fig. 6(a). TRA activity level.

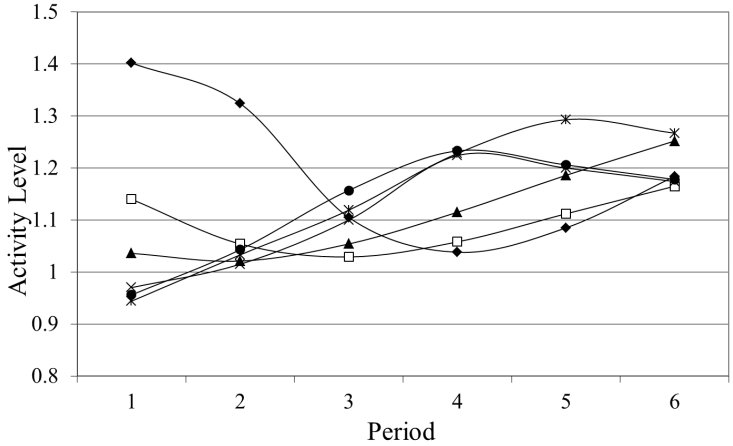

Fig. 6(b). TEL activity level.

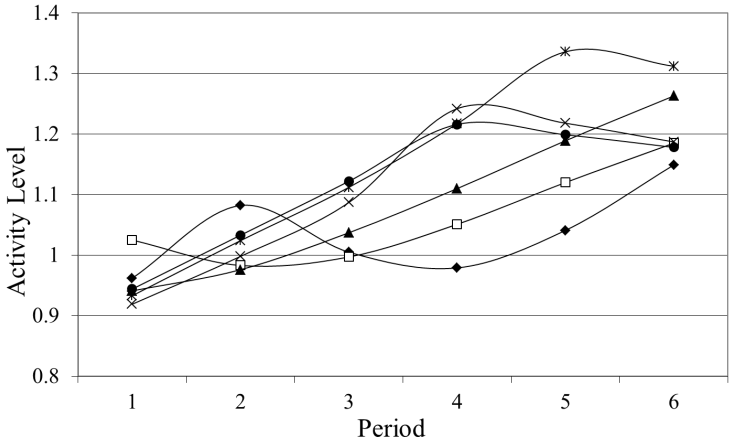

Fig. 6(c). PWR activity level.

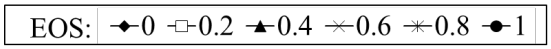

Fig. 6. The impact of EOS on the activity level over time.

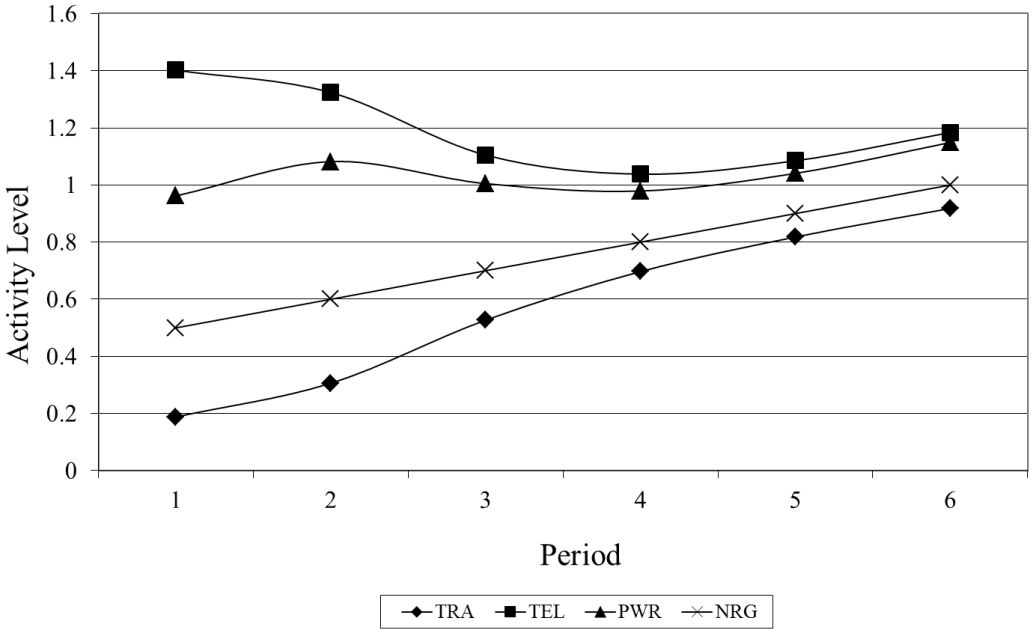

Fig. 7(a). Evolution of activity level under static EOS value.

