Dynamic Game Theoretic Model of Multi-Layer Infrastructure Networks

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Abstract

Due to similarities in terms of network structure and interactions among them, most infrastructure systems can be viewed as coupled layers of a generalized transportation network in which the passenger, freight, data, water, and energy flows are the commodities in the different layers. The coupling is due to the varying degrees of interactions among these layers in terms of shared physical networks, budgetary constraints, socio-economic environments, environmental concerns, information/other resources, and in particular, functional interdependencies. However, these interactions are normally ignored in the engineering planning, design and analysis of infrastructure systems. Identifying and understanding these interactions using a holistic perspective can lead to more efficient infrastructure systems. This paper presents a preliminary network flow equilibrium model of dynamic multilayer infrastructure networks in the form of a differential game involving two essential time scales. In particular, three coupled network layers – automobiles, urban freight and data – are modeled as being comprised of Cournot-Nash dynamic agents. An agent-based simulation solution structure is introduced to solve the flow equilibrium and optimal budget allocation problem for these three layers under the assumption of a super authority that oversees investments in the infrastructure of all three technologies and thereby creates a dynamic Stackelberg leader-follower game.

Keywords: Infrastructure Networks, Agent-based Simulation, Game Theory, User Behavior, Dynamic Flows

1. Introduction

Infrastructure systems that carry passenger, freight, data, water, and energy flows are key functional elements of a society. Their efficient operation is critical to the enabling of economic and social activities, quality of life, mobility, and as highlighted in recent years, national security and disaster response. Most infrastructure systems involved in generalized transportation are organized as networks. There is a growing awareness that these *infrastructure networks* (INs) are interdependent, and can be thought of as coupled layers of a *generalized transportation network* (GTN). Therefore, the flow patterns, system performance, and investment decisions for these systems can be analyzed as integrated *multi-layer infrastructure network* (MIN) problems in which certain types of flows (or *commodities*) are transported.

In the Transportation arena, studies exist since the early 1970s that cursorily acknowledge the similarities between transportation and telecommunication networks. Dafermos (1972) developed a traffic assignment model that is capable of handling several user classes in the same transportation network using the concept of generalized flows. Each user class has an

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individual cost function and contributes to cost functions of other classes as well. The study indicates that the model can be applied to telecommunication networks in addition to traditional traffic networks, but does not consider interactions. Also, while cost functions are explicitly incorporated, other physical and/or behavioral characteristics of these flows are ignored.

In the late 1970s, the concept of "hypernetwork" was introduced by Sheffi (1978) and Sheffi and Daganzo (1978) to explicitly represent the interactions between multiple transportation modes. The interactions are modeled as sequences of discrete choices when individuals face the route/mode travel decisions on the hypernetwork. Contrary to previous works, this approach starts from a disaggregate level, and then aggregates across individuals to evaluate system-wide performance.

The literature on the MIN problem is rather sparse. However, the importance of this problem has been recognized in recent years due to heightened security concerns, and in particular, the operational linkages across critical infrastructure systems. In the United States, this has led to the formation of a Homeland Security department that coordinates the security-related functions of multiple individual federal agencies that previously operated without explicit coordination. Rinaldi et al. (2001) discuss critical infrastructure interdependencies by highlighting some examples of cascading failure phenomena, whereby the malfunctioning of one infrastructure system can have severe negative consequences on other systems. They discuss various dimensions of infrastructure interdependencies by identifying the types of interdependencies, infrastructure operation environment, degrees of coupling, infrastructure characteristics, and types of failures. They also identify some modeling and simulation challenges for addressing the infrastructure dependency problem. Heller (2001) summarizes some recent studies on interdependencies across civil infrastructure systems. She emphasizes the importance of information infrastructure in the operation of various infrastructure systems, and proposes the concept of "integrated information infrastructure systems" and "meta-infrastructure systems". However, both these articles are descriptive and do not propose modeling approaches.

Haimes and Jiang (2001) propose a Leontief-based input-output model to formulate the interdependencies of interconnected critical infrastructures in terms of failure risk. They consider an "economy" consisting of n critical infrastructure systems that are thought of as interconnected production sectors in the economy. In this system, the output of each sector is the risk of "inoperability" of the associated infrastructure network, and the input to the sector can be in terms of failures due to accidents, natural hazards, or acts of terrorism, in addition to the negative impact from the failure of another sector. The quantity, quality and likelihood of failures are converted into an expected level of failure, and this risk is measured in monetary terms. A preliminary study on the dynamics of such risk is also discussed using a Leontief-based dynamic model. By applying a Leontief input-output model, which is the classical approach to formulate interdependencies among interconnected sectors in an economic system, the study introduces a potentially powerful tool to analyze the interactions among critical INs. However, it focuses on the interactions of failure risk among the various INs, which is only one aspect of infrastructure interdependencies.

Friesz et al. (2001) introduce the concept of multi-layer infrastructure networks that involves generalized transportation. They study the interdependencies among the different layers by using a spatial computable general equilibrium (SCGE) model. The concept of interdependency is generalized, and five sources of interdependencies are identified. The various layers of the MINs are formulated as the transportation sectors connecting multiple regions in an economy, in which each layer carries a particular type of commodity. When the markets of all commodities are cleared and the flows in each sector are distributed in the corresponding network, a general equilibrium is reached in the economic system. In addition, a MIN capital budgeting model is proposed for the budget allocation problem in which the impacts of inter-layer interdependencies are explicitly considered.

Nagurney and Dong (2002) propose the concept of a unified “supernetwork” to capture the interactions among transportation, telecommunication, energy, and financial “subnetworks”. This approach is mathematically substantially similar to the Sheffi-Daganzo hypernetwork perspective but stresses different types of infrastructure rather than different transportation modes. Nagurney and Dong illustrate their perspective through case studies using subsets of these infrastructure systems in application domains such as supply chain modeling, telecommuting, teleshopping, and electronic commerce. However, this work does not deal with the key questions of multiple time scales and linkage constraints (beyond the standard notions of flow conservation). The work is largely static in perspective, and considerations of dynamics are based on the notion of projection of trajectories onto constraint boundaries, so that all agents follow constraint boundaries and do not visit the interior of the relevant feasible region.

Some relevant studies in a more narrow and limited context consider only a subset of these infrastructure systems or certain aspects of the interdependence. These include the telecommuting problem (Salomon, 1997; Mokhtarian and Meenakshisundaram, 1999; Choo et al., 2001), shared inter-state resources problem (Apogee Research, 1996), and the telecommunications needs for intelligent transportation systems deployment (Gianni and Moore, 1997; Johnson and Thomas, 2000). They provide analytical/practical results as well as behavioral insights by addressing only limited aspects of the infrastructure interdependency problem.

These studies and real-world events suggest that identifying and understanding the interactions among infrastructure systems using a holistic perspective can potentially lead to more efficient infrastructure systems. However, these interdependencies are generally ignored in engineering practice, which typically addresses infrastructure systems in isolation. There are several reasons for this common perspective. First, engineering infrastructure systems are complex even at an individual level leading to a significant degree of difficulty if the scope is broadened to include multiple systems. Second, different INs are planned, designed and operated by different public, private and/or public-private sectors without explicit coordination. Third, the degree of coupling across INs can vary substantially implying weak interactions in some cases and strong ones in others. However, events in the recent past suggest that the explicit consideration of multiple INs simultaneously can be essential to circumventing unintended catastrophic consequences, even if the interactions between any subset of them are weak. For example, this scenario can manifest as cascading failures of IN components that cripple essential societal functions across several of them. Also, a key economic benefit of considering multiple INs simultaneously is in terms of enabling more informed engineering decisions and/or resource allocation strategies.

The expected benefits of considering infrastructure interdependencies are many-fold. Recent events suggest that addressing INs individually may lead to wasted resources, operational inefficiencies, and at times cripples some subnetworks completely. The California energy crisis in the summer of 2000 is an excellent case study in this context. The deregulation of the power industry, aided by the lack of a holistic perspective, led to severe power shortages in that state when extreme weather conditions coincided with high power needs leading to debilitating cascading effects on the telecommunication and water networks. The rolling power cuts in the Silicon Valley region crippled data networks and Internet functionality affecting businesses nation-wide. Another example of cascading effects across infrastructure systems is the port union strike on the U.S. west coast in 2002. This had a ripple effect on the air, rail and road transportation systems, and on commerce and e-commerce. It is an example of functional interdependencies across in infrastructure systems.

If such linkages are identified in the planning stage, cascading phenomena can be circumvented or at least planned for in terms of contingency measures. An important caveat that further emphasizes the need for capturing IN interdependencies is that some of the cascading catastrophes may not manifest when an IN is being analyzed in isolation. This implies that system failure and the impacts of catastrophic phenomena can be alleviated to some extent by explicitly considering the linkages, even in the operational stage. For example, the terrorist attack in New York in September 2001 led to the shutdown of several infrastructure systems due to the cascading effects across INs. This highlights the notion that when multiple INs fail, there may be strong interdependencies across these failures. By contrast, a severe road traffic accident may lead to the partial failure of the road IN. This indicates the need to introduce redundancies in individual INs to alleviate the cascading effects on other INs. For example, disaster response strategies involving the road infrastructure subnetwork should focus on strengthening multiple critical routes based on factors other than population coverage alone. These routes should also factor in access to critical infrastructure systems such as power and water systems to dilute the cascading effects across other INs.

The consideration of multiple INs simultaneously entails the addressing of three key aspects in a research context. First, there is a need to identify the types and degree of the interactions among different INs. A specific IN can have varying types and degrees of interactions with other INs. For example, an automobile transportation subnetwork has physical interdependencies with a road-based freight transportation subnetwork in terms of shared right of way. This is a strong interaction as travel delay characteristics are highly correlated. By contrast, the interdependency between the automobile and water subnetworks in a region is typically much weaker though they may have similar network structures. Further, the level of interactions between the same IN pair can themselves vary with other factors. For example, in areas with dense population, the characteristics of the telecommunication systems may have a significant impact on the performance of the transportation system through phenomena such as telecommuting and teleshopping. By contrast, such interactions may be less significant in a sparsely populated area.

