Information-based network control strategies consistent with estimated driver behavior

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Abstract

This study proposes a fuzzy control based methodology to determine information-based network control strategies that are consistent with the controller's objectives and its estimation of driver response behavior. It is the core of the broader problem where the objective is to enhance the performance of a vehicular traffic system through real-time information-based network control strategies. The controller seeks behavior consistency by solving a fixed-point problem that estimates drivers' likely reactions to the controller-proposed information strategies while determining them. Experiments are performed to evaluate the effectiveness of the proposed methodology. The results suggest the importance of using a behavior-consistent approach to determine the information-based network control strategies. That is, the effects of driver response behavior to information provision may require more meaningful strategies than those provided under the traditional dynamic traffic assignment models to reliably estimate or control system performance.

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Keywords: driver route choice; fuzzy control based optimization; information-based control; behavior consistency.

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1. Introduction

Deployment effectiveness of information-based network control strategies in congested vehicular traffic networks entails the robust modeling of traffic flow dynamics and driver behavior. Existing approaches, broadly addressed under the umbrella of dynamic traffic assignment (DTA), primarily focus on robustly capturing traffic flow dynamics (Peeta and Ziliaskopoulos, 2001). However, their driver behavioral assumptions can be restrictive for real-time deployment (Peeta and Yu, 2006). This motivates the development of a route guidance paradigm that integrates information-based network control strategies and realistic driver behavior representation.

Driver behavior is a fundamental factor and a key source of complexity in predicting traffic network states unfolding over time. However, most DTA models are based on a rigid framework; they either pre-specify behavior of drivers and/or assume rigid compliance characteristics. Few DTA models consider heterogeneity among drivers. Even these models assume that driver behavior classes can be pre-specified. In addition, they assume *a priori* knowledge of the driver behavior class fractions in the ambient traffic stream. This rigidity raises issues related to the realistic modeling of the driver behavior and consequently, of the effectiveness of the information-based network control strategies. A detailed discussion of the behavioral limitations of DTA models is presented in Peeta and Yu, (2006).

Incorrect prediction of traffic system states based on the aforementioned assumptions can negatively impact the validity and effectiveness of the information-based network control strategies and potentially deteriorate system performance. In reality, driver route choice decisions, even under information provision, are based on the driver's innate behavioral tendencies, past experience, situational factors (such as time-of-day, weather conditions, and trip purpose), and the ambient traffic conditions encountered (Peeta and Yu, 2004). This is true irrespective of the type of information, the strategy used to deploy it, or whether drivers receive no information.

While information provision and content can be used as control variables to influence system performance, they cannot imply perfect or pre-specified rates of compliance by the drivers to the supplied information, as is predominantly done in the DTA arena. From the traffic controller perspective, providing personalized, generic or class-specific information based on a better understanding of driver response tendencies and ambient traffic conditions could generate a more effective control paradigm. It would determine what information to provide to whom, based on the system controller objectives and the controller's estimation of the driver behavior.

This study is motivated by the issues raised heretofore that reflect a practical need: how do we bridge the realism gap between existing DTA models proposed for network-level deployment and the need to incorporate driver response behavior adequately while reconciling them with reasonable expectations in terms of data availability? We propose to address this through a conceptual extension of the traditional DTA-based approach, labeled behavior-consistent realtime traffic routing. Behavior-consistent information-based control strategies imply that the likely (controller-estimated) response behavior of drivers to these strategies is explicitly factored in determining them. The use of a controller-estimated driver response behavior model in this study is a formal recognition of the limitations on data availability in a deployment context. Ideally, the controller would like to have full knowledge of each driver's behavior. But, this may not be practically possible for a variety of reasons. However, the controller can construct an estimated model of driver behavior based on historical data, field sensor data, surveys, and insights from past behavioral studies. Such a model can further be fine-tuned over time.

Existing DTA models solve for some controller objective(s) under pre-specified driver response behavior characteristics and use the resulting route assignment proportions directly as the information-based control strategies to be deployed (Bottom, 2000; Peeta and Yu, 2006). By contrast, since the behavior-consistent approach accounts for the likely driver response in determining the control strategies, the associated route assignment proportions recommended to drivers are different than those based on the specific DTA objective. For example, the DTA approach for a system optimal (SO) objective will use the SO route proportions "as is" to provide route recommendations to drivers. Under the behavior-consistent approach, the SO route proportions tend to become the controller's goal, and the controller recommends, based on an estimated driver behavior model, more or less proportions of drivers to take specific routes so as to approach as close as possible to the SO proportions. This implies a fixed-point problem where the information-based control strategies depend on the estimated driver response and vice versa. Fig. 1 conceptually shows the traditional DTA-based and the proposed behavior-consistent approaches. Fig. 1(a) indicates that the traditional DTA-based approaches use the DTA solutions directly as information-based network control strategies (for example, Peeta and Mahmassani, 1005). As discussed and hence the proposed behavior-consistent

directly as information-based network control strategies (for example, Peeta and Mahmassani, 1995; Lo et al., 1996; Nakayama et al., 1999). As discussed earlier, this approach has behavioral limitations in the deployment context. Fig. 1(b) illustrates that the proposed approach uses a fuzzy control mechanism to determine the behavior-consistent information-based network control strategies based on a DTA solution and the controller's estimation of driver behavior. This enables the controller to ensure consistency between its objectives, the information strategies, and the drivers' likely reactions to the information provision. Hence, unlike the traditional DTA deployment strategy, the proposed approach prevents the under- or over-recommendation of routes, or the recommendation of routes that are not considered by the drivers. This is because the controller factors the drivers' likely reactions to the information strategies while determining them. It should be noted here that the controller could use other objectives, such as the user equilibrium (UE) solution, as the desired goal instead of the SO objective within this framework.

The remainder of this paper is organized as follows. Section 2 summarizes some characteristics of information strategies and uses them to define driver information classes. Section 3 describes the problem and Section 4 formulates it. Section 5 presents the solution concept used to determine the behavior-consistent information-based control strategies. Section 6 discusses experiments and analyzes their results. Section 7 presents some concluding comments.

2. Modeling of information characteristics

2.1. Information type

From the information type perspective, information can be categorized as: (i) descriptive information where instantaneous or projected traffic conditions are provided, and (ii) prescriptive information where specific routes are recommended to the drivers, typically UE or SO routes based on instantaneous or projected travel times. In this context, information can also be categorized as: (a) quantitative information which consists of numeric information related to the network and/or route conditions such as expected travel times, and (b) qualitative information which consists of linguistic labels describing route conditions. Therefore, we could consider

information-based control strategies in terms of descriptive quantitative information, descriptive qualitative information, and prescriptive information.

Most DTA models view and/or pre-specify driver behavior in terms of objectives such as UE, SO, stochastic user equilibrium (SUE), or bounded rationality (BR) under the descriptive information type, and in terms of compliance characteristics under the prescriptive information type. As discussed in Section 1, such a modeling approach is restrictive in depicting realistically both information characteristics and driver response behavior. In addition, these models are unable to adequately handle and process linguistic variables (Peeta and Yu, 2004). In our study, behavior is not pre-specified or restrictive, and the information-based network control strategies are modeled to be consistent with the real-world information types.

DTA models typically provide link/route travel times from a descriptive perspective or the recommended route in a prescriptive context. The study approach provides more realistic information content. That is, the models used to determine information strategies and estimate the driver's likely response behavior enable the determination and processing of linguistic messages such as "heavy traffic ahead" under the descriptive qualitative information type, specific route recommendations under the prescriptive information type, or both simultaneously. Hence, we focus on personalized information that can be descriptive qualitative and/or prescriptive. Both these information types are simultaneously determined by the behavior-consistent approach, as discussed in Section 3 and later. Descriptive qualitative information implies linguistic messages describing traffic conditions downstream of the current location for the current set of routes that a driver is considering to his/her destination. Prescriptive information implies the specific route recommended to the driver.

2.2. Information class modeling

In our study, only drivers with suitably-equipped devices can receive personalized information; other drivers receive no information. The personalized information received by the equipped drivers is viewed as being part of an information service market which provides prescriptive information, descriptive linguistic information, or both as three different subscribed products. If a driver subscribes to prescriptive information, he/she may or may not be provided a route at various decision points by the behavior-consistent approach as discussed in Section 1. By contrast, a driver subscribing to descriptive linguistic information always receives it at various decision points though it is also determined by the behavior-consistent approach. In this context, the behavior-consistent approach determines whether a stronger or weaker linguistic message achieves the desired proportions. Based on the above discussion, we define four driver information classes.

The first class of drivers (u = 1) subscribe to prescriptive information only. These drivers may receive specific routes at times and no information at other times during their trip depending on the time-dependent behavior-consistent network control strategy used. In a pre-trip context, a subset of these drivers is recommended to take specific routes based on the proportions suggested by the behavior-consistent strategy. The remaining prescriptive class drivers are recommended pre-trip routes based on the controller-estimated driver behavior model.

The second class of drivers (u = 2) subscribe to descriptive linguistic information only. Drivers in this class receive time-dependent linguistic information about downstream conditions for their current sets of alternative routes. The third class of drivers (u = 3) subscribe to both prescriptive and descriptive linguistic information, and can process both types of information simultaneously. As in class 1, prescriptive information may or may not be provided to specific drivers depending on the behavior-consistent strategy. However, as in class 2, these drivers always receive linguistic information. The fourth class of drivers (u = 4) do not receive information implying that these drivers have not subscribed to an information service.

3. Problem description

The behavior-consistent information-based control strategies problem is defined as follows. A system controller (or information service provider) seeks to determine information-based network control strategies that are consistent with driver behavior while addressing its objective of enhancing system performance. The approach used by the controller is to influence driver route choice decisions by providing routing information (both linguistic and prescriptive) in such a way that the proportions of drivers taking specific routes are close to the corresponding proportions under a system-wide objective, the SO solution. Thereby, the SO routes are defined as the controller-desired routes, and the corresponding route assignment proportions are labeled controller-desired proportions. To achieve this consistency, the controller estimates the driver route choice decisions using an estimated driver route choice behavior model, and uses it to determine the appropriate behavior-consistent information-based network control strategies. The methodology to obtain these strategies is the focus of this paper. Hence, this study addresses a key sub-problem of the broader problem that seeks to minimize system travel time while minimizing the difference between the controller-desired and actual proportions of drivers choosing routes.