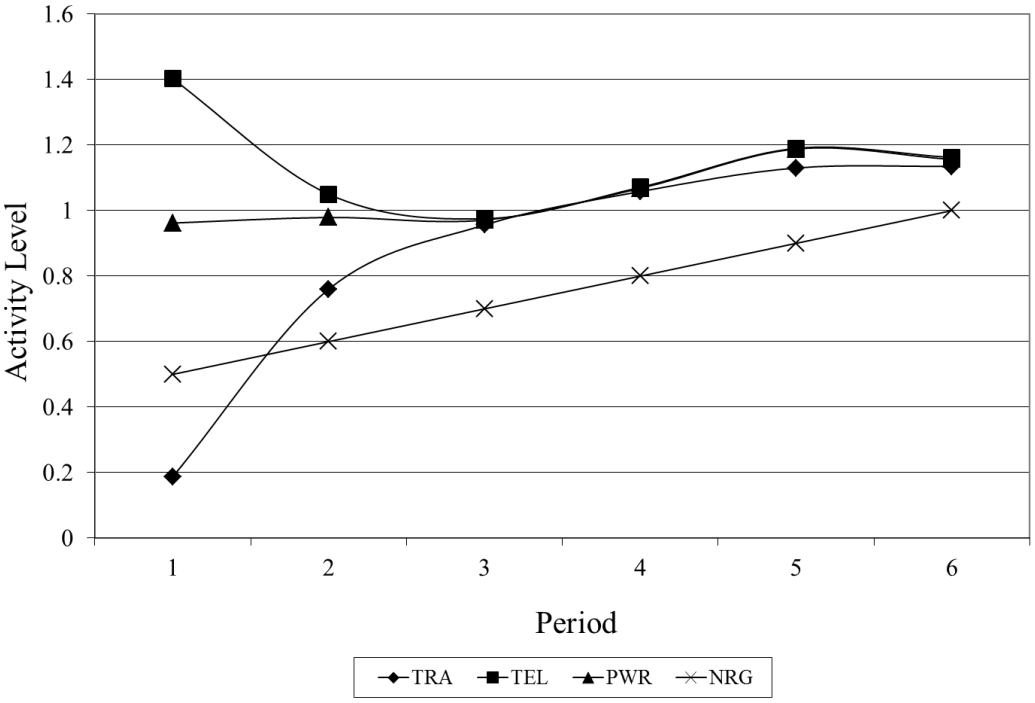

Fig. 7(b). Evolution of activity level under dynamic EOS values.

Fig. 7. System evolution under static and dynamic EOS values.