The second research aspect is the need to develop a new generation of methodological constructs that can explicitly capture the interactions among INs and analyze their impacts. Currently, systematic methods to address MIN problems do not exist. This manifests as the need to model flow dynamics across the different layers of MINs to capture the various interactions. In addition, there is a need to develop broad-based resource allocation procedures that capture these interactions vis-à-vis investment decision-making. This is important because investment and improvements in one IN may influence the performance of other INs. For example, in some regions, improving the telecommunication infrastructure may induce more people to work from home, reducing the demand and increasing the service levels on the transportation network.

The third research aspect is the need to develop efficient solution methodologies for the MIN problems. The modeling of a large-scale individual IN is inherently complex due to the large problem size, stochasticity in user behavior, and existence of disturbances. The MIN problem combines multiple INs under a single framework. The explicit consideration of coupling among these INs, in addition to the dynamic nature and the nonconvexity/nonlinear properties of the various MIN components, adds an extra dimension of complexities to the problem. These prevent the effective application of traditional analytical methods to solve the problem. This entails the consideration of non-traditional computational intelligence techniques and simulation-based approaches that go beyond the traditional methodologies typically applied in the civil infrastructure systems domain.

Studying the interdependence of infrastructure networks is a relatively new and complex research domain, and many issues need to be addressed in order to reveal the nature and engineering significance of interdependencies in the MINs. This paper is a preliminary effort to introduce some basic concepts and methodologies to analyze IN interdependencies. We propose a generic framework to address MIN problems, and focus on the network flow equilibrium analysis problem. Other aspects, such as risk management and emergency response can also be addressed using this framework, but are not the focus of this paper. Section 2 introduces the sources and types of infrastructure interdependencies, and lists key challenging issues in the problem context. Section 3 presents a preliminary three-layer (auto, urban freight, and data) flow dynamics MIN model based on existing single-layer subnetwork flow dynamics models assuming the telecommunication (data) sector as the leader in the Stackelberg game and the authority controlling the data/information network. This authority is also informed on the auto and urban freight networks. Section 4 proposes an agent-based modeling and solution framework for the MIN model, and discusses some preliminary experimental insights. Concluding comments are presented in Section 5.

2. Basic Concepts

In this section, we introduction some basic concepts related to multi-layer infrastructure networks. Section 2.1 introduces the similarities among different infrastructure systems in terms of structure, flow characteristics and system operation. Section 2.2 identifies six types of interdependencies that exist among INs. Section 2.3 lists some challenging issues for MIN problems.

2.1. Similarities in Infrastructure Networks

The various infrastructure systems in a GTN can be structurally characterized as networks. This can be a key source of interdependencies among them.

2.1.1. Structural Characteristics. Infrastructure systems are organized as networks. They are composed of basic network elements such as nodes, links, and paths. For transportation networks, the nodes are traffic intersections, activity (residence, business, shopping, recreation, etc.) centers, cities, logistic origins/destinations/hubs, railway stations/yards, airports, seaports, etc. The links are the highways, freeways, city streets, railways, air lines and seaways. For telecommunication systems, the nodes are computers (network servers, routers, and terminal clients), telecommunication exchange stations, satellites, etc., while the links are telephone lines, cables, fiber optic cables, wireless radio/microwave linkages, etc. Water (drinking and sewage) systems consist of processing stations, pump stations, and storage towers as nodes, and pipelines as links. In energy systems, nodes are electricity plants, transformer substations and gasoline stations, while the links are electricity transmission wires, pipelines, or roads.

Viewed in a regional context, the various infrastructure systems typically serve the same economy, society, and population. Therefore, their physical and operational structures are constructed to accommodate the common demand characteristics and activities. For instance, if the population and/or activity levels of a region are dense, it may contain more nodes and links of every IN type.

2.1.2. Flow Characteristics. Infrastructure networks carry different types of flows. In transportation systems, the flows are vehicles (passenger and freight), trains, airplanes, and ships. In telecommunication systems, the flows are message and data (text, voice, video, image, etc.). In energy systems, the flows are electricity and gasoline. These flows share some common properties. First, the demand generation of these flows is highly dependent on the network user needs and decisions. Second, randomness in flow pattern exists for most INs, though the degree of randomness may vary. The randomness in user behavior causes demand fluctuation, while recurrent flow patterns also exist due to specific behavioral tendencies. In addition, the system performance can also be random due to unpredictable factors such as debilitating events and/or severe weather conditions. Third, the design and analysis of INs typically assume that flows intend to (or can be controlled to) reach equilibrium states and/or satisfy some controller objectives. Fourth, the methodologies, tools, models and algorithms used to study flow equilibria across different INs are similar. For example, graph theory, queuing theory and optimization methods are widely used in the design and operations of INs. Next, all INs have capacity constraints, highlighting the importance of the resource allocation problem. Finally, all INs are susceptible to failure. This has key ramifications for interdependencies across INs and emphasizes the need for redundancy, reliability and robustness.

2.1.3. System Operational Characteristics. INs are essential to the fundamental economic and social activities of a region, and are hence closely related to public good. Therefore, public agencies are involved to some extent in the investment, planning, design, maintenance, management, and/or operation of most INs. The privately owned INs, such as telecommunication, energy and water, are normally oligopolistic markets with some major service providers in each region due to the huge investment needed and the large-scale nature of these systems. Most IN owners/operators are guided by caveats such as profit maximization, capacity maximization, or delay minimization in their planning and operational procedures. The INs also interact with the socio-economic environments in which they operate. The IN system users are influenced, to varying degrees, by the operators. However, the operators do not have full control on user decisions.

2.2. Types of Interdependencies in MINs

Six forms of interdependencies among the INs are identified in this study:

1. Physical interdependencies. Some networks are coupled by shared physical flow rights of way leading to joint capacity constraints. Data and telecommunications networks are an example. Infrastructure facilities may also share the same geography even though the flows do not share capacities. An example is the shared right of way between road transportation and telecommunication networks.
2. Functional interdependencies. The construction and operations of one IN may rely on the support from other INs. For instance, electrical power is needed for the functioning of most other INs. Another example is the need for data and information transmission for efficient transportation operations under advanced information systems.
3. Budgetary Interdependencies. Many infrastructure systems associated with GTN involve some degree of public financing so that the financing of one IN either directly or indirectly affects the financing of others.
4. Market interdependencies and spatial economic competition. With the increasing globalization of the world's economy and the trend toward ever more intelligent infrastructure, spatially separated supplies and demands for the services and goods exchanged over INs generally from a single global competitive market and, thereby, influence one another even when other explicit interdependencies are not manifest. Moreover, because of the public good aspect of many INs, numerous governmental regulations exist and are

emerging that control both intra- and inter-layer aspects of the spatially extended economic competition that occurs via INs.

5. Information Interdependencies. With recent advances in enabling information technology, comprehensive data and information infrastructures are commonly available. As a consequence, database sharing and information exchange among individual INs provides synergism and cost-efficiency. For example, urban water and energy utilities may share information on the socioeconomic characteristics of individual households to more consistently predict future demands.

6. Environmental Interdependencies. The increasing reflection of environmental issues in infrastructure policy decisions, coupled with the direct impacts of the ambient environment on various INs, indicates potential environmental interdependencies among various INs. For example, hazardous material spills can manifest as short-term effects on the flow of goods and passengers, and potentially long-term effects on nearby water networks and/or ecosystems.

2.3. Key Modeling Issues

The conceptual identification of interactions among the different layers of a MIN is reasonably straightforward. However, their quantification and systematic formulation is substantially more involved. Some of the key modeling issues include:

1. The modeling issues of MINs depend on the objectives of the associated problems. As illustrated by the literature review in Section 1, research efforts in the infrastructure interdependencies focus on two major aspects: disequilibrium analysis (Rinaldi et al., 2001; Haimes and Jiang, 2001) and equilibrium analysis (Friesz et al., 2001; Nagurney and Dong, 2002). The former emphasizes the phenomena of cascading failures or risk transmission caused by interdependent infrastructures, while the latter aims to develop general network flow equilibrium models for multiple systems, considering their flow interaction and other interdependencies. Both classes of problems study the evolving flow patterns or system states based on the physical and/or functional linkages among IN layers, and the behavioral characteristics of participating agents. Due to the behavioral implications, the equilibrium analysis problems are most naturally formulated as games with multiple self-interested players.

In addition to the equilibrium/disequilibrium (descriptive) analysis, prescriptive problems can also be formulated for multilayer infrastructure networks. Prescriptive problems, such as budget allocation, network planning and design, risk management and emergency response problems, aim to optimize the overall network performance based on system-level criteria such as cost minimization, social surplus maximization, risk minimization, or recovery time minimization after network failure. These problems are normally formulated using a top-down approach. In many cases, the equilibrium or disequilibrium analysis is the basis for the prescriptive problems.

MIN problems can also be differentiated by the implementation timescale. Typically, system optimization can be employed in both long-term planning and short-term operations of infrastructure systems. By contrast, equilibrium analysis is mostly used for long-term planning and can be the basis for prescriptive problems which contain equilibrium constraints. Disequilibrium analysis is primarily meaningful in a short-term context, and can be used to address both long-term (risk analysis and management) and short-term (emergency response) security scenarios.

2. The flow dynamics of different MIN layers can have different time scales. For example, in the MIN that includes auto, urban freight and data, the transportation networks can be characterized at a day-to-day level while the telecommunication networks have a within day basis. Changes in traffic flow patterns occur on a day-to-day scale as travelers update their

travel decisions based on the current day's experience. However, telecommunication flows change much faster and dramatically due to the "burstiness" property of data transmission demand, even compared to the within day traffic dynamics. Synchronizing the time scales of the different layers is a challenging and critical issue for flow interaction problems in a MIN.

3. The flow characteristics scale is another critical issue in the formulation of MINs. For example, when road transportation and data networks in a region are considered, the flow scales can be significantly different in terms of their influence on the corresponding IN performance. Road transportation networks can be highly congested during the peak periods of traffic flow. A small percentage reduction in this flow through telecommuting can significantly influence the traffic system performance. However, its effects on the telecommunications network are asymmetric and negligible due to the due to the relatively much higher capacities of data networks compared to data flow changes. This significantly enhances the complexity of capturing the impacts of flow changes in telecommunications networks.