This study adopts a perspective that by directing the system, to the extent possible, to a timedependent SO state, the objective of the controller to enhance system performance can be achieved in a behaviorally more realistic manner than that under the traditional DTA approaches. It should be reiterated here that behavior-consistent route proportions that move the system closer to the SO state are provided through our approach, and not the standard SO solution route proportions obtained by solving the DTA problem itself. It is well-known in the literature that the SO solution is not behaviorally sustainable. Hence, SO routes that are not considered by the drivers are not used by the controller to determine the information strategies and therefore are not recommended to the drivers. The validity of the proposed perspective has been successfully tested by Paz and Peeta (2007), where the authors expand the approach to capture the network level interactions in time and space for the real-time information-based control of a vehicular traffic network. Those results illustrated the importance of incorporating driver behavior realism in the determination of the information-based network control strategies. Significant differences in terms of system travel time savings were obtained when the behavior-consistent approach was compared to the traditional approaches (UE, SO, etc.).

Fig. 2 shows the flowchart of the proposed approach for the broader traffic routing problem in the context of real-world deployment. It is addressed in Paz and Peeta (2007), where a rolling horizon stage-based approach is used to deploy the information-based control strategies in realtime to enhance system performance for a pre-determined planning horizon. In stage number σ of the rolling horizon, the SO DTA solution for the next stage σ +1 is generated based on the current network conditions and the corresponding projected O-D demand. An iterative search based optimization procedure involving the controller-estimated driver behavior model and a fuzzy control model is then used to solve the fixed-point problem described in Section 1, to determine the behavior-consistent information-based control strategies (θ , ϕ) to provide route guidance to drivers in the roll period of the next stage. In the next stage, the controller uses these strategies to provide routes and/or descriptive information to adequately equipped drivers. The drivers use the available information and their innate behavioral tendencies to make route choice decisions. The current network conditions resulting from the actual driver decisions and the associated traffic flow interactions are then measured through sensors to complete the loop.

The iterative search based optimization procedure, which is used to solve the behaviorconsistent information strategies sub-problem of the broader problem, is the focus of this paper. It is represented by the non-shaded box located in the middle of the flowchart in Fig. 2. The subproblem, addressed by the system controller, is the determination of the proportions of drivers that should be recommended specific routes and/or the set of linguistic messages describing route conditions so that when drivers make their decisions according to the controller-estimated driver behavior model, close to SO route proportions are obtained during the roll period of the next stage.

Fig. 3 provides the details of the implementation of the rolling horizon approach. Each stage is divided into discrete time intervals of length Δ time units, and consists of *h* such units. From an implementation perspective for computational efficiency, a stage is also divided into discrete assignment intervals *w* in which the route assignment proportions are constant. The first assignment interval constitutes the roll period of the stage, and consists of *l* time units of length Δ . Hence, the stage length is a multiple of the roll period length. This is designed, without loss of generality, to simplify the formulation and solution implementation. The SO DTA solution is computed for the length of the next stage resulting in different SO proportions for each assignment interval of that stage. However, the information strategies for only the next roll period are determined using the corresponding SO assignment proportions. The computation of the SO DTA solution for the entire stage captures the effects of the projected O-D demand and the network level interactions on the information strategies for the roll period. This is because the SO proportions corresponding to the roll period of the next stage are affected by the projected conditions and/or assignments for the rest of the stage. The next section presents the formulation of the problem using this approach.

4. Problem formulation

4.1. Notation and terms

4.1.1. Notation

Description
Set of nodes in the network
Set of links in the network
Subscript for a link in the network, $a \in A$
Subscript for a linguistic message
Set of origins in the network
Set of destinations in the network
Subscript for an origin node, $i \in I$
Subscript for a destination node, $j \in J$
Roll period of stage σ ; corresponds to $\tau = (\sigma-1) \cdot l + 1, \dots, \sigma \cdot l$
Superscript for a departure time interval in stage roll period $\rho(\sigma+1)$

arphi	Number of time units (in terms of Δ) required to compute the SO solution and information strategies for $\rho(\sigma+1)$			
	Superscript for the time interval in which the computation of the SO solution and			
υ	information strategies for the next roll period begins, $v = (\sigma \cdot l - \varphi)$			
K_{ij}	Set of routes connecting O-D pair <i>ij</i>			
k	Subscript for a route in the network, $k \in K_{ij}$			
U	Set of driver classes in terms of information availability, $U \equiv \{1, 2, 3, 4\}$			
u	Superscript for driver information class, $u \in U$			
$\hat{R}^{u au}_{ij}$	Forecasted O-D demand for the next roll period, expressed as the set of drivers of class u who wish to depart from i to j in time interval τ , $\tau = \sigma \cdot l + 1, \dots, \sigma \cdot l + l$			
$S^{u au}_{ij}$	O-D demand in the next roll period representing the set of drivers of class u that departed in time interval $\tau = 1,, \sigma \cdot l$ and did not reached their destinations at the end of the current roll period, and are departing from the first intermediated node i to destination j at some time interval in the next roll period $\tau = \sigma \cdot l + 1,, \sigma \cdot l + l$			
r	Superscript for an individual driver in the network, $r \in \{\hat{R}_{ij}^{u \tau} \cup S_{ij}^{u\tau}\}$			
PK_{ij}^r	The set of preferred routes connecting O-D pair <i>ij</i> for driver <i>r</i> , $PK_{ij}^r \subseteq K_{ij}$			
<i>PK_{ij}</i>	Driver-preferred routes connecting O-D pair <i>ij</i> , $PK_{ij} = \left\{\bigcup_{r} PK_{ij}^{r}\right\} \subseteq K_{ij}$			
$D V \rho(\sigma)$	Set of controller-desired (SO) routes connecting O-D pair <i>ij</i> in roll period $\rho(\sigma)$ of stage			
$DK_{ij}^{r(c)}$	$\sigma, DK_{ij}^{\rho(\sigma)} \subseteq K_{ij}$			
$CK^{\rho(\sigma)}$	Set of controllable routes connecting O-D pair <i>ij</i> in roll period $\rho(\sigma)$ of stage σ ,			
CK _{ij}	$CK_{ij}^{\rho(\sigma)} \equiv \{ DK_{ij}^{\rho(\sigma)} \cap PK_{ij} \}$			
Ω^{ur}	$CK_{ij}^{\rho(\sigma)} \equiv \{ DK_{ij}^{\rho(\sigma)} \cap PK_{ij} \}$ Driver-information class relationship; 1 if driver <i>r</i> belongs to class <i>u</i> , and 0 otherwise			
$\frac{\Omega^{ur}}{SO_{ijk}^{\rho(\sigma)}}$	$CK_{ij}^{\rho(\sigma)} \equiv \{ DK_{ij}^{\rho(\sigma)} \cap PK_{ij} \}$ Driver-information class relationship; 1 if driver <i>r</i> belongs to class <i>u</i> , and 0 otherwise SO proportion of drivers assigned to route <i>k</i> in roll period $\rho(\sigma)$ of stage $\sigma, k \in DK_{ij}^{\rho(\sigma)}$			
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$\begin{array}{c} C R_{ij} \\ \hline \Omega^{ur} \\ S O_{ijk}^{\rho(\sigma)} \\ \hline \delta_{ijk}^{r\tau} \\ \hline \delta_{ijk}^{r\tau} \\ \hline \delta_{ijk}^{r\upsilon} \\ \hline \delta_{ijk}^{r\upsilon} \\ \hline \theta_{ijk}^{\rho(\sigma)} \\ \hline \theta_{ijk}^{\rho(\sigma)} \\ \hline \theta_{\omega} \\ \hline \end{array}$	$CK_{ij}^{\rho(\sigma)} \equiv \{DK_{ij}^{\rho(\sigma)} \cap PK_{ij}\}$ Driver-information class relationship; 1 if driver <i>r</i> belongs to class <i>u</i> , and 0 otherwise SO proportion of drivers assigned to route <i>k</i> in roll period $\rho(\sigma)$ of stage σ , $k \in DK_{ij}^{\rho(\sigma)}$ Controller-estimated route choice dummy; 1 if driver <i>r</i> leaving from <i>i</i> to <i>j</i> in time interval τ is estimated to take route <i>k</i> , and 0 otherwise, $k \in PK_{ij}^r$ Dummy variable for current route of driver; 1 if driver <i>r</i> is traveling on route <i>k</i> from <i>i</i> to <i>j</i> in time interval <i>v</i> , and 0 otherwise, $k \in PK_{ij}^r$ Controller-estimated behavior-consistent proportion of drivers taking route <i>k</i> in roll period $\rho(\sigma)$ of stage σ , $k \in PK_{ij}$ Descriptive qualitative information defined as the linguistic message describing traffic conditions for route <i>k</i> in roll period $\rho(\sigma)$ of stage σ , $k \in CK_{ij}^{\rho(\sigma)}$ Prescriptive information defined as the proportion of drivers that must be recommended to take route <i>k</i> in roll period $\rho(\sigma)$ of stage σ , $k \in CK_{ij}^{\rho(\sigma)}$ Linguistic message; "Very Light Traffic" if $\omega = 1$, "Light Traffic" if $\omega = 2$, "Moderate Traffic" if $\omega = 3$, "Heavy Traffic" if $\omega = 4$, and "Very Heavy Traffic" if $\omega = 5$			
$\begin{array}{c} \nabla R_{ij} \\ \hline \Omega^{ur} \\ SO_{ijk}^{\rho(\sigma)} \\ \hline \delta_{ijk}^{r\tau} \\ \hline \delta_{ijk}^{r\tau} \\ \hline \delta_{ijk}^{r\sigma} \\ \hline \theta_{ijk}^{\rho(\sigma)} \\ \hline \theta_{ijk}^{\rho(\sigma)} \\ \hline \theta_{ijk}^{\rho(\sigma)} \\ \hline \theta_{\omega} \\ \hline \Phi \\ \hline \end{array}$	$CK_{ij}^{\rho(\sigma)} \equiv \{DK_{ij}^{\rho(\sigma)} \cap PK_{ij}\}$ Driver-information class relationship; 1 if driver <i>r</i> belongs to class <i>u</i> , and 0 otherwise SO proportion of drivers assigned to route <i>k</i> in roll period $\rho(\sigma)$ of stage $\sigma, k \in DK_{ij}^{\rho(\sigma)}$ Controller-estimated route choice dummy; 1 if driver <i>r</i> leaving from <i>i</i> to <i>j</i> in time interval τ is estimated to take route <i>k</i> , and 0 otherwise, $k \in PK_{ij}^r$ Dummy variable for current route of driver; 1 if driver <i>r</i> is traveling on route <i>k</i> from <i>i</i> to <i>j</i> in time interval <i>v</i> , and 0 otherwise, $k \in PK_{ij}^r$ Controller-estimated behavior-consistent proportion of drivers taking route <i>k</i> in roll period $\rho(\sigma)$ of stage $\sigma, k \in PK_{ij}$ Descriptive qualitative information defined as the linguistic message describing traffic conditions for route <i>k</i> in roll period $\rho(\sigma)$ of stage $\sigma, k \in CK_{ij}^{\rho(\sigma)}$ Prescriptive information defined as the proportion of drivers that must be recommended to take route <i>k</i> in roll period $\rho(\sigma)$ of stage $\sigma, k \in CK_{ij}^{\rho(\sigma)}$ Linguistic message; "Very Light Traffic" if $\omega = 1$, "Light Traffic" if $\omega = 2$, "Moderate Traffic" if $\omega = 3$, "Heavy Traffic" if $\omega = 4$, and "Very Heavy Traffic" if $\omega = 5$ Set of linguistic messages, $\Phi \equiv \{\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5\}$			
$\begin{array}{c} C R_{ij} \\ \hline \Omega^{ur} \\ S O_{ijk}^{\rho(\sigma)} \\ \hline \delta_{ijk}^{r\tau} \\ \hline \delta_{ijk}^{r\tau} \\ \hline \delta_{ijk}^{r\upsilon} \\ \hline \delta_{ijk}^{r\upsilon} \\ \hline \theta_{ijk}^{\rho(\sigma)} \\ \hline \theta_{ijk}^{\rho(\sigma)} \\ \hline \theta_{ijk}^{\rho(\sigma)} \\ \hline \theta_{\omega} \\ \hline \Phi_{\omega} \\ \hline \Phi_{\omega} \\ \hline \end{array}$	$CK_{ij}^{\rho(\sigma)} \equiv \{ DK_{ij}^{\rho(\sigma)} \cap PK_{ij} \}$ Driver-information class relationship; 1 if driver <i>r</i> belongs to class <i>u</i> , and 0 otherwise SO proportion of drivers assigned to route <i>k</i> in roll period $\rho(\sigma)$ of stage $\sigma, k \in DK_{ij}^{\rho(\sigma)}$ Controller-estimated route choice dummy; 1 if driver <i>r</i> leaving from <i>i</i> to <i>j</i> in time interval τ is estimated to take route <i>k</i> , and 0 otherwise, $k \in PK_{ij}^r$ Dummy variable for current route of driver; 1 if driver <i>r</i> is traveling on route <i>k</i> from <i>i</i> to <i>j</i> in time interval <i>v</i> , and 0 otherwise, $k \in PK_{ij}^r$ Controller-estimated behavior-consistent proportion of drivers taking route <i>k</i> in roll period $\rho(\sigma)$ of stage $\sigma, k \in PK_{ij}$ Descriptive qualitative information defined as the linguistic message describing traffic conditions for route <i>k</i> in roll period $\rho(\sigma)$ of stage $\sigma, k \in CK_{ij}^{\rho(\sigma)}$ Prescriptive information defined as the proportion of drivers that must be recommended to take route <i>k</i> in roll period $\rho(\sigma)$ of stage $\sigma, k \in CK_{ij}^{\rho(\sigma)}$ Linguistic message; "Very Light Traffic" if $\omega = 1$, "Light Traffic" if $\omega = 2$, "Moderate Traffic" if $\omega = 3$, "Heavy Traffic" if $\omega = 4$, and "Very Heavy Traffic" if $\omega = 5$ Set of linguistic messages, $\Phi \equiv \{ \Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5 \}$ Dummy variable for route recommendation for driver <i>r</i> leaving from <i>i</i> to <i>j</i> in time			

