A Data-Consistent Fuzzy Approach for On-Line Driver Behavior under Information Provision

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Submitted for presentation at the 81th Annual Meeting of the Transportation Research Board, January 7-11, 2002, Washington D.C., and for publication in Transportation Research Record.

Manuscript Length:	6,248
5 Figures:	1,250
2 Tables:	500
Total:	7,998

ABSTRACT

This paper proposes a fuzzy approach to predict on-line driver routing decisions under information provision. The problem is characterized by subjectively interpreted and/or linguistic data, limited data on qualitative variables, and the need for computational efficiency to enable real-time deployment in a traffic control framework. The study highlights the appropriateness of a fuzzy model to address these characteristics. The proposed model enables theoretical consistency with the underlying behavioral mechanisms revealed through the available data. This is done by transforming the probabilities obtained from the data to possibilities, and then constructing membership functions. An S-shaped curve is proposed here as a more robust behavioral indicator compared to the oft-used triangular and trapezoidal shapes for these functions, while retaining their analytical tractability, computational efficiency, and ease of construction. In general, the approach suggests membership functions that are consistent with actual data. The model is analyzed using on-site stated preference data on VMS driver route diversion attitudes, and compared with a probabilistic discrete choice model for the same data. The results indicate that the fuzzy model performs more robustly for this data. When its computational efficiency and modularity are factored in, it may represent a better alternative to model on-line driver routing behavior in the context of real-time deployment.

Key words: variable message signs, fuzzy logic and control, transformation between probability and possibility, S-shaped curve

INTRODUCTION

Driver response attitudes to the supplied information are critical to the effectiveness of Advanced Traveler Information Systems (ATIS). Variable message signs (VMS) represent a generic media to disseminate routing information to drivers. The messages displayed through real-time VMS are non-personalized and, ideally, anticipatory of the conditions that drivers will likely encounter. Hence, the effectiveness of VMS is dependent on driver response and compliance with the information provided [*1*]. In addition, studies have suggested a strong correlation between the content of messages displayed through VMS and driver response behavior in terms of route diversion decisions [*2*]. While a driver's personal attributes, the information provided through VMS, and the ambient traffic conditions influence diversion decisions, in most situations drivers' latent attitudes/preferences towards diversion are not accurately measured by models due to the lack of sufficiently precise input data. They are significantly influenced by past experience, intuition, and subjective evaluation of traffic information displayed on VMS, which differ across drivers. For instance, a driver may trust his/her perception of a traffic situation more than the VMS information displayed. Thus, most input data for modeling VMS driver diversion decisions are characterized by subjectivity, uncertainty, and/or vagueness. In general, capabilities to model the real-time behavior of drivers under information provision, and/or calibrate the associated parameters are essential to the operation of traffic systems equipped with advanced information systems. Schofer et al. [*3*] conclude that while drivers value real-time traffic information they seek to incorporate their own knowledge and perspectives into the development of route plans, and expect these to be superior to those prepared through a navigation computer. Hence, the effectiveness of route guidance systems depends significantly on developing reliable real-time driver behavior models.

Driver travel decisions have traditionally been modeled using discrete choice theory [*4*], which relies heavily on the econometric theory of model estimation and testing. Abdel-Aty et al. [*5*] developed logit models using stated preference survey data for commuter route choice that included the effects of traffic information. They investigated the influence of travel time variability and traffic information on route choice, and the role of information in reducing travel time uncertainty. The results indicate the significance of travel time reliability to route choice, and the potential of ATIS to influence commuters through route advisories. However, the effect of information on reliability is elicited only for specific hypothetical situations. Jha et al. [*6*] modeled the uncertainty associated with drivers' travel time estimates using a stochastic dynamic framework and incorporated it into a travel choice model. A Bayesian updating model was developed to capture the mechanism by which travelers update their day-to-day travel time perceptions in light of ATIS information and their previous experience. It addressed the effect of the quality of information on drivers' reliance on the information system. Also, it models the inaccuracies in the information provided using probabilistic distributions. Srinivasan and Mahmassani [*7*] examine driver en-route route choices under real-time information provision using two underlying behavioral mechanisms, compliance and inertia. The compliance mechanism reflects a driver's propensity to comply with the information provided, while the inertia mechanism represents the driver's tendency to favor his/her current path. A multinomial probit model with a nested choice structure was used for the analysis. The results show that compliance and inertia change dynamically, and that network conditions and the past experience of drivers influence them. Lam and Chan [8] investigated the effects of the dynamic travel time

information conveyed via VMS. The perceived link travel time errors of drivers and the detector link travel time errors were assumed to be normally distributed.

Probabilistic route choice models assume well-defined probability distributions to capture the perceived randomness in driver behavior. However, these assumptions may be restrictive, especially in the context of on-line driver behavior under information provision, where several qualitative and/or difficult-to-measure variables such as inertia, network familiarity, trust in information, travel information quality, and/or compliance govern driver responses. In addition, the actual data on the measurable variables may be inconsistent with their assumed properties in these models. Nakayama and Kitamura [*9*] suggest that most existing models do not necessarily have psychological underpinnings, and can address only some cognitive processes underlying driver routing behavior. Also, they indicate that simple heuristics based on a cognitive process called induction may suffice, in which drivers' experiences and hypotheses formed are applied when similar situations occur. In the VMS context, situational factors such as time-of-day, weather conditions, destination, and ambient traffic conditions, can significantly influence the driver response due to the generic nature of the messages displayed. Hence, on-line computational tractability is a key desirable characteristic of on-line driver behavior models under information provision, both to predict the driver response by processing relevant real-time data and to periodically calibrate/update the associated model parameters to ensure consistency with the actual traffic conditions. Probabilistic discrete choice models are not easily amenable to efficient computation under both these contexts.