4. The performance characteristics scale can also introduce a significant complexity to the formulation of MIN problems. As illustrated by the MIN problem involving road transportation and telecommunications networks, the magnitude of performance measures are much more perceptible for transportation networks than for telecommunication networks under normal conditions. Hence, formulating robust performance operators for telecommunications networks is significantly more difficult than for transportation networks.

5. An additional source of complexity for the MIN budget allocation problem is the difficulty in enabling coordinated investment decisions due to the disparate nature of the ownership of the different IN layers. A simple mechanism is to assume a super authority that makes coordinated resource allocation decisions. However, the formulation can be significantly more complex when public and private operators co-exist in a MIN problem, and or oligopolistic entities exist within individual layers. This is because different entities can have different goals, strategies, and financial capabilities.

3. Flow Dynamics Models

The modeling of flow dynamics and equilibrium tending flows are key issues of the MIN problems. In traditional formulations, the equilibrium tending flows are considered one network at a time. However, subnetworks are not isolated in a multi-layer IN framework. The various interdependencies must be explicitly represented in terms of flow interactions, shared capacity constraints, and/or combined budget constraints.

In this section a preliminary formulation is presented for a three-layer MIN flow dynamics problem. First, the single-network flow dynamics models are introduced for auto, urban freight, and data subnetworks. Then, these models are combined to obtain a three-layer MIN model. This modeling uses a game-theoretic approach because the auto and freight flow dynamics are based on a fixed-point formulation of a Cournot-Nash equilibrium of games while travelers, travel information providers, freight shippers, and carriers are treated as self-interest players in the games.

3.1. Notation

We use the standard notation of equilibrium models in the three single-layer and three-layer network modeling formulations.

A	the set of arcs, $\lvert A\rvert = m$;
N	the set of nodes, $\lvert N\rvert = n$;
$N_o \subseteq N$	the set of nodes which are trip origins;

$N_D \subseteq N$	the set of nodes which are trip destinations;
P_{ij}	the set of paths for origin-destination (O-D) pair *i*, *j*;
P	the complete set of network paths indexed by *p*;
w_{ij}^p	an element of the path-(O-D) pair matrix; specifically, ω_{ij}^p =1 if path *p* connects O-D pair (*i*,*j*) and ω_{ij}^p =0 otherwise;
$W = (w_{ij}^p)$	the path-(O-D) pair matrix;
γ_{ap}	an element of the arc-path incidence matrix; specifically γ_{ap} = 1 if arc $a \in p$ and γ_{ap} = 0 otherwise;
$\Gamma = (\gamma_{ap})$	the arc-path incidence matrix;
$h_p(t)$	the flow on path *p* at time *t*, measured as the flow at the entrance of the first arc of path *p* at time *t*;
$h(t)$	the full vector of path flows at time *t*;
$f_a(t)$	the commodity flow on arc *a* at time *t*; $f_a(t)$ and $h_p(t)$ are related by the identity $f(t) = \sum_{p \in P} \gamma_{ap} h_p(t)$;
$f(t)$	the full vector of arc flows at time *t*;
$u_{ij}(t)$	the travel cost estimated by the ATIS for origin *i* and destination *j* on day *t*;
$u(t)$	the full vector of estimated travel cost on day *t*;
$c_p[h(t)]$	the unit cost of flow on path *p* on day *t*, as a function of the full vector of path flows *h(t)*;
$c[h(t)]$	the full vector of path costs on day *t*;
$T_{ij}[u(t)]$	the travel demand between *i* and *j* on day *t*;
$T[u(t)]$	the full vector of travel demands on day *t*;
$\Theta_{ij}(T[u(t)])$	the inverse travel demand between *i* and *j* on day *t*;
$\Theta(T[u(t)])$	the full vector of inverse travel demands on day *t*;
$\pi_i(t)$	the supply price of commodity *i* in location *r*;
π	the vector of commodity supply price on day *t*;
$d_i^r(\pi)$	the demand function in location *r* for commodity *i*;
$a_{ij}^{rs}(\pi, u)$	the input-output coefficient of productive activity *j* in location *s* relative to input commodity *i* produced in location *r*;
$A(\pi, u)$	the price and transport cost dependent activity analysis matrix;
$\bar{S}_i[\pi(t)]$	the effective commodity supply function for market *i*;
$\bar{S}[\pi(t)]$	the vector of effective commodity supply;
$k_i(t)$	the capacity of the industry in region *i* on day *t*;
$k(t)$	The vector of capacity on day *t*.

In order to formulate the multi-layer model, extra superscripts are needed for some variables. The superscript *A* refers to auto, *F* to urban freight, and *D* to data. For example, the notations h^A and h^F are vectors of path flows in auto and urban freight networks, respectively. An additional set of variables *y* is introduced to represent the improvements on each layer through investment. That is, y^A, y^F, and y^D are the vectors of improvements made to the auto, urban freight, and data subnetworks through investment.

3.2. Single-layer Dynamic Network Equilibria Models

Single-layer dynamic network equilibria models address one network at a time. The flow dynamics models of auto, urban freight, and data subnetworks are discussed in this subsection. These single-layer models are based on Friesz et al. (1994), Friesz et al. (1998) and Friesz et al. (2004).

3.2.1. Auto Layer Flow Dynamics Submodel. A day-to-day dynamic network equilibrium model is briefly introduced in this subsection. A detailed description of this model can be found in Friesz et al. (1994). It assumes that drivers change their behavior (travel demand and route choice) on a day-to-day basis based on information on network conditions provided by an advanced traveler information system (ATIS) on each day. Such behavioral change causes the adjustment of network flows from one disequilibrium state to another following the traditional Wardropian user equilibrium principle.

At the start of each day for the period of interest, the ATIS provides each driver the estimated travel cost on the various routes for that day. The driver uses this information to obtain the equilibrium costs on the paths connecting the O-D pair of his/her interest. Based on this perceived information, each driver decides whether to make the trip and the trip route. The aggregate of the decisions of all drivers determines the amount and distribution of flows on each link. This is equivalent to the *demand* in an economic system, and can be represented as an inverse demand function:

$$\Theta_{ij}^{A}(T^{A}[u^{A}(t)]) = \Theta_{ij}^{A}(\sum_{p\in P_{kl}} h_p^A : k \in N_o^A, l \in N_d^A) \equiv \Theta_{ij}^{A}(h^A) \tag{1}$$

where $h^A = (h_p^A : i \in N_o^A, j \in N_d^A, p \in P_{ij}^A)$ is the full vector of path flows. This implies that the travel costs estimated and disseminated by the ATIS are ultimately a function of the actual flows in the network. The *excess travel cost*, measured as the difference between the drivers' current actual average travel cost and the cost for their O-D pairs reported by the ATIS, is expressed as:

$$ETC_p[h^A(t)] \equiv c_p^A[h^A(t)] - \Theta_{ij}^A[h^A(t)] \tag{2}$$

The difference between the number of drivers who would have traveled (based on the inverse cost function) had the estimated O-D travel cost been realized and the total actual path flow for a given O-D pair is treated as *excess demand*. The ATIS thereby makes an adjustment on the estimated travel cost for the following day in a way that as excess demand increases (decreases), the broadcasted O-D travel cost increases (decreases) for the next day to reflect the relative scarcity (surplus) of transportation services. Thus the excess transportation demand can be expressed as:

$$ETD_{ij}[u^A(t), h^A(t)] \equiv T_{ij}^A[u^A(t)] - \sum_{p\in P_{ij}^A} h_p^A(t) \tag{3}$$

As a consequence, such a system can be readily stated as a global projective dynamic system (Smith et al., 1997). Define:

$$v_p(t) = \Pr_{\Omega}\{h_p^A(t) - \beta^A ETC_p[u_{ij}^A(t), h^A(t)]\},\ \beta^A \in \Re_+^1$$

where $\Pr_{\Omega}\{\cdot\}$ denotes an operator that projects the infeasible values onto the closed set of constraints Ω pertinent to the analysis to avoid infeasibilities. Therefore, $v_p(t)$ can be viewed as the instantaneous revision of the path preference in accordance with continuously provided excess cost information. Imposing the initial conditions, we have the full vector of excess costs as:

$$ETC[u^A(t), h^A(t)] \equiv (ETC_p[u_{ij}^A(t), h^A(t)] : i \in N_o^A, j \in N_d^A, p \in P_{ij}^A)$$

$$\max(0,v) \equiv \{v\}_+$$
$$h^A(t=0) = h^{A,0} \in \Re_+^{\rho}$$

With the coefficient vector $\eta^A \equiv (\eta_p^A : p \in P^A)$, the flow dynamics in the auto layer can be expressed as:

$$\frac{dh^A(t)}{dt} = \eta^A \{ \{ h^A(t) - \beta^A ETC[h^A(t)] \}_+ - h^A(t) \}, \ \forall t \in [0,T]$$

where

$$h^A(0) = h^{A,0}$$

and T is the period of interest.

Discretizing the above flow dynamics, and including the capacity enhancement variables, a discrete time flow dynamics can be expressed as:

$$h_{p,\tau+1}^A - h_{p,\tau}^A = \eta_p^A [\{ h_{p,\tau}^A - \beta_{ij}^A [\theta_{ij,\tau}^A (\Gamma h^A) - c_{p,\tau}^A (h_\tau^A, y_\tau^A)] \}_+ - h_{p,\tau}^A] \quad (4)$$

$$h_{p,0}^A(0) = h_p^{A,0} \quad (5)$$

$$\forall i \in N_o^A, \ \forall j \in N_D^A, \ \forall p \in P^A, \ \forall \tau \in [0, 1, 2, \ldots, T\text{-}1]$$

For details of the auto flow dynamics models, see Friesz et al. (1994).

3.2.2. Urban Freight Layer Flow Dynamics Submodel. The second layer to be discussed is the freight transportation subnetwork. Though the freight subnetwork shares the same physical network with the auto subnetwork, they have substantially different characteristics in terms of demand generation, user behavior, and decision variables. Therefore, a separate flow dynamics model is built for this layer. This model is based on Friesz et al. (1998).