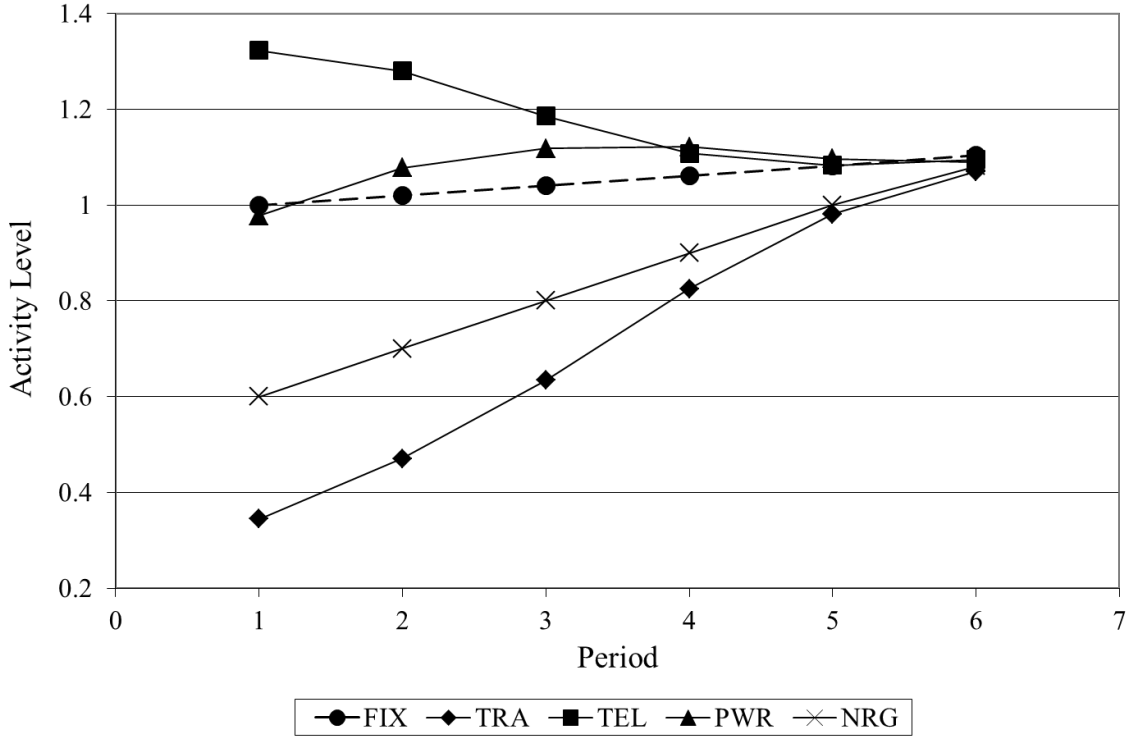

Fig. 8. Impact of reduction in NRG sector capacity on activity level of other systems over time.

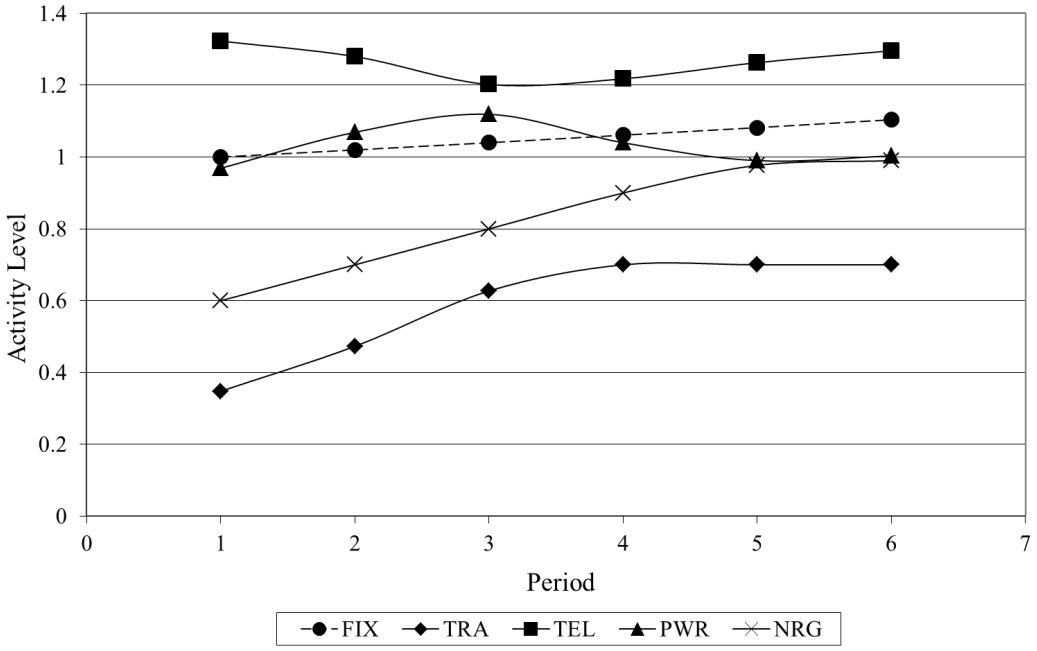

Fig. 9. Impact of disruption in NRG and TRA systems on activity level of other systems over time.