Fuzzy set theory provides a convenient mathematical handle to treat uncertainty, subjectivity, ambiguity and indetermination in practical traffic and transportation models [*10*]. A comprehensive survey of its application in the transportation arena is provided in [*11*]. Linguistic information, which represents subjective knowledge, is often used to model real-life problems that are difficult to quantify using traditional probabilistic approaches alone. As discussed earlier, the problem of on-line driver response under information provision is characterized by objective and precise knowledge that can be expressed through equations as well as subjective and ambiguous knowledge expressed linguistically. Often in such problems, drivers use subjective knowledge or linguistic information to make decisions and process it more easily than numerical information. Thus, models have been developed to use fuzzy set representations of travel alternatives, and to determine latent preferences using fuzzy logic inference systems based on linguistically expressed fuzzy rules. For example, Teodorovic and Kalic [*12*] developed a route choice model for air transportation that factors in travel costs, flight frequency, and the number of stopovers in addition to the travel time. It iteratively computes the number of passengers on the routes, compares them with the actual values, and modifies rules based on the comparisons. Lotan and Koutsopoulos [*13, 14, 15*] model driver behavior in response to traffic information using fuzzy set theory and approximate reasoning. The approach models the decision process as a non-linear combination of rules where each rule deals with a different aspect of the overall choice. The interactions between existing knowledge and new information were modeled through measures of compatibility. Lotan [*16, 17*] introduced a modeling framework for driver route choice in the presence of traffic information using fuzzy set theory, approximate reasoning and fuzzy control. A driver simulator was used to collect data on route choice behavior in the presence of information. Familiar drivers were asked to provide travel time estimates on route alternatives in the choice set and unfamiliar drivers were asked to describe their usual commuting behavior and to provide travel time estimates. By providing an option to update drivers' travel time estimates, effects of information update were also investigated. A two-stage

approach was introduced to model driver attitude and interaction with on-line information. In the first stage existing knowledge was updated based on new information, and in the second stage a decision was made based on the updated perceptions. The study provided insights on rule calibration and highlighted the importance of generating good membership functions for robust modeling. Pang et al. [*18*] used a fuzzy-neural approach to represent the correlation of the attributes with the driver's route selection. Rule-based fuzzy systems were used to represent expert knowledge transparently into the system being modeled. The route selection function was made adaptive to the driver's decision by training the fuzzy-neural network using the driver's choice. Henn [*19*] used fuzzy costs to develop a traffic assignment model that accounts for the imprecision and uncertainties in the route choice process. The predicted cost for each path is modeled using a fuzzy subset. The effect of information on drivers is modeled as a modification of the imprecision of the predicted cost of a route.

A common thread amongst existing models in the application of fuzzy logic and control to modeling driver behavior is the focus on developing more robust fuzzy *if-then* rules rather than on relating the underlying behavioral process to the fuzzy approach used. This manifests itself as the need to relate the behavioral mechanisms as well as any acquired actual data to the determination of the membership functions of fuzzy sets used in the models. While existing models use some simple oft-used shapes to represent the membership functions, these shapes are not necessarily consistent with the actual behavioral underpinnings of the problem being addressed. This is critical because the membership functions used characterize the parametric values used in the associated model, thereby defining its properties analogous to the characterization of probabilistic discrete choice models using assumptions on the error terms. Another issue with most existing fuzzy models in this context is that the shapes of the membership functions are based primarily on the modeler's intuition on the underlying process and/or subjective judgement of the field data available rather than any rigorous analysis of that data. Hence, they do not fully exploit the available data in conjunction with the subjective knowledge.

This paper uses fuzzy logic and control to model drivers' on-line route diversion process under real-time VMS information. Subjective and objective knowledge are combined to construct the membership functions used in the models. The primary focus is to exploit available information to develop an improved theoretical basis to determine the associated membership functions, thereby generating more robust models. The models are tested using stated preference VMS survey data collected in the Borman Expressway corridor in northwestern Indiana, and compared to a probabilistic discrete choice model developed previously using that data [*2*].

METHODOLOGY

Randomness and Fuzziness

Many mathematical systems have been constructed to model uncertainty which is formalized within the corresponding mathematical theories. Uncertainty has been conceived mostly in terms of probability theory. However, it is recognized that probability theory is unable to fully explain uncertainty [*20*]. The various theories are differentiated in terms of characteristics such as the interpretation mechanism, computational complexity, robustness, and generality. Each theory has some advantages and disadvantages, and no theory can be convincingly argued to be superior to the others in all cases [*21*]. Hence, the selection of an appropriate theory should be based on the problem context.

Uncertainty in the prediction of drivers' on-line behavior under information provision can be treated, among others, using random and/or fuzzy variables. The probabilistic framework, as in discrete choice theory, is well-known and widely accepted for modeling and processing randomness in real-life systems. However, its use may cause problems in certain circumstances [*22*]. In the context of driver on-line behavior modeling, probability theory may be constrained by the presence of several qualitative/subjective variables as well as difficult-to-measure variables whose associated probability distributions are unknown. Even if the probability density functions (PDFs) were known, the associated models can be analytically intractable precluding on-line implementation. Also, assumptions on these distributions to make them tractable may result in the associated models being inconsistent with the underlying behavioral mechanisms of the problem. A key feature in the VMS context that constrains probabilistic models is that the VMS messages displayed consist primarily of linguistic labels to describe traffic conditions (for example, "long delays expected", "accident ahead") or recommend alternative routes. To enable their use in probabilistic models, these linguistic expressions need to be converted into ordinal variables. Such linguistic expressions can be modeled more naturally using fuzzy logic.