vrv	Dummy variable for route recommended to driver r as of time interval v ; 1 if route k
1 ijk	was recommended, and 0 otherwise, $k \in PK_{ij}^r$
717	Linguistic message related to route k provided to driver r leaving from i to j in time
\boldsymbol{Z}_{ijk}	interval $\tau, k \in PK_{ij}^r$
$V^{r\tau}$	Controller-estimated vector of attributes for route k , excluding information, that
X_{ijk}	influence the route choice decision of driver <i>r</i> in time interval τ , $k \in PK_{ij}^r$
F	Function to denote the controller-estimated driver behavior model used to estimate the
Г	route choices of the individual drivers

4.1.2. Definition of terms

Controller-Desired Routes (DK): These are routes that the controller would like the drivers to choose. They are the time-dependent SO routes obtained by solving the SO DTA problem for a stage.

Driver-Preferred Routes (PK): These routes are preferred by the drivers and are likely to be accepted by them. The controller can generate this route set (Bekhor, et al., 2006) using historical data, travel surveys and/or technologies such as two-way communication systems and global position systems.

Controllable Routes (CK): These routes belong to both controller-desired and driverpreferred route sets. From the controller perspective, providing drivers these routes increases their likelihood of being accepted by drivers, thereby enabling the controller to better influence system performance.

Behavior-Consistency Gap: The behavior-consistency gap for controllable route k connecting O-D pair *ij* is defined as the difference between the controller-desired proportion of drivers $SO_{ijk}^{\rho(\sigma)}$ that should choose route k and the proportion of drivers $\theta_{ijk}^{\rho(\sigma)}$ that must be recommended route k in order to achieve the controller-desired proportion. Hence, more/less proportions of drivers may have to be recommended controllable routes to achieve the controller-desired proportion. Hence, more/less desired proportions depending on the traffic system dynamics and driver behavior.

4.2. Problem definition

Consider a traffic network represented by a directed graph G(N,A) where N is the set of nodes and A the set of directed arcs. A node can represent a trip origin, a destination and/or just a junction of physical links. A network with multiple origins $i \in I$ and destinations $j \in J$ is considered for generality. We are given the SO solution for the next roll period, the time-dependent O-D demand forecasts for the next roll period, the number of previously assigned drivers who are present in the network at the beginning of the next stage and their current routes, the set of driver-preferred routes and their controller-estimated vector of attributes, the information class of each driver, and the controller-estimated driver behavior model. We seek the behavior-consistent information-based network control strategies $\theta_{ijk}^{\rho(\sigma+1)}$ and $\phi_{ijk}^{\rho(\sigma+1)}$ for the next roll period that minimize the absolute difference between the SO proportions $SO_{ijk}^{\rho(\sigma+1)}$ and the controller-estimated proportions $E_{ijk}^{\rho(\sigma+1)}$, $\forall i, j, k \in CK_{ij}^{\rho(\sigma+1)}$.

4.3. Formulation

This section formulates the sub-problem described in Section 3 and defined in Section 4.2. The formulation is also a representation of the non-shaded box in the Fig. 2 flowchart.

 $\frac{\text{Given}}{(i)}: G(NA)$

(1)	G(N,A)	
(ii)	$DK_{ij}^{ ho(\sigma+1)};$	$\forall i, j$
(iii)	$SO_{ijk}^{ ho(\sigma+1)};$	$\forall i, j, k \in DK_{ij}^{\rho(\sigma+1)}$
(iv)	$\hat{R}^{u au}_{ij}$;	$\forall i, j, u, \tau = \sigma \cdot l + 1,, \sigma \cdot l + l$
(v)	$S_{ij}^{u au}$;	$\forall i, j, u, \tau = \sigma \cdot l + 1,, \sigma \cdot l + l$
(vi)	PK_{ij}^r ;	$\forall i, j, r \in \{ \hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau} \}$
(vii)	$X_{ijk}^{r au}$;	$\forall i, j, k \in PK_{ij}^r, r \in \{ \hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau} \}, \tau = \sigma \cdot l + 1, \dots, \sigma \cdot l + l$
(viii)	$Y_{ijk}^{r\upsilon}$;	$\forall i, j, k \in PK_{ij}^r, r \in S_{ij}^{u\tau}$
(ix)	$\delta_{ijk}^{\ r u}$;	$\forall i, j, k \in PK_{ij}^r, r \in S_{ij}^{u\tau}$
(x)	Ω^{ur}	$\forall u, r \in \{ \hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau} \}$
(:)	Г	

(xi) F

Objective function (controller objective):

$$\operatorname{Min.} \quad \sum_{i} \sum_{j} \sum_{k \in CK_{ij}^{\rho(\sigma+1)}} |SO_{ijk}^{\rho(\sigma+1)} - E_{ijk}^{\rho(\sigma+1)}(\theta_{ijk}^{\rho(\sigma+1)}, \phi_{ijk}^{\rho(\sigma+1)})|$$

$$\tag{1}$$

Subject to:

Controller-estimated driver behavior

$$\hat{\delta}_{ijk}^{r\tau} = F((X_{ijk}^{r\tau}, Y_{ijk}^{r\tau}, Z_{ijk}^{r\tau}); \quad \forall k \in PK_{ij}^{r}); \quad \forall i, j, k \in PK_{ij}^{r}, r \in \{\hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau}\}, \tau = \sigma \cdot l + 1, ..., \sigma \cdot l + l$$
(2)

$$E_{ijk}^{\rho(\sigma+1)} = \frac{\sum\limits_{\tau=\sigma:l+1}^{\sigma+r} \sum\limits_{r} \hat{\delta}_{ijk}^{r\tau}}{\sum\limits_{\tau=\sigma:l+1}^{\sigma+l+1} \sum\limits_{r} \sum\limits_{k \in PK_{ij}^{r}} \hat{\delta}_{ijk}^{r\tau}}; \qquad \forall i, j, k \in CK_{ij}^{\rho(\sigma+1)}$$
(3)

Demand conservation constraints $\sum_{k=1}^{\infty} \sum_{i=1}^{k} C_{i}^{kT} C_{i}^{kT} C_{i}^{kT} C_{i}^{kT}$

$$\sum_{r \in S_{ij}^{u\tau}} \sum_{k \in PK_{ij}^{r}} [\delta_{ijk}^{r\tau} \cdot \Omega^{ur}] = |S_{ij}^{u\tau}|; \qquad \forall i, j, u, \tau = \sigma \cdot l + 1, \dots, \sigma \cdot l + l$$
(4)

$$\sum_{r \in \hat{R}_{ij}^{u\tau}} \sum_{k \in PK_{ij}^{r}} [\hat{\delta}_{ijk}^{r\tau} \cdot \Omega^{ur}] = |\hat{R}_{ij}^{u\tau}|; \qquad \forall i, j, u, \tau = \sigma \cdot l + 1, \dots, \sigma \cdot l + l$$
(5)

Information-based network control constraints

$$\{\theta_{ijk}^{\rho(\sigma+1)}, \phi_{ijk}^{\rho(\sigma+1)}\} = \mathsf{g}_{\theta\phi}(SO_{ijk}^{\rho(\sigma+1)}, E_{ijk}^{\rho(\sigma+1)}(\theta_{ijk}^{\rho(\sigma+1)}, \phi_{ijk}^{\rho(\sigma+1)})); \quad \forall \ i, j, k \in CK_{ij}^{\rho(\sigma+1)}$$
(6)

$$Y_{ijk}^{r\tau} = \mathsf{g}_Y(\theta_{ijk}^{\rho(\sigma+1)}, \phi_{ijk}^{\rho(\sigma+1)}, Y_{ijk}^{r\nu}, \delta_{ijk}^{r\nu}, \Omega^{ur}); \quad \forall \ i, j, k \in PK_{ij}^r, r \in \{\hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau}\}, \ \tau = \sigma \cdot l + 1, \dots, \sigma \cdot l + l$$
(7)