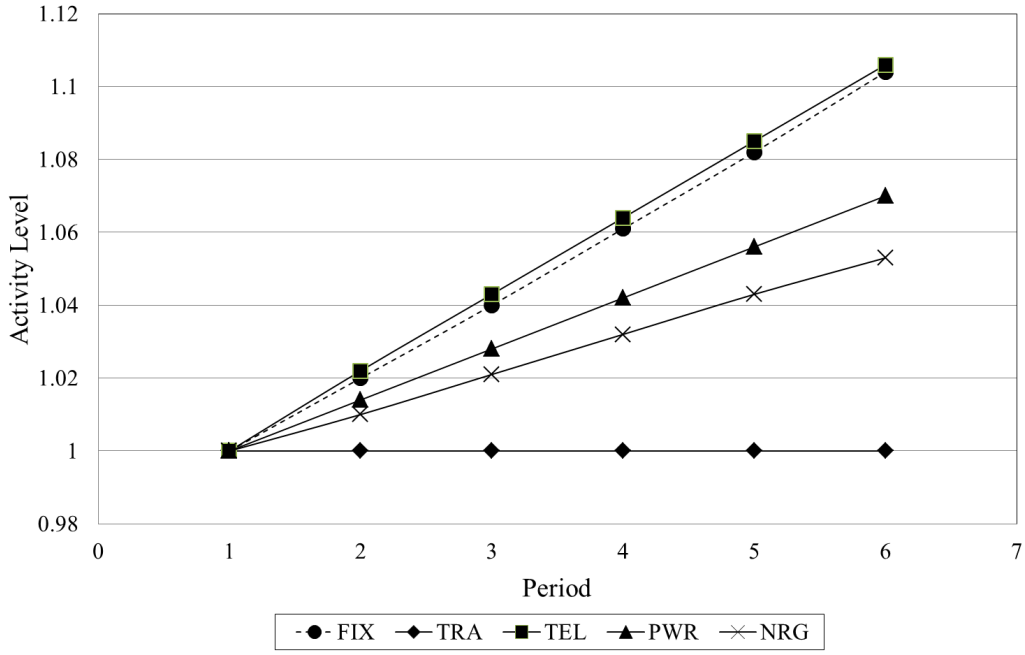

Fig. 10(a). System development over time, high EOS (EOS = 1).