The theory of possibility is analogous but conceptually different from that of probability [*22*]. While random variables are treated using probability theory, fuzzy variables can be treated through possibility theory. Probability is a measure of the frequency of occurrence of an event and generally has a physical event basis. It quantifies the frequency of occurrence of a sample in a population and focuses on random events in nature. Possibility is a mathematical representation of linguistic information. It is a quantification of the semantic, context-dependent nature of symbols. It focuses on the imprecision intrinsic in language and quantifies the meaning of events. The type and amount of information available can determine whether random or fuzzy variables should be used. If a large amount of measured data is available, it can be used to generate the probability distribution of a random variable. By contrast, if only limited data is available, or is linguistically based, or subjectively interpreted, the possibility distribution of a fuzzy variable is preferable. In many problems, random and fuzzy variables appear simultaneously, requiring a transformation between probability and possibility distributions to treat these two qualitatively different variables in a single framework. This is done by either converting the fuzzy variables to random variables or vice versa, depending on the problem characteristics. In the context of VMS on-line driver behavior, for the reasons discussed earlier, it is preferable to use fuzzy variables to address the problem. This problem is characterized by many variables which are subjective and linguistically-based, thereby naturally amenable to fuzzy logic. Hence, a transformation of several such variables to random variables may reduce the robustness of the associated probabilistic model. Therefore, the available probabilities are converted to possibilities to address the problem.

Fuzzy Logic and Control

Driver perception of the on-line information conveyed through VMS messages may be imprecise or ambiguous due to potential inaccuracies in the information conveyed and/or the linguistic nature of these messages. Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth between "completely true" and "completely false". The possibility of a fuzzy variable is a function with a value between 0 and 1 indicating the degree of evidence or belief that a certain element x belongs to a set [*23*].

The rule-based fuzzy control approach for the VMS on-line driver behavior can be described as follows. The route diversion decisions are based on a set of rules that relate driver

attributes and the displayed VMS information to their actions. A rule *i* is defined in the form of "if A_i then B_i". The left hand side (LHS) of a rule deals with driver characteristics, traffic conditions, and the content of the VMS messages while the right hand side (RHS) represents latent attitudes and propensity to divert. In the context of on-line driver behavior under VMS, it is reasonable to expect that drivers do not use very sophisticated rules in making their route diversion decisions due to dynamic nature of the problem and the associated time pressures. Hence, simple and straightforward rules consisting of one-dimensional LHS and RHS are used here.

It is important to note that in the aforementioned rule-based system, the standard inputs defined by driver decision rules do not necessarily correspond to the actual data inputs. Hence, approximate reasoning is employed to avoid a huge set of rules that coincide with all possible actual inputs. If an actual input and the LHS of rule *i* are approximately matched, a consequence may be inferred as follows:

$$\frac{\begin{array}{lll} \textit{if} \quad x \text{ is } A_i & \textit{then} & y \text{ is } B_i \\ \quad\;\; x \text{ is } A_i^* & & \end{array}}{y \text{ is } B_i^*}$$

Everything above the line is known, and below is unknown. The inferred value of B_i^* is computed based on the composition of A_i^* and an implication relation $R(x,y)$, as discussed later.

An aggregation mechanism is used to combine all consequences of driver route diversion decision rules into one set because more than one rule can be fired (used). A set of base diversion decision rules is applied to a given input and each rule yields a consequence that denotes its contribution to the driver's propensity to divert. Then, all consequences are aggregated into one fuzzy set. After the input to the driver's route diversion rules are processed by the fuzzy inference and aggregation, the result is a fuzzy output set, B^*. This output set is transformed into a crisp number representing B^* through a process called defuzzification. This single value obtained through defuzzification represents the driver's propensity to divert. The choice of the defuzzification method may have a significant impact on the accuracy of a fuzzy model.

Membership functions

A membership function of a fuzzy variable is a mapping between the fuzzy variable values and the set {0,1}, where the value in the set {0,1} indicates the possibility. As discussed earlier, construction of the membership functions is the most critical step in the design of the relevant fuzzy models, as the shapes and parameters of these functions characterize the underlying behavioral process assumed. In the past, the methods for determining membership functions have been heuristic and subjective. In most cases, triangular and trapezoidal shapes have been used because they are easy to construct and adjust. If only limited subjective information or expert knowledge are available, heuristic methods are a reasonable mechanism to determine membership functions. But, if the amount of information available is statistically significant, a more rigorous statistical method can be used to determine the membership functions because the fuzzy sets can then be obtained directly from the probability distributions [*24*]. This is done by transforming the probabilities to possibilities.

Transformation from probability to possibility

This transformation is used to construct membership functions from the probability distribution functions. Several conditions are required for the transformation, in order to construct a reasonable membership function based on the statistical data [*23, 24*]:

1) Consistency principle:

$p(a) \le \mu(a), \quad \forall a \subset X$.

2) Preference preservation:

$\mathrm{p(a)} < \mathrm{p(b)} \;\Rightarrow\; \mu(\mathrm{a}) < \mu(\mathrm{b})$,

$\mathrm{p(a)} = \mathrm{p(b)} \;\Rightarrow\; \mu(\mathrm{a}) = \mu(\mathrm{b}), \quad \forall \mathrm{a}, \mathrm{b} \in \mathrm{X}$.

3) Maximal specificity principle:

$\int_{-\infty}^{\infty} \mu(x)dx$ has the minimum value.