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$$\sum_{\substack{k \in CK_{ij}^{\rho(\sigma+1)} \\ ij}} Y_{ijk}^{r\tau} \leq 1; \qquad \forall i, j, r \in \{ \hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau} \}, \tau = \sigma \cdot l + 1, \dots, \sigma \cdot l + l \qquad (8)$$

$$\sum_{\substack{k \in CK_{ij}^{\rho(\sigma+1)} \\ ij}} \theta_{ijk}^{\rho(\sigma+1)} \leq 1; \qquad \forall i, j \qquad (9)$$

$$\forall i, j$$
 (9)

$$\begin{array}{l}
\theta_{ijk}^{\rho(\sigma+1)} = 0; \quad \forall i, j, k \notin CK_{ij}^{\rho(\sigma+1)} \quad (10) \\
Z_{ijk}^{r\tau} = \phi_{ijk}^{\rho(\sigma+1)} \Leftrightarrow \left[\sum_{u=2}^{3} \Omega^{ur} = 1\right] \\
Z_{ijk}^{r\tau} = \left\{\right\}, \text{ otherwise} \quad \left\{; \quad \forall i, j, k \in CK_{ij}^{\rho(\sigma+1)}, r \in \left\{\hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau}\right\}, \\
\tau = \sigma \cdot l + 1, \dots, \sigma \cdot l + l
\end{array}$$

$$\forall i, j, k \in CK_{ij}^{\rho(\sigma+1)}, r \in \{\hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau}\},$$

$$\tau = \sigma \cdot l + 1, \dots, \sigma \cdot l + l$$

$$(11)$$

$$\forall i, j$$
 (12)

$$k \in CK_{ij}^{\rho(\sigma+1)} \Leftrightarrow k \in \{DK_{ij}^{\rho(\sigma+1)} \cap PK_{ij}\}; \quad \forall i, j$$

$$(13)$$

$$\begin{array}{ll} 0-l \text{ variable constraints} \\ \hat{\delta}_{ijk}^{r\tau} = 0 \text{ or } 1; \\ \delta_{ijk}^{r\upsilon} = 0 \text{ or } 1; \\ \delta_{ijk}^{r\upsilon} = 0 \text{ or } 1; \\ \end{array} \qquad \qquad \forall i, j, k \in PK_{ij}^{r}, r \in S_{ij}^{u\tau} \cup S_{ij}^{u\tau} \}, \ \tau = \sigma \cdot l + 1, \dots, \sigma \cdot l + l \quad (14)$$

$$\Omega^{ur} = 0 \text{ or } 1; \qquad \forall u, r \in \{ \hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau} \}$$
(16)

$$Y_{ijk}^{r\tau} = 0 \text{ or } 1; \qquad \forall i, j, k \in PK_{ij}^r, r \in \{\hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau}\}, \tau = \sigma \cdot l + 1, \dots, \sigma \cdot l + l \qquad (17)$$

$$Y_{ijk}^{r_0} = 0 \text{ or } 1; \qquad \forall i, j, k \in PK_{ij}^r, r \in S_{ij}^{ur}$$
Linguistic variable constraints
$$(18)$$

⁺¹⁾
$$\in \Phi;$$
 $\forall i, j, k \in CK_{ij}^{\rho(\sigma+1)}$ (19)

Non-negativity constraints all quantitative variables ≥ 0

 $\phi_{iik}^{\rho(\sigma)}$

 $\theta_{ijk}^{\rho(\sigma+1)} = 0;$

 $k \in PK_{ij} \Leftrightarrow k \in \left\{ \bigcup_{r} PK_{ij}^{r} \right\};$

The above formulation is a non-linear mixed integer model with some stochastic $(\hat{\delta}_{ijk}^{r\tau})$ and linguistic $(\phi_{iik}^{\rho(\sigma)})$ variables. It has several contributions to the route guidance literature. A primary contribution is that the formulation explicitly estimates drivers' likely reactions to the information-based network control strategies while determining them, thereby circumventing realism issues with existing models that pre-specify driver response behavior. Another key contribution is the concept of route classification based on the relevance of routes to the drivers and the controller. It leads to the definition of controllable routes, which provides a realistic deployment mechanism to enhance driver compliance in a behavior-consistent manner. These two aspects enable the development of the behavior-consistent approach for information-based network control. Another contribution is the simultaneous determination of prescriptive and linguistic information that are consistent with each other. Finally, the approach enables the identification of priorities to determine whom to provide information, a significant deployment issue.

(20)

The decision variables are the set of information-based network control strategies $\theta_{ijk}^{\rho(\sigma+1)}$ and $\phi_{ijk}^{\rho(\sigma+1)}$, $\forall i, j, k \in CK_{ij}^{\rho(\sigma+1)}$. The formulation explicitly recognizes that the set of controllerdesired routes $DK_{ij}^{\rho(\sigma+1)}$ may differ from the set of driver-preferred routes PK_{ij}^r , $\forall i, j, r \in \{\hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau}\}$, leading to the concept of controllable routes.

4.3.1. Objective function

The controller objective (1) is to minimize the absolute difference between the SO proportions $SO_{ijk}^{\rho(\sigma+1)}$ for the next roll period and the corresponding controller-estimated proportion of drivers taking routes, $E_{ijk}^{\rho(\sigma+1)}$, $\forall i, j, k \in CK_{ij}^{\rho(\sigma+1)}$. The controller achieves its objective by influencing $E_{ijk}^{\rho(\sigma+1)}$ through information provision to approach $SO_{ijk}^{\rho(\sigma+1)}$, $\forall i, j, k \in CK_{ij}^{\rho(\sigma+1)}$.

4.3.2. Controller-estimated driver behavior constraints

Function F in Constraint (2) denotes the controller-estimated driver behavior model used to estimate individual driver route choices. The controller-estimated route choice for driver r (represented through dummy $\hat{\delta}_{ijk}^{r\tau}$) is a function of the route attributes $X_{ijk}^{r\tau}$, the route recommendation dummy $Y_{ijk}^{r\tau}$, and the linguistic message $Z_{ijk}^{r\tau}$, $\forall i, j, k \in PK_{ij}^r$, $r \in \{\hat{R}_{ij}^{u\tau} \cup S_{ij}^{u\tau}\}$, $\tau = \sigma \cdot l + 1, ..., \sigma \cdot l + l$. Since $Y_{ijk}^{r\tau}$ is a function of $\theta_{ijk}^{\rho(\sigma+1)}$ and $Z_{ijk}^{r\tau}$ depends on $\phi_{ijk}^{\rho(\sigma+1)}$, the constraint also implies that $\theta_{ijk}^{\rho(\sigma+1)}$ and $\phi_{ijk}^{\rho(\sigma+1)}$ simultaneously influence $E_{ijk}^{\rho(\sigma+1)}$. F can denote any model structure, such as econometric, rule-based, or hybrid. Hence, the proposed approach is independent of the behavior model structure.

This study uses a hybrid multinomial logit model as part of the controller-estimated driver behavior model, where the systematic component of the utility is determined using simple behavioral *if-then* rules. The systematic component of the utility for a route is obtained using a fuzzy logic procedure which aggregates the contribution of each route attribute to the utility. The resulting route choice probabilities/proportions are translated into the individual route choices of drivers using Monte Carlo simulation. Hence, F represents the combination of the hybrid multinomial logit model and the Monte Carlo simulation. Table 1 shows the set of the behavioral *if-then* rules used in this study. Here, the route attributes X are its expected travel time T and number of nodes NN.

Constraint (3) is a definitional constraint denoting that the controller-estimated proportion of drivers $E_{ijk}^{\rho(\sigma+1)}$ taking controllable route *k* connecting O-D pair *ij* during the next roll period is equal to the controller-estimated number of drivers taking this route divided by the total controller-estimated number of drivers making route choice decisions over all of their corresponding preferred routes $k \in PK_{ii}^r$, for that roll period, $\forall i, j, k \in CK_{ii}^{\rho(\sigma+1)}$.

4.3.3. Demand conservation constraints

Constraints (4) and (5) represent the conservation of the O-D demand for the next roll period. As discussed earlier, this demand is the sum of the numbers of previously assigned drivers that are still in the network ($|S_{ij}^{u\tau}|$) and the newly forecasted O-D demand ($|\hat{R}_{ij}^{u\tau}|$). Constraint (4) indicates that the summation, over all drivers in $S_{ij}^{u\tau}$ and the set of driver-preferred routes PK_{ij}^r , of the product of the controller-estimated route choice dummy $\hat{\delta}_{ijk}^{r\tau}$ and the driver-information class relationship Ω^{ur} , $\forall i, j, u, \tau = \sigma \cdot l + 1, ..., \sigma \cdot l + l$, should equal the cardinality of $S_{ij}^{u\tau}$. Here, the product $\hat{\delta}_{ijk}^{r\tau} \cdot \Omega^{ur}$ takes value 1 if driver *r* belongs to class *u* and the controller estimates that the summation, over all drivers in $\hat{R}_{ij}^{u\tau}$ and the set of driver-preferred routes PK_{ij}^r , of the product $\hat{\delta}_{ijk}^{r\tau} \cdot \Omega^{ur}$ takes value 1 if driver *r* belongs to class *u* and the controller estimates that the summation, over all drivers in $\hat{R}_{ij}^{u\tau}$ and the set of driver-preferred routes PK_{ij}^r , of the product of the controller estimated route $\hat{\sigma}_{ijk}^{r\tau}$ and the driver of the controller estimates $\hat{\tau}$, and 0 otherwise. Similarly, Constraint (5) indicates that the summation, over all drivers in $\hat{R}_{ij}^{u\tau}$ and the set of driver-preferred routes PK_{ij}^r , of the product of the controller-estimated route choice dummy $\hat{\delta}_{ijk}^{r\tau}$ and the driver-information class relationship Ω^{ur} , $\forall i, j, u, \tau = \sigma \cdot l + 1, ..., \sigma \cdot l + l$, should equal the cardinality of $\hat{R}_{ij}^{u\tau}$.