In this formulation, the interregional commodity flow dynamics is modeled by introducing a disequilibrium adjustment mechanism in which the commodity prices and interregional flows follow distinct signals, and constraints ensuring balanced trade flows are not enforced prior to attaining an equilibrium. In a traditional spatial price *tatonnement* process, a central auctioneer collects information from and provides information to consumers, producers and carriers. Trade cannot be realized until the equilibrium commodity prices are reached in each region. In this model the tatonnement process is modified to a *non-tatonnement* process in a sense that some feasible production and consumption will generally occur continuously along a realizable disequilibrium trajectory until the market is cleared.

The process assumes that the economy of interest is completely competitive, and for each firm in the economy there is a technologically optimal production level at which the firm produces and supplies. Consumers and producers are modeled as players with distinct goals and rules in a non-cooperative game. The transportation costs between regions are explicitly considered in addition to the spatial price differences of the commodity. The system starts from a disequilibrium state, and adjusts to the Cournot-Nash equilibrium when the market is cleared. During the adjustment process, the commodity prices in different regions respond to the excess commodity demand, which is expressed as the difference between the freight transportation demand that would have been realized based on the demand function responding to the current commodity price in each region and the effective supply function which decides the demand that actually manifests in this region, plus the difference between inflow and outflow for this region:

$$ECD_i[\pi(t), h^F(t)] = D_i[\pi(t)] - \bar{S}_i[\pi(t)] + \sum_{j \in N_d^F} \sum_{p \in P_{ij}^F} h_p^F(t) - \sum_{j \in N_o^F} \sum_{p \in P_{ji}^F} h_p^F(t) \quad (6)$$

$$\forall i \in N_d^F, \ \forall t \in [0,T]$$

where $\bar{S}_i[\cdot]$ is the effective commodity supply function for market *i*. Akin to the auto submodel, we also consider the excess price in the urban freight subnetwork. The difference is that in the auto layer the "price" is the travel cost, and in the urban freight layer it is the commodity price in the different regions. The excess delivered commodity price is expressed as:

$$EDP_p[\pi(t), h^F(t)] = \pi_i(t) + c_p^F[h^F(t)] - \pi_j(t) \quad (7)$$

$$\forall i \in N_o^F,\ \forall j \in N_d^F,\ \forall p \in P_{ij}^F$$

The transportation flows adjust in response to the delivered price, which is the summation of the commodity price in the producing node and the transportation cost that is decided by transport agents (carriers).

On the producer side, we assume that the industry in a region can only produce limited amount of the commodity. The industry capacity adjusts in response to excess industry capacity, which is measured as the difference between the present capacity and market demand under the prevailing market prices:

$$EIC_i[\pi(t), h^F(t), k(t)] = k_i(t) - \{D_i[\pi(t)] + \sum_{j \in N_d^F} \sum_{p \in P_{ij}^F} h_p^F(t) - \sum_{j \in N_o^F} \sum_{p \in P_{ji}^F} h_p^F(t)\} \quad (8)$$

$$\forall i \in N_o^F,\ \forall t \in [0, T]$$

The corresponding vectors of the above excess demand, price, and industry capacity can be expressed as:

$$ECD[\pi(t), h^F(t)] \equiv (ECD_i[\pi(t), h^F(t)] : i \in N_d^F)$$

$$EDP[\pi(t), h^F(t)] \equiv (EDP_p[\pi(t), h^F(t)] : i \in N_o^F,\ j \in N_d^F,\ p \in P_{ij}^F)$$

$$EIC[\pi(t), h^F(t), k(t)] \equiv (EIC_i[\pi(t), h^F(t), k(t)] : i \in N_o^F)$$

The initial conditions on the commodity price, path flow and industry capacity are:

$$\pi(t=0) = \pi^0 \in \Re_+^{|N^F|}$$

$$h^F(t=0) = h^{F,0} \in \Re_+^{|P|}$$

$$k(t=0) = k^0 \in \Re_+^{|N_o^F|}$$

The price, flow and capacity dynamics in the urban freight subnetwork can be expressed as:

$$\frac{d\pi(t)}{dt} = \omega\{\{\pi(t) + \alpha ECD[\pi(t), h^F(t)]\}_+ - \pi(t)\} \quad (9)$$

$$\frac{dh^F(t)}{dt} = \eta^F\{\{h^F(t) - \beta^F ETP[\pi(t), h^F(t)]\}_+ - h^F(t)\} \quad (10)$$

$$\frac{dk(t)}{dt} = \gamma\{\{k(t) - \rho EIC[\pi(t), h^F(t), k(t)]\}_+ - k(t)\} \quad (11)$$

with initial conditions

$$\pi(0) = \pi^0 \geq 0,\ h^F(0) = h^{F,0} \geq 0,\ k(0) = k^0 \geq 0.$$

By discretizing the period of interest, we get the discrete time dynamics model:

$$ECD_{i,\tau} = D_i[\pi_\tau] - \{\bar{S}_i[\pi_\tau] + \sum_{j \in N_d^F} \sum_{p \in P_{ij}^F} h_{p,\tau}^F - \sum_{j \in N_o^F} \sum_{p \in P_{ji}^F} h_{p,\tau}^F\}$$

$$\pi_{i,\tau+1} - \pi_{i,\tau} = \omega_i[\{\pi_{i,\tau} + \alpha_i ECD_{i,\tau}\}_+ - \pi_{i,\tau}]$$

$$\pi_{i,0} = \pi_i^0$$

$$EDP_{p,\tau} = \pi_{i,\tau} + c_p[h_\tau^F, y^F] - \pi_{j,\tau}$$

$$h_{p,\tau+1}^F - h_{p,\tau}^F = \eta_p^F[\{h_{p,\tau}^F - \beta^F ETP_{p,\tau}\}_+ - h_{p,\tau}^F]$$

$$h^F_{p,0} = h^{A,0}_p$$
$$EIC_{i,\tau} = k_{i,\tau} - \{D_i[\pi_\tau] + \sum_{j\in N^F_d} \sum_{p\in P^F_{ij}} h^F_{p,\tau} - \sum_{j\in N^F_o} \sum_{p\in P^F_{ji}} h^F_{p,\tau}\}$$
$$k_{i,\tau+1} - k_{i,\tau} = \gamma_i [\{k_{i,\tau} - \rho_i EIC_{i,\tau}\}_+ - k_{i,\tau}]$$
$$k_{i,0} = k^0_i$$
$$\forall i \in N^F_o,\ \forall j \in N^F_D,\ \forall p \in P^F,\ \forall \tau \in [0, 1, 2, \ldots, T\text{-}1]$$

For more details see Friesz et al. (1998).

3.2.3. Data Layer Flow Dynamics Submodel. The characteristics of the data layer are different from those of the auto and urban freight layers. First, the users of the data subnetwork cannot directly access information on the condition of the telecommunications network. Though users can feel that data transmission is “slow” or “fast”, typically they cannot access quantified transmission delay information *a priori*. Second, the users of the data subnetwork do not have to choose a route to transmit data. This function is performed by the network controller (router) based on optimizing some controller objectives. Finally, in some cases the data transmission delay is not a key concern of the data subnetwork user. For example, when sending an email, normally the sender does not know when the email reaches the receiver. In other words, the data delay does not affect the user’s decision. Due to these differences, it is difficult to develop flow dynamics models from the network user perspective unlike for other two subnetworks. However, the data flows can be represented based on how the data packages propagate in the network.

Consider a network exclusively devoted to data communication. Suppose that the scheduled demands meant to be serviced at or before pre-specified times are known with certainty for a finite time interval $[t_0, T]$. The arc delay functions, denoting the traversal time experienced by a data packet on arc a with $x_a(t)$ message volume arriving in front of the packet, take the form of:

$$D_a[x_a(t)] = A_a + \frac{B_a}{K_a - x_a} > 0,\ \forall a \in A^D \tag{12}$$

The path for data routing contains a set of arcs:

$$p \doteq \{a_1, a_2, ..., a_{i-1}, a_i, a_{i+1}, ..., a_{m(p)}\} \tag{13}$$

where $m(p) =$ the number of arcs in the data communication path *p*.

The associated flow dynamics of each arc $a_i \in p$ can be expressed as:

$$\frac{dx^p_{a_i}(t)}{dt} = g^p_{a_i}(t) - g^p_{a_{i-1}}(t) \tag{14}$$

$$x^p_{a_i}(0) = x^{p0}_{a_i} \tag{15}$$

where

$x^p_{a_i}(t) =$ the volume on arc a_i due to flow on path *p* at time *t*

$g^p_{a_i}(t) =$ the flow exiting arc a_i of path *p* at time *t*

$g^p_{a_{i-1}}(t) =$ the flow entering arc a_{i-1} of path *p* at time *t*

Note that $g^p_{a_0}$ is the flow exiting the origin node of path *p*, and is given a special symbol h^D_p to denote the flow on path *p*:

$$g^p_{a_0} \equiv h^D_p$$

The data volume on arc *a* is the summation of the contributions from the paths traversing that arc, and is given by:

$$x_a = \sum_{p\in P} \gamma_{ap}^{D} x_a^{p} \tag{16}$$

where γ_{ap}^{D} is the arc-path incidence variable:

$$\gamma_{ap}^{D} = \begin{cases} 1 \text{ if } a \in p \\ 0 \text{ if } a \notin p \end{cases}$$

We also use $\mathrm{W}^{\mathrm{D}} = \{(i, j) : i \in N_o^D, j \in N_d^D\}$ to denote the set of O-D pairs between which the data packets are moved, where

$N_o^D =$ the set of nodes from which data traffic originates

$N_d^D =$ the set of nodes to which the data traffic is destined

By carefully considering the propagation time and flow dynamics, we obtain the following proper flow progression constraints:

$$g_{a_1}(t + D_{a_1}[x_{a_1}(t)])(1 + D_{a_1}^{'}[x_{a_1}(t)]\dot{x}_{a_1}) = h_p(t) \tag{17}$$

$$g_{a_i}^{p}(t + D_{a_i}[x_{a_i}(t)])(1 + D_{a_i}^{'}[x_{a_i}(t)]\dot{x}_{a_i}) = g_{a_{i-1}}^{p}(t) \tag{18}$$

$$\forall p \in P^D,\ i \in [2, m(p)]$$

These constraints are derived so as to be completely consistent with the point queue model of arc delay.