Many methods have been suggested to map probability densities to possibility distribution functions satisfying the above principles. A common method is the truncated triangular transformation, proposed for unimodal and symmetric probability distributions [*25*]. It is a good approximation of the optimal probability-possibility transformation and is computationally efficient, but is not appropriate for non-symmetric probability laws. Another method constructs the optimal membership functions from specific PDFs [*24*]. However, it can be applied only to some of the most common PDFs and is computationally inefficient. A third method focuses on information-preservation during transformation. It assumes that probability and possibility are connected via intervals (or log-intervals) [*21*]. The quality of transformation is highly sensitive to the interval size. Also, it is difficult to build, and the optimal transformation does not satisfy the maximal specificity principle. The three methods require well-defined continuous PDFs, which are unlikely to be available in our problem context. The most reasonable assumption is the availability of the frequencies of actual observations. Hence, in this paper an alternative method, obtained by extending the method proposed by Dubois and Prade [*26*], is developed for the transformation of discretized probabilities to possibility functions, while approximately satisfying the optimality requirement for the transformation.

Dubois and Prade [*26*] suggest a mechanism for interpreting histograms to build fuzzy sets from them. This is an easy and straightforward transformation for discrete data, and satisfies the consistency principle and preference preservation. However, this transformation cannot avoid a loss in information precision, and the obtained possibility distribution consists of discrete values corresponding to the original discrete probabilities. The loss in precision can be traded-off with the gains in computational tractability since the fuzzy variables are easier to handle. The steps of the transformation are:

Step 1: Construct histograms with n intervals from the data.

Step 2: $X=\{x_i \mid i = 1\ldots..n\}$, where the x_i are ranked so that $p_1 \ge p_2 \ge \ldots\ldots\ldots \ge p_n$,

$\sum_{i=1}^{n} p_i = 1$ and p_i is a probability measure of X belonging to an interval x_i.

A_i denotes the set of events $\{x_1, x_2, \ldots\ldots, x_i\}$, $i \le n$; $A_0 = \varnothing$; and A denotes any subset of events from 1 through n.

Step 3: The degree of necessity of A is the degree of impossibility of the opposite event $\overline{A}$

$$N(A) = \sum_{x_i \in A} \max(p_i - \max_{x_k \notin A} p_k, 0) .$$

Step 4: The degree of possibility of A is:

$$\Pi(A) = 1 - N(\overline{A}),\ \forall A \subseteq X.$$

Step 5: The discretized possibility distribution is:

$$\mu_i = 1 - N(X - \{x_i\}) = 1 - N(A_{i-1}) = 1 - \sum_{j=1}^{i-1}(p_j - p_i) = i * p_i + \sum_{j=i+1}^{n} p_j, \quad \forall\ i = 1,.....n.$$

As discussed earlier, a rigorous method should be employed to derive well-defined membership functions from these discrete possibility values. Triangular- and trapezoidal-shaped membership functions have been popular because of their computational efficiency and ease of construction. However, they are not necessarily consistent with the underlying behavioral processes governing on-line routing decisions, and may have behaviorally unrealistic kinks at the junctions of two adjacent linear components.

If adequate data is available (which allows the transformation from probabilities to possibilities), then the discretized possibility distributions most closely fit an S-shaped curve. Hence, in this study, an S-shaped curve is proposed to provide the basis for the shape of the membership functions. For example, an S-shaped curve depicts the commonly-used cumulative normal distribution. This family of curves has been widely used to model various natural phenomena. Here, we propose a second-order S-shaped curve consisting of three regions, as illustrated in Figure 1. It is described in the next section. In the lower tail (first) region of the membership function of an attribute (fuzzy variable), changes in attribute values have little effect on the possibilities. This implies behaviorally that the attribute values in this region contribute very little to the explanatory power of this attribute vis-à-vis the driver on-line routing decisions. Stated differently, the driver is indifferent to the attribute values in this region. In the second region, there is a gradual increase in the explanatory power of this attribute with regard to the driver decisions. After the second region, there is an indifference region in which changes to the attributes values do not significantly affect the explanatory power. However, the attribute values in this region have the highest explanatory power, and therefore, this region is the most important one in terms of its influence on the routing decisions. Hence, the S-shaped curve is more representative of driver behavior and perception compared to the triangular and trapezoidal shapes, while retaining the same level of computational efficiency and ease of construction. In this study, the membership functions are constructed using the S-shaped and reversed S-shaped curves.

The commonly-used probit and logit probabilistic discrete choice models have S-shaped curves approximately similar to the one proposed here. However, the use of probit and logit functions to model the membership functions presents some analytical difficulties. First, the tails of their S-shaped curves are asymptotic to 0 and 1. This is problematic because the membership functions need to satisfy a normality property which requires that at least one fuzzy variable value correspond to a possibility 1 (perfect membership). Second, the inverse of membership functions and their integrals are required to perform the implication and aggregation tasks in the fuzzy approach. In this context, the probit function is difficult to manipulate analytically, and the logit function is less tractable compared to the proposed S-shaped curve. This is computationally important because the on-line behavior models entail real-time implementation.

Equations of S-shaped and reversed S-shaped curves

1) S-shaped curve:

$\mu(x) = 0, \quad \text{for } x \le \alpha$

$$\mu(x) = 2(\frac{x-\alpha}{\beta-\alpha})^2 \ , \quad \text{for } \alpha \le x \le \frac{\alpha+\beta}{2}$$

$$\mu(x) = 1-2(\frac{x-\beta}{\beta-\alpha})^2 \ , \quad \text{for } \frac{\alpha+\beta}{2} \le x \le \beta$$

$$\mu(x) = 1 \ , \ \text{for } x \ge \beta$$

2) Reversed S-shaped curve:

$$\mu(x) = 1 , \quad \text{for } x \le \alpha$$

$$\mu(x) = 1-2(\frac{x-\alpha}{\beta-\alpha})^2 \ , \quad \text{for } \alpha \le x \le \frac{\alpha+\beta}{2}$$

$$\mu(x) = 2(\frac{x-\beta}{\beta-\alpha})^2 \ , \quad \text{for } \frac{\alpha+\beta}{2} \le x \le \beta$$

$$\mu(x) = 0 \ , \ \text{for } x \ge \beta$$

The two parameters α and β are determined such that the S-shaped curve best fits the discrete values of the possibility measures obtained through the transformation of probabilities to possibilities. In estimating the parameters of the S-shaped curve, the normality of a fuzzy set should be satisfied so that β (or α for the reversed S-shaped curve) equals the fuzzy variable value beyond which the possibility is one. After β (or α for the reversed S-shaped curve) is determined, the other parameter is estimated using the least-squares curve-fitting method. This estimation method determines an S-shape (or reversed S-shape) that fits more robustly in the vicinity of $\mu(x)$=1. This is desirable because the lower tail region of the membership function is not significant for fuzzy inference and control.