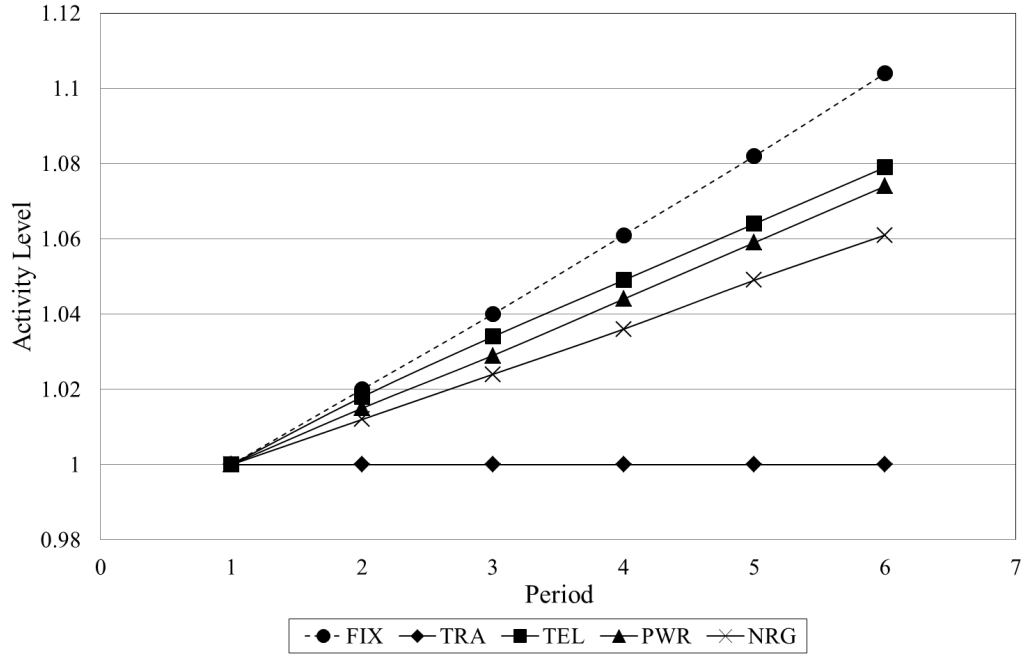

Fig. 10(b). System development over time, low EOS (EOS=0.1).

Fig. 10. System development under different EOS levels.

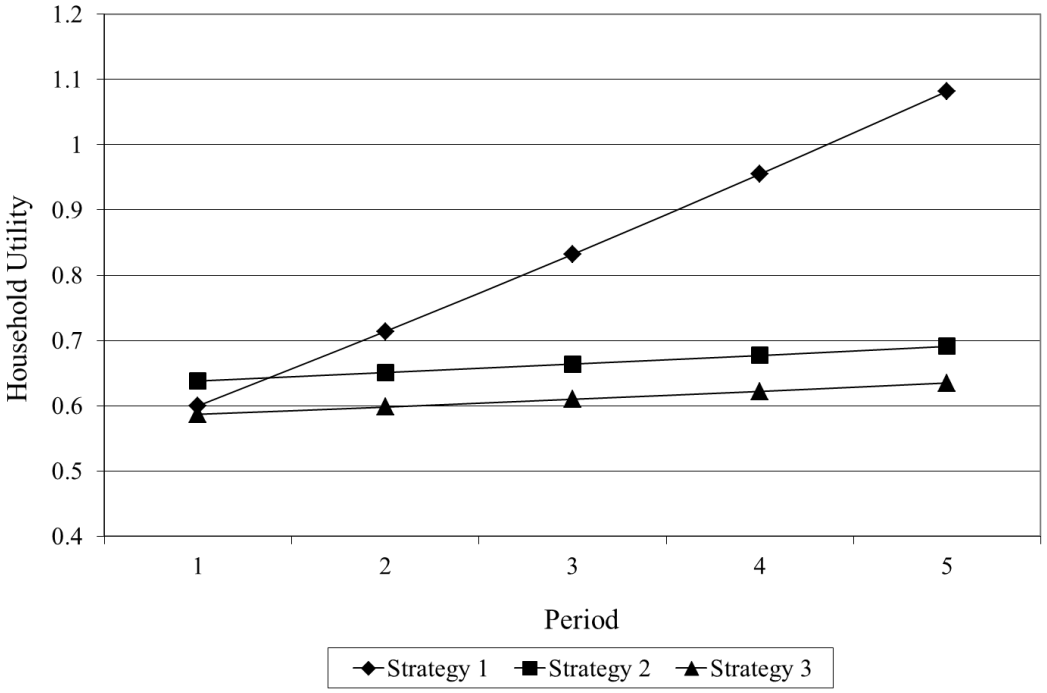

Fig. 11. Impact of recovery strategies on household utility.

Table 1

SAM Calibration Data for the Numerical Experiments

Systems	NRG	PWR	TRA	TEL	Intermediate Demand	Final Demand	Total Demand
NRG	356404	115954	70468.6	2245.1	545071.2	134508	679579
PWR	5045.4	163.4	2188.4	8328	15725.2	377032	392757
TRA	17264.1	28108.9	77655.1	13630.7	136658.8	342111	478770
TEL	4074.5	538.1	16519.4	367853	388984.9	834348	1223333
Labor	63603.6	55119.5	199263	486799			804784
Capital	233188	192873	112675	344478			883214
Total	679579	392757	478770	1223333		1687998	4462436

* Unit: Million US Dollars

** Data Source: 2005 National Social Accounting Matrix (SAM), Bureau of Economic Analysis, U.S. Department of Commerce, http://www.bea.gov/