We further define $D_p(t, x)$ as the path delay operators that tell us the delay experienced by a message packet transmitted at time *t* and encountering traffic conditions *x*:

$$D_p(t, x) \equiv D_p(t) = \sum_{i=1}^{m(p)} \delta_{a_i p} \Phi_{a_i}(t, x) > 0 \tag{19}$$

where $\Phi_{a_1}(t, x)$ are the arc delay operators obeying:

$$\Phi_{a_1}(t, x) = D_{a_1}[x_{a_1}(t)] > 0$$

$$\Phi_{a_2}(t, x) = D_{a_2}[x_{a_2}(t + \Phi_{a_1})] > 0$$

$$\Phi_{a_3}(t, x) = D_{a_3}[x_{a_3}(t + \Phi_{a_1} + \Phi_{a_2})] > 0$$

…..

$$\Phi_{a_1}(t, x) = D_{a_1}[x_{a_1}(t) + \Phi_{a_1} + ... + \Phi_{a_{i-1}}] = D_{a_i}[x_{a_i}(t + \sum_{j=1}^{i-1} \Phi_{a_j})] > 0 \tag{20}$$

We also introduce the arrival penalty operator $\Pi[t + D_p(t, x) - T_A]$ where T_A is the prescribed fixed arrival time with $T_A > T$, and the arrival penalty operator has the properties:

$$t + D_p(t, x) > T_A \Rightarrow \Pi[t + D_p(t, x) - T_A] = \chi^{L}(t, x) > 0 \tag{21}$$

$$t + D_p(t, x) < T_A \Rightarrow \Pi[t + D_p(t, x) - T_A] = \chi^{E}(t, x) > 0 \tag{22}$$

$$t + D_p(t, x) = T_A \Rightarrow \Pi[t + D_p(t, x) - T_A] = 0 \tag{23}$$

for every path $p \in P$. Consequently, the effective delay operator for each path is:

$$\Psi_p(t, x) = D_p(t, x) + \Pi[t + D_p(t, x) - T_A] > 0 \tag{24}$$

Now we are ready to express the dynamic flow routing problem as an optimal control model. Suppose that there is a single agent that sets message transmission rates and determines message routes. The objective of this agent is to minimize the total system delay for the network over the period $[0, T]$ across all O-D pairs:

$$\text{Minimize } J_1[h(t),t] = \sum_{p\in P}\int_0^T \Psi_p[t,x(t)]h_p(t)dt \tag{25}$$

We also assume that there are two types of data demands in the network: scheduled and unscheduled demands. We denote the fixed, unscheduled demand for O-D pair $(i,j)\in W$ by $Q_{ij}\in\Re^1_{++}$, and the scheduled demand for the same O-D pair by $R_{ij}(t)$ at time $t\in[0,T]$. The following flow generation and conservation constraints and non-negativity restrictions hold for every O-D pair $(i,j)\in W$:

$$\sum_{p\in P_{ij}} h_p(t) \geq R_{ij}(t) \tag{26}$$

$$\sum_{p\in P_{ij}}\int_0^T h_p(t)dt = Q_{ij}^D + \int_0^T R_{ij}(t)dt \doteq \tilde{Q}_{ij}^D \tag{27}$$

$$x\geq 0,\ g\geq 0,\ h^D\geq 0 \tag{28}$$

where

$$x \doteq (x_{a_i}^p : p\in P^D, i\in[1,m(p)]) \tag{29}$$

$$g \doteq (g_{a_i}^p : p\in P^D, i\in[1,m(p)]) \tag{30}$$

$$h^D \doteq (h_p^D : p\in P^D) \tag{31}$$

Define:

$$\Lambda_1 = \{(x,h,g) : (17), (18), (26), (27), \text{and } (28) \text{ hold}\} \tag{32}$$

as the set describing the feasible region of the omniscient controller. Then, the overall dynamic flow model can be expressed as:

$$\left.\begin{array}{ll}\min J_1[h(t),t] = \sum_{p\in P}\int_0^T \Psi_p[t,x(t)]h_p(t)dt & \\ s.t. \qquad\qquad (x,h,g)\in\Lambda_1 & \end{array}\right\} \tag{33}$$

Further details can be found in Friesz et al. (2004).

3.3. Three-layer Flow Dynamics Model

Based on the single-layer flow dynamics models discussed in Section 3.2, an integrated three-layer MIN flow dynamics model is presented. Let:

$T_{ij,\tau+1}^A =$ the auto demand during the period $[\tau,\tau+1]$ in the absence of data flows

$Q_{i\ell,j,\tau+1}^D =$ data volume between i and ℓ pertinent to employer at j and worker residing at i during the period $[\tau,\tau+1]$

$N_o^D =$ set of all data sender nodes

$N_d^D =$ set of all nodes demanding data

The effective auto demands in light of the option to work at home afforded by data flows are:

$$T_{ij,\tau+1}^E = T_{ij,\tau+1}^A - \alpha\sum_{\ell} Q_{i\ell,j,\tau+1}^D$$

where $\alpha\in\Re_{++}^A$ is an exogenous parameter. This leads to:

$$T_{ij,\tau+1}^E = T_{ij,\tau+1}^A - \alpha\sum_{\ell} Q_{i\ell,j,\tau+1}^D \tag{34}$$

$$\sum_{p\in P_{ij}^A} h_{p,\tau+1}^A = T_{ij,\tau+1}^E \tag{35}$$

$$[\theta_{ij,\tau+1}^A(T^E) - c_{p,\tau}^A(h^A,y^A)]T_{ij,\tau+1}^E = 0$$

$$\theta^{A}_{ij,\tau+1}(T^{E}) - c^{A}_{p,\tau}(h^{A}, y^{A}) \le 0$$

$\forall i \in N^{A}_{o}$, $\forall j \in N^{A}_{D}$, $\forall p \in P^{A}$, $\forall \tau \in [0, 1, 2, \ldots, T\text{-}1]$, and $(i, j) \in W^{A}$.

The flow conservation constraints for data flows are:

$$\sum_{p \in P^{D}_{i\ell}} \int_{\tau}^{\tau+1} g^{p}_{a_0}(\xi) d\xi = Q^{D}_{i\ell, j, \tau+1} \tag{36}$$

where $P^{D}_{i\ell}$ is the set of paths of the data network, ξ is a dummy variable for continuous time, and constraint (33) is stated for every $\tau = 0,1,2\ldots$. It is important to note here that the data trip matrix elements $Q^{D}_{i\ell, j, \tau+1}$ and the elastic auto demands $T^{A}_{ij,\tau+1}$ are control variables.

Equation (34) demonstrates the interaction between auto and data flows. It assumes that the flows in the two subnetworks are convertible to each other under certain circumstances. Note that $T^{A}_{ij,\tau+1}$ is the potential total auto demand. While several types of traffic activities (such as drive to work or shopping) are substitutable by data communication (such as telecommuting or teleshopping), the effective auto demand can be decided by the potential total auto demand minus the demand switched to the data subnetwork. The switched auto demand can be represented as a function of data volume $Q^{D}_{i\ell, j, \tau+1}$.

The flow conversion represented by Equation (34) can stem from several scenarios, and telecommuting is an example of such switchable flows. It is a work mode that uses the employee's home or a location close to home as the employee work space, and connects to the office, supervisor, colleagues, clients and others through a telecommunications network. Over the past two decades, it has been suggested to policy makers as an efficient way of mitigating vehicular traffic congestion problems and negative consequences thereof. The telecommuting population has been constantly increasing over the past decade due to the quantum leap in information technology development, and this increase is predicted to continue even more rapidly in the future. Telecommuting employees either telecommute on a regular basis, or often in a month. Surveys (Doherty et al., 2003) suggest that between 25 and 65% of jobs in North America and Europe are at least partly telecommutable.

The linkage between the transportation layer and telecommunication layer is in terms of the user's decision on whether to telecommute or commute based on the performance of the data and auto subnetworks. The availability of telecommuting infrastructure is an important factor that influences the choice of telecommuting. According the 1999 National Telework Survey, teleworkers spend 38% of their work time on the computer; 17% on the phone; 24% doing reading, research or analysis; and 9% on face-to-face meetings (Pratt, 1999). One cannot suppose that all the time spent on a computer is dedicated to communication. However, it is reasonable to assume that a considerable portion of communication is done through the computer due to the spawning of the Internet revolution and the versatility of computers vis-à-vis sending and receiving information in different formats such as email, fax, downloading and uploading electronic files through FTP, web browsing, online voice and image exchanging, and real-time data retrieving. A study in Los Angeles (Nilles, 1993) suggests that significant differences exist between telecommuters and non-telecommuters in terms of personal ownership and usage of advanced information technologies. It suggests that a higher percent of telecommuters owns personal computer, while the percentage of telecommuters owning a computer modem is double that of non-telecommuters. It also indicates that a higher percentage of telecommuters use other telecommunication services such as email and audio conferencing. Another study (Allenby and Roitz, 2002) lists the reasons why employees do not adopt telecommuting and why some former telecommuters quit telecommuting. The lack of efficient telecommunication capabilities is identified as the most important reason in both cases. According to a web-based survey (Sina, 2003), the lack of high-speed Internet access is one of the major causes for reduced work efficiency of telecommuters. However, the availability and quality of telecommunication services may not be the primary reason for telecommuting decisions. The technological limitations in terms of data transmission are diminishing due to the increasing availability of broadband Internet access both at home and at work. The organizational restrictions, job characteristics, need for face-to-face communication, work efficiency, and personal or family issues such as health or childcare may have greater influence on the telecommuting choice once a stable telecommunication service is established. In addition, the traffic related concerns such as travel time, gas price and other monetary costs, safety, environmental issues, and land use can be the key factors influencing the telecommuting decisions. Therefore, in the example presented in Section 4, the telecommuting decision of an auto user is primarily based on the auto subnetwork performance.