4.3.4. Information-based network control constraints

Constraints (6)-(13) represent the information-based network control constraints. Constraint (6) has a fixed point structure and denotes that the information strategies $\theta_{ijk}^{\rho(\sigma+1)}$ and $\phi_{ijk}^{\rho(\sigma+1)}$ are the outcome of a procedure $g_{\theta\phi}$ that relates them to the SO proportions $SO_{ijk}^{\rho(\sigma+1)}$ and the controller-estimated proportions (obtained using F) of drivers taking routes $E_{ijk}^{\rho(\sigma+1)}$, $\forall i, j,$ $k \in CK_{ij}^{\rho(\sigma+1)}$. Constraints (2), (3), (7), and (11) together indicate that $E_{ijk}^{\rho(\sigma+1)}$ is a function of $\theta_{ijk}^{\rho(\sigma+1)}$ and $\phi_{ijk}^{\rho(\sigma+1)}$, implying the fixed point structure of (6). Constraint (6) also indicates that $\theta_{ijk}^{\rho(\sigma+1)}$ and $\phi_{ijk}^{\rho(\sigma+1)}$ are interdependent. Hence, different combinations of $\theta_{ijk}^{\rho(\sigma+1)}$ and $\phi_{ijk}^{\rho(\sigma+1)}$ may minimize the objective function, implying the potential for multiple solutions.

In our study, the fuzzy control model in Fig. 2 represents $g_{\theta\phi}$. An advantage of the fuzzy logic methodology in this context is that it facilitates the simultaneous determination of prescriptive $\theta_{ijk}^{\rho(\sigma+1)}$ and descriptive $\phi_{ijk}^{\rho(\sigma+1)}$ information strategies. This is because it can enable a *many-to-many* mapping from the SO solution, the controller-estimated driver behavior, and the information provided to the drivers, to the information strategies.

Constraint (7) states that the value of the dummy variable $Y_{ijk}^{r\tau}$ for the route recommended by the controller to driver *r* is the result of the discretization of the aggregate proportions $\theta_{ijk}^{\rho(\sigma+1)}$ through the procedure g_Y . It is also dependent on $\phi_{ijk}^{\rho(\sigma+1)}$ because the recommended proportions for a route should be consistent with the linguistic message provided for it. It further depends on the past route recommendation (up to interval *v*) for a driver Y_{ijk}^{rv} and the route taken by that driver δ_{ijk}^{rv} as these characteristics can be used to devise a behavior-consistent priority scheme for the future route recommendation. For example, drivers who subscribe to a premium information provision service and request information in the next roll period from the controller can receive the highest priority. Given that these drivers are requesting information, they are more likely to accept the provided controllable routes.

In the priority scheme used in this study, drivers considered to receive recommendation for route k are categorized in priority subgroups based on their existing routes, prior route recommendations, and their responses to these recommendations. The first sub-group consists of drivers that were recommended to take route k in the previous stage and are currently traveling on it $(Y_{ijk}^{rv}=1, \delta_{ijk}^{rv}=1)$. This is because the controller seeks to prevent route switching, if possible, to enhance driver valuation of information; frequent switch recommendations may cause drivers to increasingly ignore the recommended to take route k in the previous stage and are not currently traveling on it $(Y_{ijk}^{rv}=0, \delta_{ijk}^{rv}=0)$. This is designed to attract drivers who are currently traveling on one of their preferred routes which are not controllable. The drivers of the third sub-group are those that were not recommended to take route k in the previous stage but are traveling on it $(Y_{ijk}^{rv}=0, \delta_{ijk}^{rv}=0)$. Akin to first sub-group, this is to prevent route switching if possible. Within a sub-group, the selection of drivers is performed randomly. Finally, for drivers not belonging to any of these sub-groups, this selection is randomly done.

Constraint (8) ensures that no more than one route is recommended to a driver, as per the strategy employed in this study. That is, $Y_{ijk}^{r\tau}$ can only take value 1 for at most one route in $CK_{ij}^{\rho(\sigma+1)}$, depending on whether the controller chooses to recommend a route to that driver based on the behavior-consistent approach.

Consistent with Constraint (8), Constraint (9) indicates that the total proportion of drivers receiving route recommendations cannot exceed 1. That is, the controller cannot recommend routes to more than hundred percent of the drivers.

Constraint (10) states that routes that do not belong to the set of controllable routes $(k \notin CK_{ii}^{\rho(\sigma+1)})$ are not recommended to drivers.

Constraint (11) indicates that the linguistic message $Z_{ijk}^{r\tau}$ for route *k* provided to driver *r* in time interval τ is equal to the descriptive information $\phi_{ijk}^{\rho(\sigma+1)}$ for route *k* if and only if the driver has access to such information (u = 2 or 3). If the driver does not have access to descriptive information, $Z_{iik}^{r\tau}$ is defined by the null set { }.

Constraint (12) states that route k belongs to the set of driver-preferred routes PK_{ij} if and only if it belongs to the set represented by the union of the preferred route sets of all individual drivers going from *i* to *j*. Constraint (13) states that route k belongs to the set of controllable routes $CK_{ij}^{\rho(\sigma+1)}$ if and only if it belongs to both the controller-desired and driver-preferred route sets. Constraints (2), (3), (12) and (13) together enable the control of a system where the set of driver-preferred routes and the corresponding controller-estimated proportion of drivers taking routes are defined considering the entire set of driver-preferred routes.

4.3.5. 0-1, qualitative, and non-negativity variable constraints

Constraints (14)-(18) restrict specific variables to take a value 0 or 1. Constraint (19)

indicates that the descriptive information $\phi_{ijk}^{\rho(\sigma+1)}$ for controllable route *k* must belong to the set of available linguistic messages. Constraint (20) is the non-negativity constraint for all quantitative variables.

5. Problem solution

It is difficult to solve the formulation described in Section 4 using traditional hard computing techniques such as non-linear optimization or traditional control theory. A key issue is their limited ability to handle the imprecision, uncertainty and subjectivity associated with incomplete data and/or qualitative/linguistic variables (ϕ). Linguistic variables are important in this problem context because they enable the modeling of information provision strategies used in the real world; qualitative messages such as "heavy traffic ahead" or "minor delays."

In this study, the formulation is solved using a fuzzy logic based optimization framework. Fuzzy logic allows some tolerance to imprecision, uncertainty and/or partial truth, while enabling a more tractable and computationally efficient solution mechanism (Tsoukalas and Uhrig, 1997). Computational efficiency is important in the deployment context as the control strategies are needed in sub-real time. Other advantages of using a fuzzy logic framework in this problem context include: (i) the knowledge/experience of traffic control personnel can be incorporated in control if-then rules. and (ii) the framework enables the simultaneous the processing/determination of quantitative and qualitative traffic information.

An iterative search based optimization procedure, briefly mentioned in Section 3 and illustrated by the non-shaded box in Fig. 2, is used to solve the formulation (1)-(20). It is shown in detail in Fig. 4 and consists of the controller-estimated driver behavior model and a fuzzy control model in an iterative search process for an O-D pair. It seeks to determine the information-based control strategies that minimize the difference between the SO proportions and the corresponding controller-estimated proportions of drivers taking routes.

First, the controller-estimated driver behavior model (Fig. 4) is used to forecast driver route choice decisions using an initial set of information-based control strategies (described in Section 5.2.1), and the prioritization scheme (described in Constraint (7)) and driver information class. If these controller-estimated proportions in relation to their corresponding SO proportions satisfy a convergence criterion (illustrated in Section 5.2.3), the search procedure terminates. If convergence is not yet achieved, the fuzzy control model (illustrated by the non-shaded boxes in Fig. 4 and described in Section 5.1) is used to update the information-based control strategies (θ , ϕ) for the next iteration so as to further reduce the difference between the SO and controller-estimated proportions. Hence, the fuzzy control model represents the update mechanism (direction-finding and step-size) for the optimization framework. The iteration counter is updated and the current information-based control strategies are used to determine the controller-estimated route proportions to close the loop. This iterative search procedure is summarized in Section 5.2.

Although the search procedure is conducted for all O-D pairs within a rolling horizon stage as shown in Fig. 2, the time and O-D pair dimensions (superscript $\rho(\cdot)$ and subscripts *ij*) are ignored hereafter without loss of generality to simplify the notation.

5.1. Fuzzy control model

The fuzzy control model consists of three components, as shown by the non-shaded boxes in Fig. 4. The first component is the input (denoted by the dotted box). The second component represents the decision processing steps (denoted by the three solid boxes) and consists of the control *if-then* rules based inference step, the aggregation step, and the defuzzification step. The third component is the output (denoted by the dashed box). The model is described in detail hereafter.

5.1.1. Variables and notation

Additional variables used in the iterative search based optimization procedure are:

Variable	Description		
$\hat{\eta}$	Number of iterations in the iterative search procedure		
η	Superscript to denote the iteration number, $\eta = 1,, \hat{\eta}$		
RP	Number of control <i>if-then</i> rules for prescriptive information		
RD	Number of control <i>if-then</i> rules for descriptive information		
R	Total number of control <i>if-then</i> rules, $R = RP + RD$		
α	Superscript to denote a control <i>if-then</i> rule, $\alpha = 1,,R$		
E_k^η	Controller-estimated proportion of drivers taking route k in iteration η , $k \in PK$		
e_k^η	Error in iteration η defined as the difference between SO_k and E_k^{η} , $k \in CK$		
• 77	Change in error in iteration η , defined as the difference between the current iteration		
$\Delta e_k''$	error e_k^{η} and the previous iteration error $e_k^{\eta-1}$, $k \in CK$		
07	Proportion of drivers that must be recommended (prescriptive information) to take		
Θ_k^{\prime}	route k in iteration η , $k \in CK$		
$\pmb{\phi}_k^\eta$	Linguistic descriptive information for route k in iteration η , $k \in CK$, $\phi_k^{\eta} \in \Phi$		
$\widetilde{\pmb{\phi}}_k^{\;\eta}$	Crisp value associated with descriptive information for route k in iteration η , $k \in CK$		
$\Lambda \rho \eta$	Change in the proportion of drivers that must be recommended (prescriptive		
$\Delta \theta_k^{\prime\prime}$	information) to take route k in iteration η , $k \in CK$		
$\Delta \widetilde{\phi}^{\eta}$	Change in the crisp value associated with descriptive information for route k in		
$\Delta \varphi_k$	iteration $\eta, k \in CK$		
$\Delta \theta_{\mu}^{\alpha \eta^{*}}$	Fuzzy outcome for change in prescriptive information for route k in iteration η		
Δo_k	obtained by fuzzyfing the inputs using rule α , $k \in CK$, $\alpha = 1,, RP$		
$\Delta \widetilde{\phi}_{k}^{\ lpha \eta^{*}}$	Fuzzy outcome for change in descriptive information for route k in iteration η		
· K	obtained by fuzzyfing the inputs using rule $\alpha, k \in CK, \alpha = RP+1,, R$		
$\Lambda \Omega \eta^*$	Fuzzy outcome for change in prescriptive information for route k at iteration η		
$\Delta \theta_k^{\prime}$	resulting from the aggregation of the fuzzy outcomes of an the fulles $(\alpha - 1,, KF)$,		
	Fuzzy outcome for change in descriptive information for route k at iteration n		
$\Delta \widetilde{\phi}_k^{\eta*}$	resulting from the aggregation of the fuzzy outcomes of all the rules ($\alpha = RP+1$ R)		
	$k \in CK$		
$\gamma_k^{\alpha\eta}$	Degree at which rule α for route k is activated in iteration η , $k \in CK$		
μ	Membership function for a fuzzy set		

$S(\cdot)$	Function to calculate the area of the membership function for a fuzzy set
$\overline{\mathbf{A}}^{\alpha}$	Centroid of the fuzzy set for prescriptive information given by the consequent (right
0	hand side) of rule α , $\alpha = 1,,RP$
$\overline{A} \alpha$	Centroid of the fuzzy set for descriptive information given by the consequent (right
φ	hand side) of rule α , $\alpha = RP+1,,R$

5.1.2. Input

The vectors of inputs for iteration η are defined by:

$$e_k^{\eta} = SO_k - E_k^{\eta} \text{ and } \Delta e_k^{\eta} = e_k^{\eta} - e_k^{\eta-1} \qquad \forall k \in CK$$
(21)

They are used to determine the update $(\Delta \theta, \Delta \phi)$ to the current solution. The role of Δe_k^{η} is to smoothen the search process by precluding potential oscillatory behavior in the decision variables (θ, ϕ) that can occasionally arise by considering only the current error e_k^{η} in the update mechanism.