APPLICATION OF THE PROPOSED MODEL

Data on Driver VMS Route Diversion Attitudes

Stated preference survey data from an on-site survey [*2*] conducted in the Borman Expressway corridor in northwestern Indiana is used to analyze the proposed fuzzy model, and compare its performance to that of a probabilistic discrete choice model [*2*]. The survey questionnaire consists of questions on drivers' socioeconomic characteristics, their attitudes towards traffic information provided, and their propensity to divert under different VMS message contents. The data consists of 1984 observations (248 people responded to 8 types of VMS messages). Three VMS route diversion behavior models are developed. They include a "General" model that incorporates all survey data, a "Truck Drivers" model based on data obtained from truck drivers, and a "Non-truck Drivers" model using data from non-truck drivers. This is primarily because the Borman Expressway corridor has a high percentage of truck traffic. A binary choice process that predicts whether the driver will divert or will not divert is used to analyze the performance of the fuzzy and the probabilistic discrete choice models. In the survey questionnaire, diversion intentions are indicated on a five point Likert scale from 1 to 5, where 1 indicates the least willingness to divert and 5 indicates the highest willingness to divert. However, in the discrete choice model, respondents who answered 4 or 5 are assumed to divert, while those who answered 1, 2, or 3 are assumed not to divert [*2*], so as to map the responses to the binary choice process. Since the fuzzy model generates continuous values between 1 and 5, these values also

need to be converted to binary choices. This is done by using the value 3.5 as the threshold beyond which drivers are assumed to divert.

Route Diversion Rules

In the proposed approach, a driver's route diversion decisions are assumed to be made based on some simple considerations. Hence, each diversion rule should have a simple and straightforward reasoning. The diversion decision rules applied here are based on the questions in the VMS survey and have the form of "if A then B". Three main categories of rules are defined: 1) driver attributes: gender, age, education level, household size, familiarity, trust on information, sensitivity to delay, freeway preference, and consideration of neighborhood; 2) content of VMS messages; and 3) situational characteristics: weather, time-of-day, and trip purpose. The route diversion rules related to the situational characteristics are not included in the models because the survey data does not provide driver responses to VMS in specific situational settings. Table 1 shows the set of base rules used in this study.

Determination of Membership Functions

The shape of the membership function and the amount of overlap between adjacent sets characterize the membership function of each fuzzy variable (attribute) used to determine a driver's propensity to divert under VMS information. Each set is modeled as an S-shaped curve and has a potential overlap with neighboring sets. To validate and calibrate the fuzzy model, the model output and the driver's willingness to divert based on the survey are compared. Thus, the minimum and the maximum willingness to divert are set as 1 and 5, respectively. Figure 2 shows the probabilities from actual data, the transformed possibilities, and the S-shaped membership function estimated, for the fuzzy set "driver will divert". Figure 3 depicts three membership functions with regard to the driver willingness to divert, which are constructed through the transformation. As discussed previously, the S-shaped membership functions (S-shaped curve, reverse S-shaped curve, and combination of these two) are more representative of driver on-line VMS response behavior compared to the triangular functions shown in Figure 4 that are generated through some heuristic. The construction of membership functions using the transformation of probabilities to possibilities improves the model performance as shown in Table 2, where the prediction rates for the triangular and transformation approaches are 54.10% and 58.52%, respectively, for the General model.

Implication Operator

The choice of an appropriate implication operator is problem-dependent and an important step in building fuzzy models [*23*]. For each driver, driver attributes, the contents of information, and situational characteristics are used against every diversion decision rule. Each rule results in a corresponding consequence, determined by the amount of overlap between the current input and the standard input of the rule. The execution of each rule *i* is based on γ_i, the overlap between the standard input of the ith rule, A_i, and the current input A_i*. B_i* is determined using two types of implication operators, Mamdani and Larsen, as shown in Figure 5. The Mamdani Min implication operator is defined as:

$$\mu_{B_i*}(y) = \min(\gamma_i, \mu_{B_i}(y)),$$

while the Larsen Product implication operator is defined as:

$$\mu_{B_i*}(y) = \gamma_i \cdot \mu_{B_i}(y),$$

where γ_i is the degree of overlap between A_i, and A_i*, given by:

$$\gamma_i = \max_{x \in X} \min(\mu_{A_i*}(x), \mu_{A_i}(x)).$$

The Mamdani operator makes B_i equal to B_i* if A_i equal to A_i*, which is a desirable property of implication. The Larsen operator preserves the original shape of the membership function of the set B_i, because the membership function of the RHS value B_i* is scaled by the γ_i of each rule *i*. All diversion decision rules are applied to the current input *A**, consisting of driver characteristics and the contents of VMS messages. The corresponding consequence that each rule *i* yields, B_i*, depends on the γ_i of each rule *i*. Based on the analysis for the current dataset, the Larson Product is slightly better here, as shown by the prediction rates in Table 2.