The auto and data layers can be seamlessly combined under a single day-to-day dynamics framework because the users only need to make a decision on whether to commute or telecommute at the beginning of each day. Once the decision is made, he/she will become a user of the corresponding subnetwork for that day. Therefore, to address the time scale issue, the time scales of the three subnetworks can be treated on a day-to-day basis.

The three-layer model is based on the assumption that a super authority is responsible for the provision of information to commuters and urban freight agents in order to minimize the total social costs of congestion. This minimization is constrained by the equilibrium tending behavior of commuters and urban freight agents. The fiction of the super authority allocating information to minimize social costs is employed in order to calculate the most efficient information-passenger-freight flow patterns. We will use the Pareto-optimal efficient flow patterns found from this model in the next phase of our research (not reported here) to evaluate capital investments.

Thus the problem becomes an optimal control problem with the objective function:

Minimize $(w_1 J_1 + w_2 J_2 + w_3 J_3)$

where J_1, J_2, and J_3 are the subnetwork delay based on the delay operators of auto, urban freight, and data layers, and w_1, w_1, w_1 are corresponding weights. These weights are varied parametrically to generate the set of Pareto optimal (or non-dominated) solutions that may be used in various multi-objective decision making (MCDM) schemes to evaluate the social desirability of a given information allocation by the super authority. Our intent in this paper is merely to show how Pareto optimal alternatives are generated using the original dynamic Stackelberg game-theoretic model we have proposed, and so we do not elaborate on the MCDM techniques that would use Pareto optimal solutions to arrive at a best compromise information allocation plan.

With auto, urban freight, data dynamics, and coupling constraints, a three-layer fast-slow model (where "fast" corresponds to the data subnetwork flow dynamics and "slow" corresponds to the other two subnetworks) can be expressed as:

Minimize $(w_1 J_1 + w_2 J_2 + w_3 J_3)$ (37)

Subject to

$$h^A_{p,\tau+1} - h^A_{p,\tau} = \beta[\theta^A_{ij,\tau}(\sum_{p\in P^A_{ij}} h^A_{p,\tau}) - c^A_{p,\tau}(\sum_{p\in P^A_{ij}} h^A_{p,\tau}, y^A)] \quad (38)$$

$$h^A_{p,0}(0) = h^{A0}_p \quad (39)$$

$$h^F_{p,\tau+1} - h^F_{p,\tau} = R^F_p(h^F, y^F) \quad (40)$$

$$h^F_{p,0}(0) = h^{F0}_p \quad (41)$$

$$\pi_{i,\tau+1} - \pi_{i,\tau} = \omega_i[\{\pi_{i,\tau} + \alpha_i ECD_{i,\tau}\}_+ - \pi_{i,\tau}] \quad (42)$$

$$\pi_{i,0} = \pi_i^0 \tag{43}$$

$$k_{i,\tau+1} - k_{i,\tau} = \gamma_i[\{k_{i,\tau} - \rho_i EIC_{i,\tau}\}_+ - k_{i,\tau}] \tag{44}$$

$$k_{i,0} = k_i^0 \tag{45}$$

$$\frac{dx_{a_i}^p}{dt} = g_{a_{i-1}}^p - g_{a_i}^p \tag{46}$$

$$x_{a_i}^p(0) = x_{a_i}^{p0} \tag{47}$$

$$\sum_{p \in P_{i\ell}^D} \int_\tau^{\tau+1} g_{a_0}^p(\xi) = Q_{i\ell,j,\tau+1}^D \tag{48}$$

$$[\theta_{ij,\tau}^A(\sum_{p \in P_{ij}^A} h_{p,\tau}^A) - c_{p,\tau}^A(h^A, y^A)] \sum_{p \in P_{ij}^A} h_{p,\tau}^A = 0 \tag{49}$$

$$c_{p,\tau}^A(h^A, y^A) - \theta_{ij,\tau}^A(\sum_{p \in P_{ij}^A} h_{p,\tau}^A) \geq 0 \tag{50}$$

$$T_{ij,\tau+1}^A - \alpha \sum_{p \in P_{ij}^A} Q_{i\ell,j,\tau+1} D = \sum_{p \in P_{ij}^A} h_{p,\tau}^A \tag{51}$$

$$(i, j) \in W^A \tag{52}$$

$$p \in P_{ij}^A \tag{53}$$

$$\tau = 0,1,2,3\ldots \tag{54}$$

$$\text{All variables} \geq 0 \tag{55}$$

In addition to the MIN flow dynamics model presented here, a three-layer MIN budget allocation model has also been developed that includes combined budget constraints and an objective function that explicitly considers the overall benefits across the three layers. It adds additional dimensions of complexity to the problem. This is a practically important strategic infrastructure planning problem, especially for decision-making under a centralized authority. However, due to space limitations, this model is not presented here.

The above model can be used to capture several of the interdependencies discussed in subsection 2.2. Since the auto and freight subnetworks share the same physical facility, the cost function of the transportation system depends on both the auto and freight flows. This represents physical interdependency. Also, since the flow can be diverted between data and auto network, the two subnetworks compete for the same group of users. This can be treated as a market interdependency. The model (37)-(55) can also be extended to reflect more infrastructure interdependencies under the proposed modeling framework. For instance, the performance of transportation system can also be the function of data system performance due to the impact of an advanced traveler information system. This can be viewed as a functional interdependency. If a budget allocation model is considered and resources are to be assigned to the various subnetworks, a budgetary interdependency manifests. We do not incorporate these components here because the focus of this paper is on the network flow equilibrium analysis only.

The key modeling issues discussed in subsection 2.3 are also addressed. Model (37)-(55) is a mathematical program with equilibrium conditions, represented by the flow dynamics for the individual layers. The network flow dynamics of the different MIN layers are formulated using separate constraints without intervention, and the flow interactions only manifest through the flow conversion operators. This implies that flow characteristics such as time scale and cost functions can be addressed based on the current available models for the individual infrastructure networks. Budgetary issues can also be addressed under different scenarios. First, a common budgetary resource can be available from public agencies (such as federal/local government) to improve the performance of the individual infrastructure networks. In such a case, a central budgetary constraint is considered for all layers. Second, when the public/private budget is only available to a specific infrastructure layer, a budget constraint is added to that layer, and the resource is distributed to the different components of the layer.

4. Solution Procedure and Preliminary Experiments

As discussed in Section 3, the three-layer flow dynamics MIN model formulated here is based on the three single-layer flow dynamics models. Akin to the single-layer models, the three-layer MIN model can be represented using a system of combined differential and/or difference equations under behavioral assumptions for the various players in each layer. The proposed formulation assumes simple behavioral rules. If it also uses simplified performance operators, desirable mathematical properties such as linearity and convexity are preserved. Such a system of equations can be solved using commercial software. However, when the problem is scaled to a real-world MIN system and incorporates the more involved behavioral tendencies of real-world players and realistic performance operators, key concerns arise in terms of the problem complexity, solution accuracy and computational efficiency, precluding successful implementation of the traditional methods. First, this system is unavoidably non-convex; the non-convexity arises from the non-linear equality constraints that express the coupling of the various IN layers. Second, the accuracy of the flow dynamics and system evolution process of a MIN is highly dependent on the ability to realistically replicate the behavior of various players in each subnetwork. The involvement of human decisions in this process introduces an important source of stochasticity due to the wide variations in user preferences, perceptions and information accessibility, and the learning capabilities of all players. Third, the number of variables and constraints needed to model a large-scale MIN is significant. Hence, the solution for such a complex system using traditional analytical approaches could be prohibitively expensive. Finally, the presence of explicit path variables in the model formulation introduces further complexity; this is a direct result of its dynamic nature and the fact that the generalized transportation demands naturally occur at the origin-destination level. Therefore, non-traditional computational intelligence techniques and/or simulation-based solution approaches are appropriate for the generalized MIN problem. We propose an agent-based simulation (ABS) approach as a general solution mechanism for the generalized MIN problem.

4.1. Agent-based Simulation Solution Approach

Agent-based modeling is a well-established and active branch of artificial intelligence. An agent, by definition, is a computational entity that can be viewed as perceiving and acting upon its environment, and that is autonomous in that its behavior at least partially depends on its own experience (Weiss, 2000). In ABS modeling, different players in a system are represented as intelligent agents interacting with each other and with the environment. The intelligent agents perceive information and pursue specific goals by performing certain actions.

In recent years, ABS has attracted increasing attention in diverse domains such as sociology, economics, engineering, and science as it offers several inherent advantages compared to traditional analytical approaches for problems that lack well-behaved mathematical properties and/or are difficult to represent analytically. Due to its robust ability to handle large-scale problems involving complex behavior, interaction, dynamics, stochasticity, learning, rationing, and decision-making, ABS offers a promising and innovative way to understand, manage and simulate behavior of users in distributed, open, and heterogeneous systems. Another key advantage of ABS is its ability to provide transparent behavioral interpretations for model parameters. By contrast, traditional approaches that rely on regression analysis of aggregate data to estimate model parameters often lack satisfactory behavioral interpretations for complex systems. In addition, ABS can typically represent the behavior of large-scale stochastic systems using a small set of explanatory variables (Parunak et al., 1998).

ABS has several advantages for solving the MIN problem, especially in the context of the tatonnement process that describes the dynamic system evolution process. First, the agents in an ABS are autonomous; given rules and goals, they can "behave" by themselves, and improve their knowledge through a learning process. The behavior preferences, objectives, actions and constraints of agents can be conveniently encapsulated into their rules. This enables a simulator to replicate the system from an individual agent level rather than from a "centralized control" perspective; that is, using a "bottom-up" approach rather than a "top-down" one. This perspective is more consistent with reality. For example, in a transportation system, each traveler typically makes his/her own decisions in a non-cooperative manner based on past experience and perceived system conditions, rather than being tightly controlled by a central authority, even under information provision. Second, the learning capability of agents provides a convenient tool to robustly model system dynamics. Agents learn from their past experience, as well as by sharing information with each other and interacting with the environment in which they live. These characteristics provide flexibility vis-à-vis modeling realism for complex systems such as the MIN problem. This is especially important for the game-theoretic approach used to solve the day-to-day evolution process in the MIN system. Third, ABS allows the convenient representation of a hierarchy structure of various agents and different subsystems. In other words, multiple interacting systems can be modeled as individual agents, each comprising a set of sub-agents. For instance, the individual infrastructure systems such as auto, freight, and data subnetworks can be modeled as agents interacting with each other. At the same time, each individual IN can itself contain a set of intelligent sub-agents whose performance and/or behavior are independent of each other while inheriting common features from a higher class. This enables us to conveniently simulate multi-level multi-class agents with distinct behavioral characteristics. Fourth, ABS can be easily combined with other optimization and solution methods, such as simulated annealing, to improve the computational efficiency. Thus, a hybrid approach can potentially be used to more efficiently search for solutions. Finally, ABS has the capability to easily implement different time scales for different agents. This can aid in solving the complex time-scale issue in generalized MIN problems.