5.1.3. Decision processing component

The processing component can be summarized as follows. In the first step, the inputs are mapped to appropriate membership functions to obtain the fuzzy outcomes according to the control *if-then* rules. The second step aggregates the outcomes of all fired (used) rules. In the final step, a defuzzification scheme is used to determine updates to the decision variables. Sections 5.1.3.1 and 5.1.3.2 describe the *if-then* rules and the corresponding membership functions, respectively. The three steps of the decision process are described in Section 5.1.3.3.

5.1.3.1. Control if-then rules

If-then rules are logical statements where the *if* part is called the "antecedent" and the *then* part is called the "consequent". They can entail multiple dimensions to enable the mapping of many inputs to many outputs. In this study, these rules are simple two-dimensional rules obtained from observed patterns and problem characteristics. For example, if the error is positive for a given route (antecedent), the number of drivers taking this route should increase implying that the route must be recommended to more drivers (consequent). Since the controller does not know the ideal level of response ($\Delta\theta$, $\Delta\phi$) in relation to the magnitude of the inputs (*e*, Δe), there is uncertainty on the response magnitudes based on the inputs. Hence, multiple rules are defined to account for the various input-response possibilities. The fuzzy control logic is used to identify the rules which are fired and the degree of their contribution so as to elicit the best response through the iterative search based optimization procedure. The relative magnitudes (positive small, negative large, etc.) of the inputs and outputs are handled using membership functions whose parameters can be field-calibrated through optimization (Paz and Peeta, 2008a). However, as discussed in Section 5.1.3.2, these parameters need not be calibrated as they only influence computational efficiency and not the update magnitudes ($\Delta\theta$, $\Delta\phi$).

An example of a control *if-then* rule is as follows:

if [*e* is NS and Δe is PL] *then* [$\Delta \theta$ is PS]

In this rule (also shown in Fig. 4), if the error e is negative small (NS) and the change in error Δe is positive large (PL), then the information strategy θ is increased by a positive small (PS) quantity $\Delta \theta$. This outcome is aggregated along with the outcomes of all other rules to generate the crisp composite output.

The antecedents or left hand side (LHS) of the rules correspond to the inputs and the consequents or right hand side (RHS) to the outputs. The LHS and RHS are characterized by the following five fuzzy sets: "Negative Large (NL)", "Negative Small (NS)", "Zero (Z)", "Positive Small (PS)" and "Positive Large (PL)" error and change in error. Thus, e_k^{η} , Δe_k^{η} , $\Delta \theta_k^{\eta}$ and $\Delta \tilde{\phi}_k^{\eta} \in [$ "NL", "NS", "Z", "PS", "PL"]. Table 2 presents the set of control *if-then* rules used by the fuzzy control model in our study.

5.1.3.2. Membership functions

The membership functions (μ) are used to handle the imprecision, uncertainty and/or partial truth of inputs and their associated consequences on the outputs. Fuzzy logic uses membership functions to enable reasoning with variables that are vague in nature, such as language-based descriptors (e.g. congestion ahead, the error is NL). Corresponding to the five fuzzy sets, there are five triangular membership functions each for e, Δe , $\Delta \theta$, and $\Delta \phi$ as indicated in Fig. 5; these functions are independent of η and k. In addition, there are five membership functions associated with the five messages for the descriptive information ϕ . Based on this, there are three membership functions associated with each control *if-then* rule, one each for the inputs e_k and Δe_k , and one for the output (either $\Delta \theta_k$ or $\Delta \phi_k$).

The set of membership functions associated with an input/output are designed to cover the range of its domain as illustrated in Fig. 5; in this study, their parameters evenly cover the range. For e, Δe , and $\Delta \theta$, the corresponding domains [-1,1] have direct physical interpretations based on the values they can take. For $\Delta \phi$, the domain is divided into five equal parts, each of which corresponds to a linguistic message. The advantage of using a triangular shape is its simplicity which aids computational efficiency as the membership function can be fully defined using only three parameters, its modal point, and its lower and upper half-widths. Paz and Peeta (2008a) use an off-line H-infinity filter based approach to optimize the membership function parameters specifically to enhance on-line computational efficiency. That is, the optimized parameters provide the same solution as the default parameters but in lesser computational time.

5.1.3.3. Decision process

For each iteration η , the max-min composition operator and Larsen product implication operator are used for the fuzzy inference step, and the center of gravity method is used for the defuzzification step (see Tsoukalas and Uhrig, 1997). The current inputs, e_k^{η} and Δe_k^{η} , are matched against the *R* control *if-then* rules to determine the corresponding degrees of activation. The degree at which each rule is activated is obtained using the relevant components of e_k^{η} and Δe_k^{η} , and the max-min operator:

$$\gamma_k^{\alpha\eta} = \max_{z \in \mathbb{Z}} \min(\mu_e^{\alpha}(z), \mu_{\Delta e}^{\alpha}(z)) \qquad \forall k \in CK, \alpha$$
(22)

where Z represents the universe of the domains of the fuzzy sets e_k^n and Δe_k^n . The membership functions of the fuzzy outcomes $\Delta \theta_k^{\alpha \eta^*}$ and $\Delta \tilde{\phi}_k^{\alpha \eta^*}$ for each rule are then obtained using the Larsen product implication operator as:

$$\mu_{\Delta\theta_{k}^{\alpha\eta^{*}}} = \gamma_{k}^{\alpha\eta} \cdot \mu_{\Delta\theta}^{\alpha} \qquad \forall k \in CK, \, \alpha = 1, \dots, RP$$
(23)

and

$$\mu_{\Delta\phi_{k}^{\alpha\eta*}} = \gamma_{k}^{\alpha\eta} \cdot \mu_{\Delta\widetilde{\phi}}^{\alpha} \qquad \forall k \in CK, \, \alpha = RP+1, \dots, R$$
(24)

To aggregate the outcomes from all rules for each route k, the following scheme (Zadeh, 1996) is used:

$$\mu_{\Delta\theta_k^{\eta^*}} = \sum_{\alpha=1}^{RP} \mu_{\Delta\theta_k^{\alpha\eta^*}} \qquad \forall k \in CK$$
and
$$(25)$$

$$\mu_{\Delta \widetilde{\phi}_{k}^{\eta^{*}}} = \sum_{\alpha=RP+I}^{R} \mu_{\Delta \widetilde{\phi}_{k}^{\alpha \eta^{*}}} \qquad \forall k \in CK$$
(26)

The center of gravity method is then used to defuzzify the fuzzy aggregate outcomes $\Delta \theta_k^{\eta^*}$ and $\Delta \tilde{\phi}_k^{\eta^*}$ represented by the membership functions in (25) and (26), respectively, to generate the crisp outcomes of the decision variables as follows.

$$\Delta \theta_{k}^{\eta} = \frac{\sum\limits_{\alpha=1}^{RP} \overline{\theta}^{\alpha} \cdot S(\mu_{\Delta \theta_{k}^{\alpha \eta^{*}}})}{\sum\limits_{\alpha=1}^{RP} S(\mu_{\Delta \theta_{k}^{\alpha \eta^{*}}})} \qquad \forall k \in CK$$
(27)

and

$$\Delta \widetilde{\phi}_{k}^{\eta} = \frac{\sum_{\alpha=RP+I}^{R} \overline{\phi}^{\alpha} \cdot S(\mu_{\Delta \widetilde{\phi}_{k}^{\alpha \eta^{*}}})}{\sum_{\alpha=RP+I}^{R} S(\mu_{\Delta \widetilde{\phi}_{k}^{\alpha \eta^{*}}})} \qquad \forall k \in CK$$
(28)

5.1.4. Output

The crisp results, $\Delta \theta_k^{\eta}$ and $\Delta \tilde{\phi}_k^{\eta}$, are used to update the information-based traffic control

strategies in iteration η :

$$\theta_k^{\eta} = \theta_k^{\eta-1} + \Delta \theta_k^{\eta} \qquad \forall k \in CK$$
(29)

and

$$\widetilde{\phi}_{k}^{\eta} = \widetilde{\phi}_{k}^{\eta-1} + \Delta \widetilde{\phi}_{k}^{\eta} \qquad \qquad \forall k \in CK$$
(30)

where $\theta_k^{\eta-1}$ and $\tilde{\phi}_k^{\eta-1}$ are the crisp values for the information strategies in the previous iteration (η – 1). Hence, $\Delta \theta_k^{\eta}$ and $\Delta \tilde{\phi}_k^{\eta}$ represent the combined search direction and step size of the iterative search based optimization procedure.

For prescriptive information, θ_k^{η} is directly used as output from the fuzzy control model. However, since descriptive information is linguistic, an additional step is required to transform the continuous crisp value $\tilde{\phi}_k^{\eta}$ to a discrete message:

$$\phi_k^{\eta} = \{ \Phi_{\omega} \mid \min_{\omega} (|\phi_k^{\eta} - \overline{\Phi}_{\omega}| \quad \forall \ \omega = 1, \dots, 5) \} \qquad \forall \ k \in CK$$
(31)

It corresponds to selecting the fuzzy set (linguistic message) with the largest mapping with $\tilde{\phi}_k^{\eta}$ (degree of membership) among the possible fuzzy sets; it implies that the selected fuzzy set has the closest centroid $(\overline{\Phi}_{\omega})$ to $\tilde{\phi}_k^{\eta}$. Here, the use of the continuous variables $\tilde{\phi}_k^{\eta}$ in the fuzzy control model rather than the direct use of the discrete linguistic messages is to achieve smooth convergence by reducing jumps in the objective function that can result from the use of the discrete variables. Hence, the descriptive information variable is viewed in our approach as the outcome of continuous crisp values.