Defuzzification Methods

The membership function of the total output *B** is the union of all individual outputs. Two defuzzification methods are employed and compared to test their significance. The Center of Area (COA) defuzzification method is defined as:

$$u^* = \frac{\sum_{i=1}^{N} u_i \cdot \mu_{union}(u_i)}{\sum_{i=1}^{N} \mu_{union}(u_i)}$$

where μ_{union} is defined by the union of all B_i^* contributions. This is a well-known and popular defuzzification method, but it favors the central point of μ_{union}. If the areas of two or more firing rules overlap, this method counts the overlapping area only once, which may be problematic in certain situations because the importance of the overlapping area is underestimated. The Center of Sums (COS) is used to take care of this potential drawback of COA. It counts the overlapping areas of multiple rules as many times as the overlaps occur. COS is given by:

$$u^* = \frac{\sum_{i=1}^{N} u_i \cdot \sum_{j=1}^{K} \mu_{B_j^*}(u_i)}{\sum_{i=1}^{N} \sum_{j=1}^{K} \mu_{B_j^*}(u_i)}$$

The performance of these defuzzification methods depends on the characteristics of the problems being modeled and the datasets being used. The two approaches are compared in terms of prediction rates for the current data. As illustrated in Table 2, COS performs slightly better here.

Adjustment of Weights of Rules

Driver route diversion decisions are predicted based on the evaluation of all attributes represented as *if-then* rules. A defuzzified crisp value, obtained by evaluating an *if-then* rule, indicates driver willingness to divert based on that specific attribute. A driver's willingness to divert is then determined using a combination of all defuzzified crisp values. A weighted sum is determined using the weights of *if-then* rules as follows:

$$u^* = \sum_{i=1}^{N} w_i u_i^*$$

where w_i is the weight of rule category *i* and u_i is the crisp output obtained by evaluating the ith rule. A weighted sum is considered to be reasonable because the importance and contribution of each attribute is different. For instance, the contents of VMS messages might be more dominant than a driver's age in driver route diversion decisions. A simple and straightforward calibration method introduced by Lotan and Koutsopoulos [*13*] is employed in this study to obtain the relative weights of individual rules. Suppose there are *N* observations of route diversion choices and *K* *if-then* rules. Then,

$$w_i = \frac{\sum_{j=1}^{N} \gamma_i^j \delta^j - \sum_{j=1}^{N} \gamma_i^j (1-\delta^j)}{\sum_{j=1}^{N} F_i^j}, \text{ for } i = 1, \ldots, K$$

where,

M^n = decision obtained from the model (model output) for the nth observation

O^n = stated decision in the survey for the nth observation

$$\delta^n = \begin{cases} 1, & if \;\; M^n = O^n \\ 0, & otherwise \end{cases}$$

γ_i^n = the degree of overlap between A_i, and A_i* defined in the ith rule for the nth observation

$$F_i^n = \begin{cases} 1, & \text{if } \gamma_i^n > 0 \\ 0, & \text{otherwise} \end{cases}$$

The weight w_i determines the amount of contribution that the model output of the ith rule makes to the final decision. Hence, the weights of rules can identify unimportant or problematic *if-then* rules. The performance of the model that includes calibration of rule weights is contrasted against that of the model which assumes uniform weights. Table 2 indicates the improvement in model performance under the calibrated weights.

For the survey data in the current study, the fuzzy model which uses transformation of probabilities to possibilities, the Larsen implication operator, the COS defuzzification method, and the calibrated weights for *if-then* rules, performs the best in predicting driver route choice decisions.

Comparison with a Probabilistic Discrete Choice Model

A binary logit model [*2*] is used to predict the driver VMS route diversion decisions using the same survey data used to develop the fuzzy model. In general, it is difficult to find consistent measures to compare the performance of the two models, since they are based on different theoretical approaches. For a binary logit model, ρ^2 is often used as an informal goodness-of-fit index. However, there is no corresponding measure for a fuzzy model. Hence, the percentage of correct predictions is used as a common measure here to compare the fuzzy and logit models.

The binary logit model used to predict the probability of a driver diverting under a VMS message is as follows [*2*]:

$$V = (V_{in} - V_{jn}) = ONE + \eta X + \omega VMS$$

where,

ONE = alternative specific constant corresponding to divert,

X = vector of those explanatory variables other than VMS message type that may influence a driver's decision to divert,

η = vector of estimated parameters corresponding to *X*,

VMS = vector of dummy explanatory variables representing each of the VMS message types provided to drivers, and

ω = vector of estimated parameters corresponding to *VMS*.

After the parameters of the logit model are estimated, the choices predicted by it are generated in order to compare it with the fuzzy model. The output of the logit model is not a discrete choice, but the probability that a driver will divert. Therefore, this probability needs to be converted into

discrete choices, “divert” or “not divert”, for each driver. Monte Carlo simulation is used to generate this choice from the driver’s route diversion probability predicted by the logit model. A uniform random number generator is used to generate values between 0 and 1. If the number generated is smaller than the diversion probability, the driver is assumed to divert. For the “General” model, this process is repeated for all 1984 observations. A vector of diversion decisions for the 1984 observations represents one realization of the Monte Carlo simulation. 50 such realizations are generated, and the percentage of correct predictions is computed for each realization. The use of several realizations ensures that the diversion decisions converge to the diversion probabilities. The average over the 50 realizations is used to represent the prediction rate for the binary logit model. The prediction rates using the binary logit models are 56.85%, 63.25%, and 60.51% for the General, Truck Driver, and Non-truck Driver cases, respectively. By comparison, the corresponding values for the fuzzy models are 62.77%, 70.59%, and 67.36%. Hence, for the current dataset, the fuzzy models perform better than the binary logit models for all cases. The VMS study uses only stated preference data. Further, this data does not explicitly provide a binary diversion decision and uses a Likert scale instead. If the stated binary diversion decisions are known, or data is available through revealed preferences or a driving simulator, the prediction power can potentially be further enhanced.