4.2. ABS Solution for the MIN Problem

An ABS solution procedure is proposed to solve the three-layer flow dynamics model on a day-to-day basis. Table 1 illustrates the agents of the highest hierarchy in the ABS modeling for the three-layer MIN problem solution procedure. The agents/environments of auto, urban freight, and data subnetworks, along with the actions, rules and information these agents receive and/or provide, are listed in the table. The actions decide the functions the agents perform in the flow dynamics model, and can be represented by the combined differential and difference equations. Information is the input and output of the actions performed by each agent. Different agents retrieve and provide different information based on the subnetwork they belong to and the actions they can take. Rules are the objectives and/or the behavioral basis for the actions of agents.

Figure 1 shows the interactions between the various agents and environments (subnetworks) on a day-to-day basis for the three-layer MIN flow dynamics problem. For the auto subnetwork, ATIS operators collect information on travel costs from the installed sensors and the traffic demand through interactions with the drivers. They provide the estimated travel costs to drivers whose travel decisions manifest as the traffic demand for the current day.

Based on the estimated travel cost in the traffic subnetwork and the level of service in the data subnetwork, the potential driver chooses to drive to work or telecommute. If a potential driver telecommutes, he/she becomes a data user. In the data subnetwork, the router decides the data routing mechanism based on the prevailing data communication demand and the associated telecommunication network structure. In the urban freight subnetwork, the auctioneer collects consumption and production plans from consumers and producers, respectively, in each region, and computes the excess demand/supply in the market. The carriers obtain the freight transportation demands from the market, decide the freight routing plan over the traffic subnetwork, and estimated the shipping costs to auctioneer. The auctioneer determines the delivered price for a commodity in each region based on the excess demand/supply and shipping costs. The delivered price information is provided to consumers and producers, who in turn modify their consumption, production, and/or capacity change plans. This iterative day-to-day procedure continues until the market is cleared.

The proposed ABS structure coincides with that of the MIN flow dynamics model introduced in Section 3, though only a simplified example is demonstrated in the numerical experiments in Subsection 4.3. The flow dynamics in the MIN are realized by the tatonnement-like behavior of participating agents. This enables the circumvention of optimization-based centralized flow assignment approaches, which lack behavioral basis for MINs, especially for descriptive cases.

4.3. Numerical Experiments

Some preliminary insights are generated on ABS-based solution procedures for MIN problems by conducting numerical experiments for a two-layer MIN consisting of the auto and data subnetworks. Without loss of generality, the freight subnetwork is not considered in these experiments. The experiments are conducted for a small-scale artificial two-layer network under simple behavioral assumptions. Results from a telecommuting scenario are provided to illustrate the potential of ABS for addressing MIN problems.

4.3.1. Experimental Setup. The experiments use a two-layer system consisting of two nodes, three traffic links and one data link, as illustrated in Figure 2, to analyze the capabilities of ABS vis-à-vis articulating the MIN system evolution for a day-to-day time scale. As illustrated in Table 1, the players such as the auto user, data user and traffic network ATIS operator are represented as self-interested, rule-based agents in the simulation. The traffic and data networks are treated as the external environment. Agents collect information such as the perceived traffic costs from the environment. The performance of the environment is influenced by the decision of the agents. Agents also interact with each other. For example, the ATIS operator in the traffic network can disseminate predicted traffic cost information to the auto agents. In our experiments, 10,000 potential auto agents are simulated for each day, and they can be converted to data agents based on the prevailing traffic condition. A single traffic ATIS operator agent is considered.

The performance operators for the traffic subnetwork are the link travel cost functions of the form:

$$c_a(f_a) = A_a + B_a\left(\frac{f_a(t)}{K_a}\right)^4$$

where

$c_a(f_a) =$ travel cost of link a for day t

$f_a(t) =$ traffic flow on link a for day t

A_a, B_a, and K_a = link-specific coefficients

Specifically, the cost functions used for the three traffic links are:

$$c_1(f_1) = 10[1 + 0.15(\frac{f_1}{20})^4]$$

$$c_2(f_2) = 20[1 + 0.15(\frac{f_2}{40})^4]$$

$$c_3(f_3) = 25[1 + 0.15(\frac{f_1}{30})^4]$$

for each solution iteration (day).

Experiment 1: Single-layer Network Experiment

To compare the ABS performance for the single-layer and two-layer networks, a single-layer network experiment is conducted to generate a benchmark. Here, the auto network is used for the single-layer case and user equilibrium is assumed to be the system objective. The total traffic demand is fixed at 100 units. At the beginning of each iteration (day), the agents on the non-optimal routes switch to the best route with a probability that is dependent on the travel cost difference between their current route and the best route for the previous iteration (day). The travel route switching probability for an agent is assumed to be a negative exponential function and is given by:

$$P = e^{\frac{-\theta}{c_{current} - c_{best}}}$$

where

P = the probability of switching from the current route to the best route for that agent

c_{best} = travel cost on the best route

$c_{current}$ = travel cost on the current route

θ = resistivity parameter

The resistivity parameter θ is a measure of the sensitivity of an agent to switching. A higher value of θ translates to a low sensitivity implying that larger travel time savings are required to compel this agent to switch. The value of the resistivity parameter depends on several factors such as the driver's socio-economic characteristics, trip purpose, familiarity to traffic network, and time of day.

As shown in Figure 3.1, with the resistivity parameter θ = 40, the link flows approach equilibrium after 15 iterations using the ABS solution procedure. With 100 traffic demand units, the user equilibrium flows on the three links are 36, 47 and 17 units, respectively.

Experiment 2: Two-layer Network Experiment

In this experiment, a two-layer network containing the auto and data subnetworks is considered. Here, each agent is a single system user who will decide at the beginning of each day (iteration) whether to drive using the traffic links or telecommute using the data link. This decision is based solely on his/her travel experience on the previous day. If the agent experiences long travel delays on the previous day, his/her probability to telecommute increases. If the agent decides to drive to work, he/she also decides which traffic link to take based on the travel costs for the link he/she took and the link with minimal cost on the previous day, as described in Experiment 1.

We assume that the total number of system users is fixed. However, the auto subnetwork user is sensitive to the travel costs because he/she can choose to telecommute. This subnetwork choice is determined using a binary logit model of the form:

$$\pi_A = \frac{e^{-\rho c^A}}{e^{-\rho c^A} + e^{-\rho c^D}}$$

where

π_A = probability of using auto subnetwork

c^A = travel cost on auto subnetwork

c^D = cost in data subnetwork, converted to units of travel cost

ρ = positive scale parameter, which is set to 0.1 for the study experiments

In the study experiments, the data subnetwork cost c^D is assumed to be a constant reflecting that the data generated by the switched users who telecommute is a negligible component of the total data flow.

Note that a simple behavioral rule is used here to represent the user decision of choosing between auto and data networks. As a result, the data network can be thought of as an alternative route in a generalized transportation network, and the above model is equivalent to a mode choice model with route choice decisions. This is because the primary focus of the paper is to propose a generic modeling framework for multiple interacting infrastructure networks rather than on the sophistication of the behavioral models. If available, more robust behavioral models can be plugged into the current framework. For example, in many instances, the decisions to telecommute are not only influenced by the technological factors, but also restricted by organizational issues such as the need for face-to-face communication or physical presence for some jobs. These factors can be incorporated into the current modeling framework by further classifying the potential drivers into different groups.

In summary, the two-layer MIN flow dynamics problem is a hierarchical decision-making problem. At the upper level, the user decides whether to telecommute or drive. If the user decides to drive, then the route choice represents the second-level of decision-making. Figure 3.2 illustrates the equilibrium tending flows for the two-layer network in terms of traffic link flows. The system approaches equilibrium after about 50 iterations (days). With 100 traffic demand units, the user equilibrium flows on the three links are 38, 50 and 9 units, respectively. The remaining demand is serviced by the data link.

4.3.2. **Results and Insights.** Figure 3 illustrates that the system flows tend to reach user equilibrium in both the single- and two-layer network scenarios. With identical values for the common parameters, the single-layer network reaches the equilibrium state much faster than the two-layer network. This highlights the interactions across the two INs considered, and suggests that interactions can significantly affect the individual network performance, and more broadly, influence the budget allocation process which is based on system performance characteristics. In the two-layer network, there is an additional choice alternative, leading to switching between the auto and data subnetworks. Hence, it takes more iterations for the system to reach equilibrium under the two-layer scenario. However, the two-layer network has better worst-route (in terms of travel cost) performances in the disequilibrium states. Figure 4 illustrates the travel costs on the worst routes for the auto subnetwork in single-layer and two-layer networks for the first 20 iterations. The results indicate that the travel costs on the worst routes in the two-layer network are typically lower than that in the single-layer network, especially during the initial iterations. This is because users taking a “bad” traffic link have a higher probability to choose telecommuting in the next iteration, thereby reducing the traffic subnetwork demand. However, this potential benefit is only an upper bound on the network performance improvement because from a transportation system analysis perspective, improved network performance may induce more traffic from other sources.