As indicated in the decision process, both types of information strategies are computed simultaneously and for all controllable routes as they are mutually dependent. This is necessary and adds several dimensions of complexity to the problem. Some drivers may have access to both types of information and use them to make their routes choice decisions. Therefore, the effect of one strategy influences the effect of the other on the entire set of drivers choosing routes. Further, information on a route directly affects the proportion of drivers taking that route as well as the other routes because the information results in driver switching from some routes to others. These interdependencies are illustrated through the experiment results presented in Section 6.

5.2. Iterative search based optimization procedure

First, the set of controllable routes for each O-D pair for the next roll period are determined. It is possible that no controllable routes exist for some O-D pairs, in which case no search is conducted for them. For the next roll period $\rho(\sigma+1)$ and for an O-D pair *ij* with controllable routes, the algorithmic steps of the iterative search based optimization procedure (represented by Fig. 4) for the next roll period are as follows.

5.2.1. Step zero: initialization

Set the iteration counter, $\eta = 1$. If the set of controllable routes for the next roll period (based on $SO_{ijk}^{\rho(\sigma+1)}$) are identical to those in the current roll period, initialize the information strategies $(\theta_k^{\eta}, \phi_k^{\eta})$ for them to the ones in the current roll period. If these controllable route sets are different, set $\theta_k^{\eta} = 0$ and $\phi_k^{\eta} = \Phi_3$, $\forall k \in CK_{ii}^{\rho(\sigma+1)}$.

5.2.2. Step one: controller-estimated behavior-consistent proportions

Use the controller-estimated driver behavior model F to compute the controller-estimated behavior-consistent proportions of drivers taken routes E_k^{η} based on the information-based network control strategies θ_k^{η} and ϕ_k^{η} , $\forall k \in CK_{ij}^{\rho(\sigma+1)}$.

5.2.3. Step two: convergence check

Check for convergence using (32). χ is the number of iterations used for averaging to check for convergence. First, compute the difference between the $SO_{ijk}^{\rho(\sigma+1)}$ and E_k^{η} to generate e_k^{η} . Then, use it along with \tilde{e}_k^{η} , the average value of the error over the last χ iterations for route k in iteration η , and the errors for the last χ iterations (e_k^m is the error in iteration m for route k), to determine the value to compare with ϖ , a pre-specified small constant indicating the required accuracy. If the number of iterations at convergence is less than χ , the corresponding number of iterations is used for the averaging.

$$\sqrt{\frac{1}{\chi} \left(\sum_{m=\eta-\chi+1}^{\eta} \left(e_k^m - \widetilde{e}_k^\eta \right) \right)^2} < \varpi \qquad \qquad \forall \ k \in CK_{ij}^{\rho(\sigma+1)}$$
(32)

Terminate the iterative search procedure if the inequality in (32) is satisfied for all controllable routes for O-D pair *ij*. Otherwise, go to Step three.

5.2.4. Step three: update the information strategies

Use the fuzzy control model to update the information strategies θ_k^{η} and ϕ_k^{η} based on the SO proportions $SO_{ijk}^{\rho(\sigma+1)}$ and the corresponding controller-estimated behavior-consistent proportions E_k^{η} , $\forall k \in CK_{ij}^{\rho(\sigma+1)}$. Update the iteration counter, $\eta = \eta + 1$, and go to Step one.

6. Experiments

Experiments are designed to evaluate the performance of the fuzzy control model and illustrate the significance of behaviorally-consistent approaches to determine information-based network control strategies. Three sets of experiments are conducted to evaluate the performance

of the fuzzy control model under various driver classes (in terms of information type, information access, and their level of responsiveness to information).

6.1. General experimental setup

The Borman expressway corridor network shown in Fig. 6 is used to conduct the experiments. Located in northwest Indiana, it consists of a sixteen-mile section of interstate I-80/94 (called the Borman expressway), I-90 toll freeway, I-65, and the surrounding arterials and streets. It has 197 nodes and 460 links. While the proposed methodology can be used to determine the information strategies for multiple O-D pairs where different drivers have different sets of preferred routes, a single O-D pair and a single set of driver-preferred routes (all driver have the same set of preferred routes) are used here to illustrate the key methodological insights associated with the behavior-consistent approach. As shown in Fig. 6, there are four driver-preferred routes (zigzag lines) and four controller-desired routes (dashed lines) connecting the selected O-D pair, but only three of them fully overlap. The controller seeks to achieve the SO proportions only on the set of controllable routes (the three routes that fully overlap). Thus, controller-desired routes 1, 2 and 3 are defined as the controllable routes in these experiments. The SO proportions are 49%, 26% and 10% for routes 1, 2 and 3, respectively.

As shown in Table 1, two types of drivers are considered in the controller-estimated driver behavior model based on their level of responsiveness to information provided. The first type of drivers, labeled as "more responsive" to the information strategies, are more likely to be influenced by the information provided. The second type of drivers, categorized as "less responsive", are less likely to be influenced by the information provided. They rely more on their past experience and perceptions to make route choice decisions. Drivers that are not influenced at all by the information are viewed here as drivers without information.

Note that the actual driver behavior may be different. However, as discussed earlier and illustrated in Fig. 2, the actual driver behavior is addressed only in the overall framework of Fig. 2, and not in the sub-problem addressed in this paper. Paz and Peeta (2007) analyze the performance of the overall framework.

In terms of drivers' access to prescriptive and/or descriptive information, four driver classes are considered as discussed in Section 2.2. For the experiments involving descriptive information, $\omega = 1,...,5$ is used to represent the linguistic messages defined in Section 4.1.1.

The experiments are conducted for only the first stage of the rolling horizon; this is based on the objectives of this paper of investigating the effectiveness of the behavior-consistent approach rather than a network-level analysis. In the figures illustrating the results, which correspond to the first iteration of the first stage, the points on the y-axis are based on the initial information-based control strategies ($\theta_k^{\eta} = 0$ and $\phi_k^{\eta} = \Phi_3$, $\forall k \in CK_{ij}^{\rho(\sigma+1)}$).

6.2. Experiments: prescriptive information only

6.2.1. Specific objectives and design

The objective of these experiments is to evaluate the ability of the fuzzy control model to generate effective prescriptive information strategies under the two classes of driver responsiveness to information. To illustrate insights, it is assumed that all drivers have access to prescriptive information, but only a subset of them receive route recommendations depending on

the behavior-consistent strategy used (and the priority scheme discussed in Constraint (7)). The remaining drivers do not receive information, and hence, their route choice decisions are assumed to be without the influence of information for the roll period of that stage. The decision variable is the vector θ that represents the proportions of drivers that must be recommended to take specific routes.

6.2.2. Experiment results and analysis

Fig. 7 presents the results of these experiments. Fig. 7(a) shows the controller-estimated proportion (fraction) of drivers taking routes in each iteration of the search procedure under the currently calculated vector of information strategies. It can be noticed that the controller can achieve close to the SO proportions (shown by the three horizontal lines in the figure for the three routes) through its information provision strategies. However, it achieves a faster convergence when all drivers are more responsive to the information strategies. This is because when drivers are more likely to make route choice decisions consistent with the recommendation, the controller can achieve its objective with fewer recommendations.

Fig. 7(b) shows the proportion of drivers that must be recommended to take each route in order to achieve the desired (SO) proportions. The values of the information strategies at convergence indicate that more recommendations are required for one of the three routes under the more responsive behavior scenario when compared to the less responsive behavior scenario. This may seem counterintuitive since it is expected that fewer recommendations are necessary to achieve the desired proportions under more responsive behavior. However, note that under this type of behavior (more responsive), the iterative search procedure achieves its objective in fewer (about 5) iterations. After 5 iterations in Fig. 7(a), the estimated proportions are almost constant, but the recommended proportions in Fig. 7(b) still have substantial variability. This implies the existence of multiple solutions, due to the interdependencies discussed in Section 5.1.4. For example, in Fig. 7(b), in the neighborhood of 6 iterations (when the desired proportions are achieved for the more responsive case), the controller still needs to provide less information under the more responsive case compared to the less responsive case for route 1, which is the route that requires more recommendations around iteration 50 for the more responsive behavior.

The results from Fig. 7(b) indicate that there are significant behavior-consistency gaps in all cases. That is, there are significant differences between the controller-desired proportion of drivers choosing routes and the proportions of drivers that must be recommended to take the routes in order to achieve the desired proportions. Some of the behavior-consistency gaps are negative, while others are positive. Hence, the experiment results highlight the importance of using a behavior-consistent approach to determine the information-based network control strategies to achieve the controller-desired proportions.

6.3. Experiments: descriptive information only

6.3.1. Specific objectives and design

The objective of these experiments is to evaluate the ability of the fuzzy control model to generate effective descriptive information under the two classes of driver responsiveness to information. Here, all drivers receive descriptive information only. The decision variable here is the vector ϕ that represents linguistic labels describing route conditions.

6.3.2. Experiment results and analysis

Fig. 8 shows the experiment results. Fig. 8(a) shows the controller-estimated proportion of drivers choosing routes for each iteration of the search procedure under the currently calculated vector of information strategies. As indicated in this figure, the controller can achieve close to the desired proportions. The model achieves a faster rate of convergence and values slightly closer to the desired proportions when all drivers are less responsive. This is because when all drivers receive information, the change from one message to another produces a larger discrete effect in the proportion of drivers choosing routes under the more responsive case. Therefore, typically the controller has reduced ability to get closer to the desired proportion due to the large effects of information provision under more responsive behavior.

Fig. 8(b) shows the vector of information values ϕ , the set of messages that the controller provides to the drivers. As illustrated, the procedure converges to a stable set of messages. The messages at convergence indicate that stronger messages are required under less responsive behavior compared to those under the more responsive case. This result is intuitive because stronger messages are needed to compensate the fact that drivers are less influenced by the messages in the less responsive behavior case.

It is not possible to define behavior-consistency gaps for linguistic information because each message represents an unknown proportion of drivers choosing routes. This is another important reason to use a behavior-consistent approach to determine the information-based network control strategies. Traditional approaches cannot incorporate the linguistic nature of information strategies.

6.4. Experiments: prescriptive, descriptive, prescriptive and descriptive, and no information

6.4.1. Specific objectives and design

The objective of these experiments is to evaluate the ability of the fuzzy control model to simultaneously generate effective prescriptive and descriptive information under the two classes of driver responsiveness to information. In these experiments, 25% of the drivers can only access prescriptive information; 25% of the drivers only receive descriptive information; 25% of the drivers have access to prescriptive information and receive descriptive information; and the remaining 25% of the drivers cannot access prescriptive information and do not receive descriptive information. Hence, the information-based traffic control strategies here are the vector ϕ of messages describing routes conditions and the vector θ of proportions of drivers that must be recommended to take routes.