CONCLUDING COMMENTS

This paper presents a fuzzy logic and control based model to predict on-line driver VMS route diversion decisions. It focuses on developing an approach that enables theoretical consistency with the underlying behavioral process revealed through the available data. This is done by transforming the probabilities obtained from the data to possibilities. The possibility distribution is used to determine the shape of the associated membership functions rather than the arbitrary use of some standard shapes, in contrast to existing models in this area. The S-shaped curve is proposed here as a more robust behavioral indicator compared to the commonly-used triangular and trapezoidal shapes, while retaining their analytical tractability, computational efficiency, and ease of construction. However, the proposed approach does not limit the membership functions to S-shaped curves; rather it seeks the shapes that achieve the best fit with the possibility distributions obtained. In the absence of adequate data on some variables, the transformation from probabilities to possibilities is not robust. In such situations, the S-shaped curve is suggested for the membership functions rather than the use of the triangular or trapezoidal shapes. The proposed approach is modular, implying that *if-then* rules corresponding to attributes can be added or removed with ease to generate better model explanatory power. This is especially important in the on-line driver behavioral context where the effects of situational factors vary dynamically.

The on-line driver routing behavior problem under information provision is characterized by several variables that are subjectively interpreted and/or linguistically oriented. Limited data may be available, especially on the subjective variables. Also, computational efficiency is a key requirement for the use of the associated models in real-time traffic control frameworks under advanced information and sensor systems, both for the determination of real-time routing strategies and the on-line calibration of the behavioral parameters to enable consistency with the unfolding traffic conditions. These desired characteristics motivate the use of the proposed fuzzy approach to develop on-line driver response models under a broad gamut of information dissemination systems. The approach is computationally efficient on-line, can incorporate non-

quantitative data, and is more amenable to limited data situations. The on-line VMS driver route diversion prediction problem is used as an example to highlight the approach.

The proposed fuzzy model is analyzed using on-site survey data from the Borman Expressway corridor on VMS driver response attitudes. Insights are obtained on the various components of the model using the prediction rates as the performance measure. The results show that for the current data, the fuzzy model developed performs more robustly compared to a binary logit model developed using that data. When its computational efficiency and modularity are factored in, the proposed approach may represent a better alternative to model on-line driver routing behavior in the context of real-time deployment.

In general, the choice of the use of probability theory or possibility theory is problem-dependent. When groups of variables best characterized as random or fuzzy appear simultaneously in a problem, the transformation from possibility to probability will be useful if a probabilistic model is preferred from a modeling perspective. Fuzzy variables could also be used in probabilistic discrete choice models using the linkage between possibility and probability. In this context, hybrid approaches are currently being investigated.

ACKNOWLEDGEMENTS

This material is based upon work supported by the National Science Foundation under Grant No. 9702612. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

REFERENCES

1. Wardman, M., P. W. Bonsall, and J.D. Shires. Driver Response to Variable Message Signs: A Stated Preference Investigation. *Transportation Research*, Part C, Vol. 5, No. 6, 1997, pp. 389-405.
2. Peeta, S., J. L. Ramos, and R. Pasupathy. Content of Variable Message Signs and On-line Driver Behavior. In *Transportation Research Record 1725,* TRB, National Research Council, Washington, D.C., 2000, pp. 102-108.
3. Schofer, J. L., F. S. Koppleman, and W. A. Charlton. Perspectives on Driver Preferences for Dynamic Route Guidance Systems. In *Transportation Research Record 1588*, TRB, National Research Council, Washington, D.C., 1997, pp. 26-31.
4. Ben-Akiva, M., and S. R. Lerman. *Discrete Choice Analysis: Theory and Applications to Travel Demand*. MIT Press, Cambridge, Massachusetts, 1985.
5. Abdel-Aty, M. A., R. Kitamura, and P. P. Jovanis. Using Stated Preference Data for Studying the Effect of Advanced Traffic Information on Drivers' Route Choice. *Transportation Research*, Part C, Vol. 5, No. 1, 1997, pp. 39-50.
6. Jha, M., S. Madanat, and S. Peeta. Perception Updating and Day-to-Day Travel Choice Dynamics in Traffic Networks with Information Provision. *Transportation Research*, Part C, Vol. 6, No. 3, 1998, pp. 189-212.
7. Srinivasan, K. K., and H. S. Mahmassani. Modeling Inertia and Compliance Mechanisms in Route Choice Behavior under Real-Time Information. In *Transportation Research Record 1725,* TRB, National Research Council, Washington, D.C., 2000, pp. 45-53.
8. Lam, W. H. K., and K. S. Chan. A Model for Assessing the Effects of Dynamic Travel Time Information via Variable Message Signs. *Transportation,* Vol. 28, 2001, pp. 79-99.
9. Nakayama, S., and R. Kitamura. Route Choice Model with Inductive Learning. In *Transportation Research Record 1725*, TRB, National Research Council, Washington, D.C., 2000, pp. 63-70.
10. Teoderović, D. Fuzzy Sets Theory Applications in Traffic and Transportation. *European Journal of Operational Research,* Vol. 74, 1994, pp. 379-390.
11. Teoderović, D. Fuzzy Logic Systems for Transportation Engineering: the State of the Art. *Transportation Research*, Part A, Vol. 33, No. 5, 1999, pp. 337-364.
12. Teodorović, D., and M. Kalić. A Fuzzy Route Choice Model for Air Transportation Networks. *Transportation Planning and Technology,* Vol. 19, 1995, pp. 109-119.
13. Lotan, T., and H. N. Koutsopoulos. Models for Route Choice Behavior in the Presence of Information Using Concepts from Fuzzy Set Theory and Approximate Reasoning. *Transportation*, Vol. 20, 1993, pp. 129-155.
14. Lotan, T., and H. N. Koutsopoulos. Approximate Reasoning Models for Route Choice Behavior in the Presence of Information. *Transportation and Traffic Theory*, The 12th International Symposium on the Theory of Traffic Flow and Transportation, Berkeley, CA, 1993, pp. 71-88.
15. Lotan, T., and H. N. Koutsopoulos. Modeling Default Behavior in the Presence of Information and Its Application to the Route Choice Problem. *International Journal of Intelligent Systems,* Vol. 14, 1999, pp. 501-533.
16. Lotan, T. Effects of Familiarity on Route Choice Behavior in the Presence of Information. *Transportation Research*, Part C, Vol. 5, No. 3, 1997, pp. 225-243.