5. Concluding Comments

The interactions among various infrastructure systems are normally ignored in engineering planning, design and analysis procedures. However, identifying and understanding these interactions using a holistic perspective can lead to more efficient infrastructure systems. The interactions among the INs exist in terms of physical, functional, budgetary, market, informational, and environmental interdependencies. Three key aspects need to be addressed vis-à-vis research on IN interdependencies. They include the identification of the types and degree of IN interdependencies, the development of a new generation of methodological constructs that explicitly consider these interdependencies, and the development of efficient and robust solution methodologies. Key challenging issues in this context include the time scale of the flow dynamics, flow characteristics scale, performance characteristics scale, and the complexities in enabling coordinated investment decisions across various INs.

We propose a preliminary three-layer flow dynamics model for the automobile, urban freight and data networks based on three single-layer flow dynamics submodels. Specifically, the three coupled network layers are modeled as being comprised of Cournot-Nash dynamic agents. An agent-based simulation approach is proposed to solve the MIN problem. Experiments are conducted on a small network under simple behavioral and system performance assumptions. The results suggest that ABS has the potential to solve generalized MIN problems, as extensions to incorporate more complex behavioral rules into the decision-making process of agents are seamless.

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Table 1. Agents in the ABS Modeling for the Three-layer MIN Problem.

Sub-networks	Agents/ Environments	Actions	Rules	Information
Auto	Drivers (users)	Make travel decisions	User optimal	Get estimated travel costs; provide traffic demand
	ATIS operators	Disseminate traffic information	System optimal	Get actual travel costs and traffic demand; provide estimated travel costs
	Traffic network	Measure system performance	N/A	Get traffic demand; provide actual travel costs
Urban Freight	Consumers	Consume commodities	Utility maximization	Get commodity price; provide demand plan
	Producers	Produce commodities	Profit maximization	Get demand plan and commodity price; provide production and capacity change plan
	Carriers	Ship commodities	Delay minimization	Get demand and production plans; provide routing plans and shipping costs
	Auctioneer	Perform tatonnement process	Market clearance	Get demand, production and shipping costs; provide delivered price for the commodity
	Market	Enable commodity trading	N/A	Get demand and production plans; provide excess demand
Data	Users (drivers)	Make telecommuting decisions	Utility maximization	Get levels of services from Auto and Data; provide telecommuting decisions
	Routers	Route data	Delay minimization	Get network information; provide data routing decisions
	Data network	Measure level of service	N/A	Get data routing decisions; provide level of service information for Data

Figure 1. Agent/Environment Interactions for the Three-layer MIN Problem.

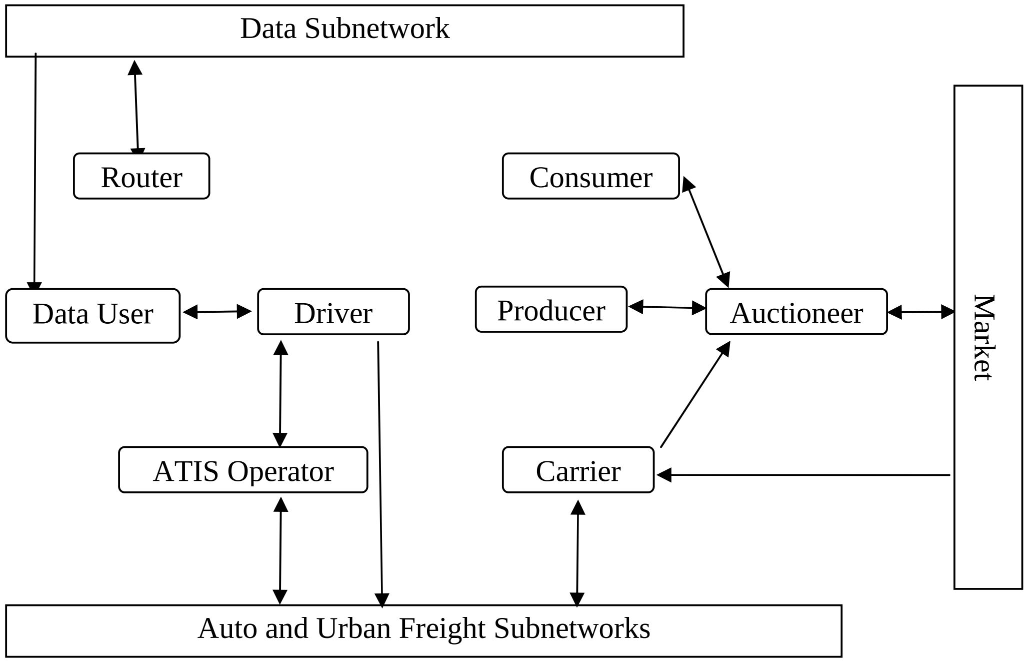

Figure 2. Two-layer IN with Traffic and Data Links.

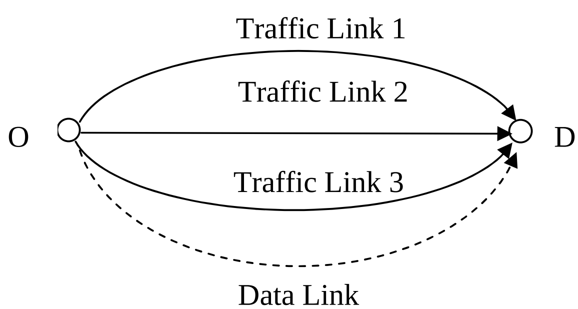

Figure 3. Equilibrium Tending Flow in Single-layer and Two-layer Networks.

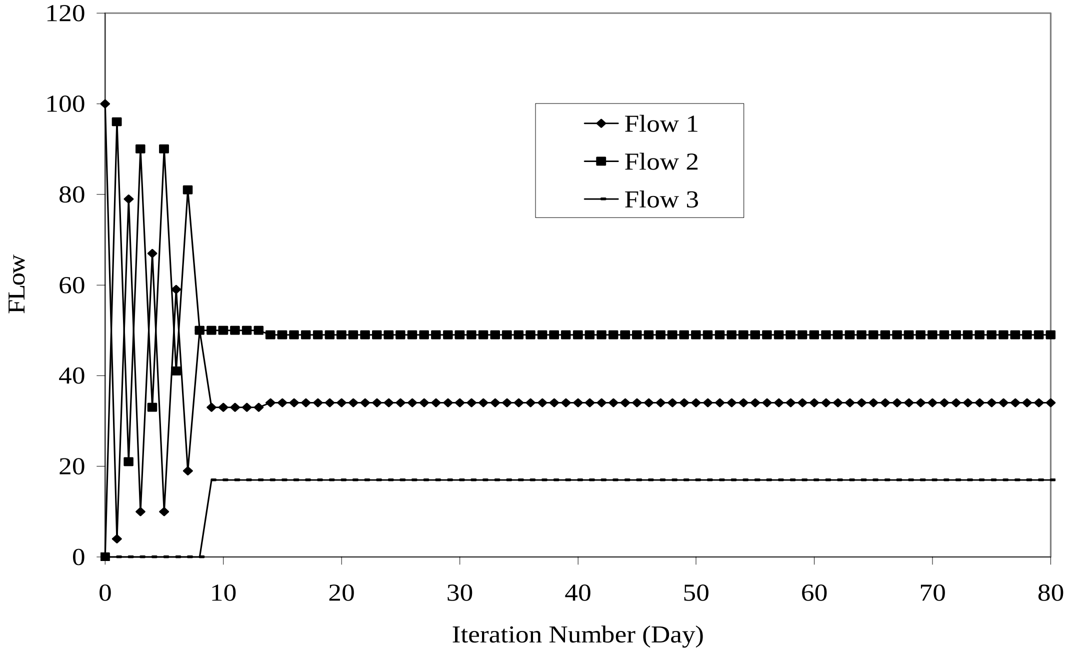

Figure 3.1. Equilibrium Tending Flow on the Auto Network (Single-layer Network).

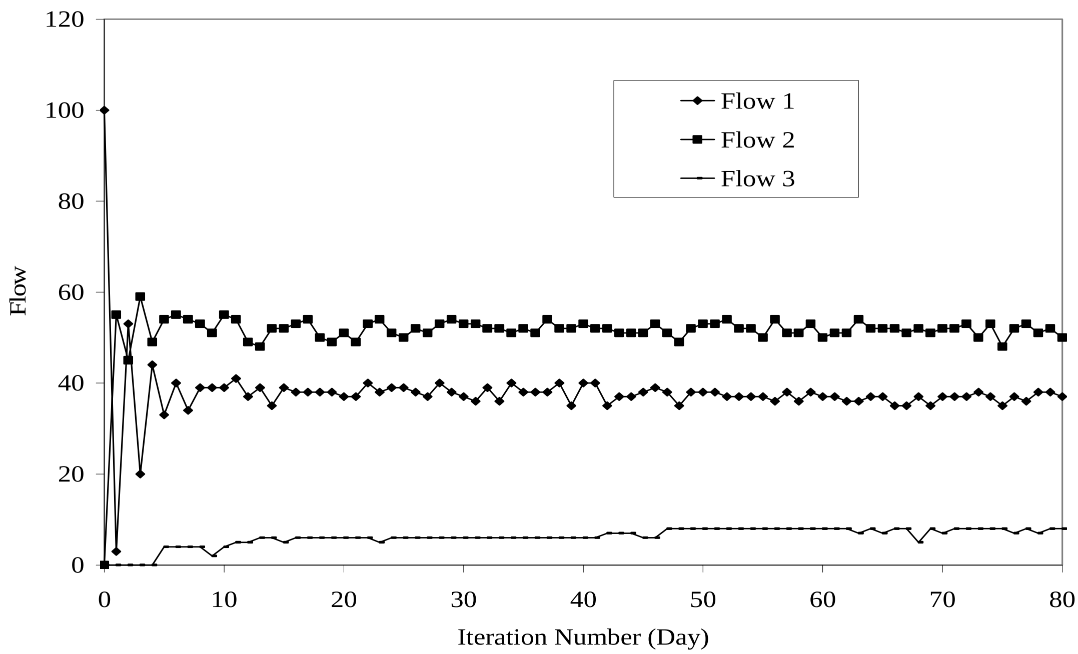

Figure 3.2. Equilibrium Tending Flow on Auto Subnetwork (Two-layer Network).

Figure 4. Travel Costs on Worst Routes in Single-layer and Two-layer Networks.