6.4.2. Experiment results and analysis

Fig. 9 shows the controller-estimated proportion of drivers taking routes for each iteration of the search procedure under the currently calculated vectors of information strategies. For both levels of responsiveness, the controller achieves close to the desired proportions. However, it achieves a smoother convergence when all drivers are less responsive to the information strategies. This is because the linguistic messages have a weaker switching effect for these drivers, reducing jumps in the objective function.

Fig. 10(a) shows the results of these experiments for the prescriptive vector of information strategies θ , the proportion of drivers that must be recommended to take specific routes. For both levels of responsiveness, the procedure converges to a relative stable set of values. The vector of prescriptive information strategies at convergence indicates that more recommendations are required for two of the three routes under the more responsive behavior case when compared to the less responsive case. The reasons for this are the same as in the first set of experiments because as shown in Fig. 9, the estimated vector of drivers taking routes reaches the SO proportions in the early iterations of the search procedure with fewer recommendations than the ones around iteration 50 for the more responsive behavior.

Fig. 10(b) shows the results of these experiments for the descriptive vector of information strategies ϕ , the set of messages that the controller provides to the drivers. In both cases, the procedure converges to a stable set of messages, and the set of messages is almost identical. However, the trajectories to achieve the final messages are different; a stronger message is required for route 2 under the less responsive behavior.

The information strategies are the outcome of complex processes resulting from the mutual dependency of prescriptive and descriptive information, as well as the presence of multiple driver classes in terms of information accessibility. These experiments highlight the complexity of the problem faced by the controller and show the effectiveness and robustness of the fuzzy control modeling approach to address the multidimensionality and nonlinearity of the problem.

7. Concluding comments

This study is the first in the literature to propose a methodology to determine behaviorconsistent information-based network control strategies, by factoring the controller's estimation of driver route choice behavior in generating these strategies. It proposes the concept of behavior-consistency gap to illustrate the need for such strategies and to highlight the behavioral inadequacies of existing DTA modeling approaches and the consequent deployment paradigms. Existing deployment mechanisms are primarily categorized as reactive (Hawas and Mahmassani, 1997; Pavlis and Papageorgiou, 1999) or anticipatory (Peeta and Mahmassani, 1995; Peeta and Zhou, 2002). While reactive approaches do not suffice for capturing the dynamics of network spatio-temporal interactions, existing anticipatory mechanisms mostly focus on the effects of high-fidelity traffic flow dynamics combined with rudimentary behavior dynamics (see Peeta and Yu, 2004, 2006). The proposed behavior-consistent approach represents an anticipatory mechanism that is robust in terms of both the traffic flow and behavioral aspects. The study also proposes the concept of controllable routes to formally incorporate driver route consideration behavioral preferences under information provision. Further, the controllable routes approach circumvents a key deployment concern expressed for traditional DTA models, the possibility that travelers are "lied to" by the traffic control center (due to its system-level objectives) and provided sub-optimal routes thereby affecting their level of trust and credibility in relation to the provided information.

The use of a fuzzy logic methodology based on simple *if-then* rules has key implications for modeling realism, deployment convenience, and computational efficiency. It simplifies the controller design and results in a computational efficient approach which is a desirable characteristic for real-time operations. The adequacy of aggregate level generic *if-then* rules based on system observation and problem characteristics for both the controller-estimated behavior modeling and the generation of control strategies circumvents many data needs that

would otherwise be required at the level of the individual driver. A synergistic advantage is that the calibration of the associated membership function parameters is not required for solution accuracy; such calibration only affects computational efficiency in terms of convergence rate. This characteristic is confirmed by Paz and Peeta (2008a) who propose an off-line H-infinity filtering methodology to optimize the membership function parameters of the fuzzy control model leading to significant computational savings. The fuzzy control model also enables the simultaneous consideration of quantitative and qualitative variables, an important characteristic of information-based route guidance. Peeta and Yu (2004) show that such a fuzzy logic based framework can also capture information-related behavioral phenomena over multiple timescales in a unified manner.

The study results highlight the complexity of the problem faced by the controller and show the effectiveness of the fuzzy control modeling approach to address the multidimensionality and nonlinearity of the problem. They also indicate the importance of using a behavior-consistent approach to determine the information-based control strategies. The iterative search procedure was found to converge always to a stable solution in terms of the proportions of drivers that must be recommended to choose routes and/or the linguistic message to provide. A detailed analysis of the experiment results suggests that many driver-preferred routes tend to have large behaviorconsistency gaps because large numbers of drivers take these routes independent of information provision. This implies that to direct the system towards desired proportions of drivers choosing routes, the controller may have to recommend more or less drivers to take some routes depending on the network dynamics and driver behavior tendencies. That is, the effects of driver response behavior to information provision may require more meaningful strategies than those provided under the traditional DTA models to have a reliable estimate/control of system performance. The direct use of the solutions from traditional DTA models (proportions of drivers to be assigned to various routes) may not result in the controller-desired solutions due to the behavior-consistency gap, and can possibly worsen conditions compared to the "no information" scenario.

The problem addressed in this paper is a conceptual sub-problem of the broader traffic routing problem that seeks to minimize system travel time in congested traffic networks. Paz and Peeta (2007) illustrate the effectiveness of the proposed behavior-consistent approach in a rolling horizon based deployment context that captures the network-level interactions in terms of traffic flow and driver behavior. They suggest that behavior-consistent information-based control strategies are superior and entail greater compliance compared to standard DTA-based UE or SO strategies. The current study requires controllable routes to have a full overlap between the controller-desired and driver-preferred routes. Paz and Peeta (2008b) propose alternative paradigms to relax this requirement to provide more flexibility in developing practical information-based control strategies. They also explore insights on directing the system towards the UE state rather than the SO state as UE routes are more likely to overlap with driver-preferred routes.

The proposed approach requires an adequate estimation of driver behavior under information provision. In this paper, the controller-estimated driver behavior model is assumed to provide reasonable estimates. In other work (Paz and Peeta, 2008c), the authors propose an on-line calibration procedure that enables the simultaneous on-line determination of behavior-consistent information strategies and the calibration of the controller-estimated model parameters.

The behavior-consistent framework focuses on personalized information. As generic information is a popular information dissemination mechanism, it would be useful to extend the framework to disseminate multiple types of information. An advantage of the proposed fuzzy

logic based methodology is its amenability to incorporating personalized and generic information simultaneously.

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Category	LHS	RHS			
Controller- estimated	If T is Very Low (VL)	Driver will choose the alternative (O)			
	If <i>T</i> is Low (L)	Driver will probably choose the alternative (PO)			
driver- expected	If <i>T</i> is Medium (M)	Driver is indifferent to the alternative (I)			
travel time (T)	If <i>T</i> is High (H)	Driver probably will not choose the alternative (PN)			
	If T is Very High (VH)	Driver will not choose the alternative (N)			
	If NN is Very Low (VL)	Driver will choose the alternative (O)			
Route	If NN is Low (L)	Driver will probably choose the alternative (PY)			
complexity	If NN is Medium (M)	Driver is indifferent to the alternative (I)			
(NN)	If NN is High (H)	Driver probably will not choose the alternative (PN)			
	If NN is Very High (VH)	Driver will not choose the alternative (N)			
Decorintivo	If Z is "Very Light Traffic" (VLT)	Driver will choose the alternative (Y)			
information	If Z is "Light Traffic" (LT)	Driver will probably choose the alternative (PO)			
(Z) For more	If Z is "Moderate Traffic" (MT)	Driver is indifferent to the alternative (I)			
responsive	If Z is "Heavy Traffic" (HT)	Driver probably will not choose the alternative (PN)			
drivers	If Z is "Very Heavy Traffic" (VHT)	Driver will not choose the alternative (N)			
Descriptivo	If Z is "Very Light Traffic" (VLT)	Driver will probably choose the alternative (PO)			
information	If Z is "Light Traffic" (LT)	Driver is indifferent to the alternative (I)			
(Z) for less	If Z is "Moderate Traffic" (MT)	Driver is indifferent to the alternative (I)			
responsive	If Z is "Heavy Traffic" (HT)	Driver is indifferent to the alternative (I)			
unvers	If Z is "Very Heavy Traffic" (VHT)	Driver probably will not choose the alternative (PN)			
Prescriptive	If <i>Y</i> is Route is Recommended (RR)	Driver will choose the alternative (O)			
(Y) for more	If <i>Y</i> is Route Was Recommended (RWR)	Driver will probably choose the alternative (PO)			
responsive drivers	If <i>Y</i> is Route is Not Recommended (RNR)	Driver will not choose the alternative (N)			
Prescriptive	If <i>Y</i> is Route is Recommended (RR)	Driver will probably choose the alternative (PO)			
(Y) for less	If Y is Route Was Recommended (RWR)	Driver is indifferent to the alternative (I)			
responsive drivers	If Y is Route is Not Recommended (RNR)	Driver probably will not choose the alternative (PN)			

Table 1Behavioral *if-then* rules for the rule-based controller-estimated driver behavior model

Table 2

Control *if-then* rules used by the fuzzy control model to determine prescriptive and/or descriptive information

		Error (<i>e</i>)				
		NL	NS	ZR	PS	PL
	NL	NL	NL	NS	NS	NS
Change	NS	NL	NS	ZR	ZR	ZR
in Error	ZR	NL	NS	ZR	PS	PL
(Δe)	PS	ZR	ZR	ZR	PS	PL
	PL	PS	PS	PS	PL	PL

where: NL = Negative large NS = Negative small ZR = Zero

PS = Positive small

PL = Positive large



Fig. 1. Conceptual framework: (a) traditional DTA-based approach, (b) proposed behaviorconsistent approach.



Fig. 2. Conceptual framework for the behavior-consistent real-time traffic routing problem under information provision.

Fig. 3. Rolling horizon deployment approach.

Fig. 4. Iterative search procedure and fuzzy control model for the determination of the behaviorconsistent information-based control strategies.

Fig. 5. Membership functions used by the fuzzy control model to determine prescriptive and descriptive information.

Fig. 6. Borman network showing the sets of driver-preferred routes (zigzag lines) and controllerdesired routes (dashed lines) for a single O-D pair.

Fig. 7. Results for 100% prescriptive information case: (a) controller-estimated proportion of drivers taking routes, (b) proportion of drivers that must be recommended to take specific routes.

Fig. 8. Results for 100% descriptive information case: (a) controller-estimated proportion of drivers taking routes, (b) messages to provide to drivers.

Fig. 9. Results for 25% prescriptive, 25% descriptive, 25% descriptive and prescriptive, and 25% no information case: controller-estimated proportion of drivers choosing routes

Fig. 10. Results for 25% prescriptive, 25% descriptive, 25% descriptive and prescriptive, and 25% no information case: (a) proportion of drivers that must be recommended to take routes, (b) messages to provide to drivers.