17. Lotan, T. Modeling Discrete Choice Behavior Based on Explicit Information Integration and Its Application to the Route Choice. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, Vol. 28, No. 1, 1998, pp. 100-114.
18. Pang, G. K. H., K. Takahashi, T. Yokota, and H. Takenaga. Adaptive Route Selection for Dynamic Route Guidance System Based on Fuzzy-Neural Approaches. *IEEE Transactions on Vehicular Technology,* Vol. 48, No. 6, 1999, pp. 2028-2040.
19. Henn, V. Fuzzy Route Choice Model for Traffic Assignment. *Fuzzy Sets and Systems,* Vol. 116, 2000, pp. 77-101.
20. Wonneberger, S. Generalization of an Invertible Mapping between Probability and Possibility. *Fuzzy Sets and Systems,* Vol. 64, 1994, pp. 229-240.
21. Geer, J. F., and G. J. Klir. A Mathematical Analysis of Information-Preserving Transformation between Probabilistic and Possibilistic Formulations of Uncertainty. *International Journal of General Systems*, Vol. 20, 1992, pp. 143-176.
22. Lapointe, S., and B. Bobée. Revision of Possibility Distributions: A Bayesian Inference Pattern. *Fuzzy Sets and Systems,* Vol. 116, 2000, pp. 119-140.
23. Tsoukalas, L. H., and R. E. Uhrig. *Fuzzy and Neural Approaches in Engineering*. John Wiley and Sons Inc., New York, 1997.
24. Civanlar, M. R., and H. J. Trussell. Constructing Membership Functions Using Statistical Data. *Fuzzy Sets and Systems,* Vol. 18, 1986, pp. 1-13.
25. Mauris, G., V. Lasserre, and L. Foulloy. Fuzzy Modeling of Measurement Data Acquired from Physical Sensors. *IEEE Transactions on Instrumentation and Measurement*, Vol. 49, No. 6, 2000, pp. 1201-1205.
26. Dubois, D., and H. Prade. Unfair Coins and Necessity Measures: Towards a Possibilistic Interpretation of Histograms. *Fuzzy Sets and Systems,* Vol. 10, 1983, pp. 15-20.

LIST OF TABLES AND FIGURES

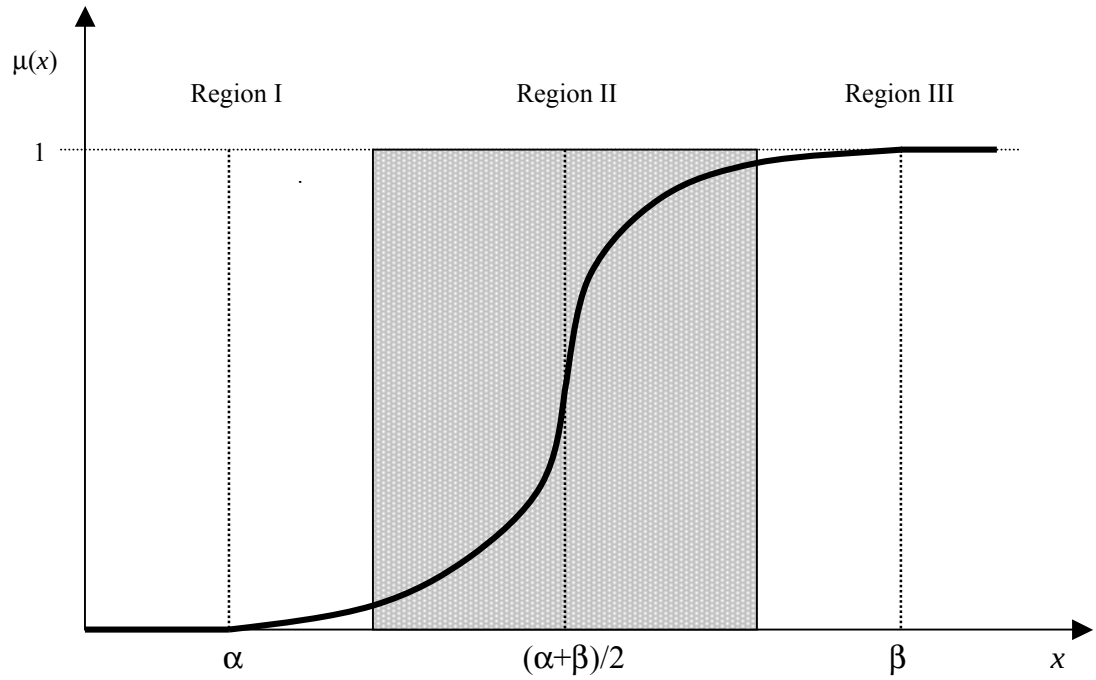

FIGURE 1 The proposed S-shaped curve.

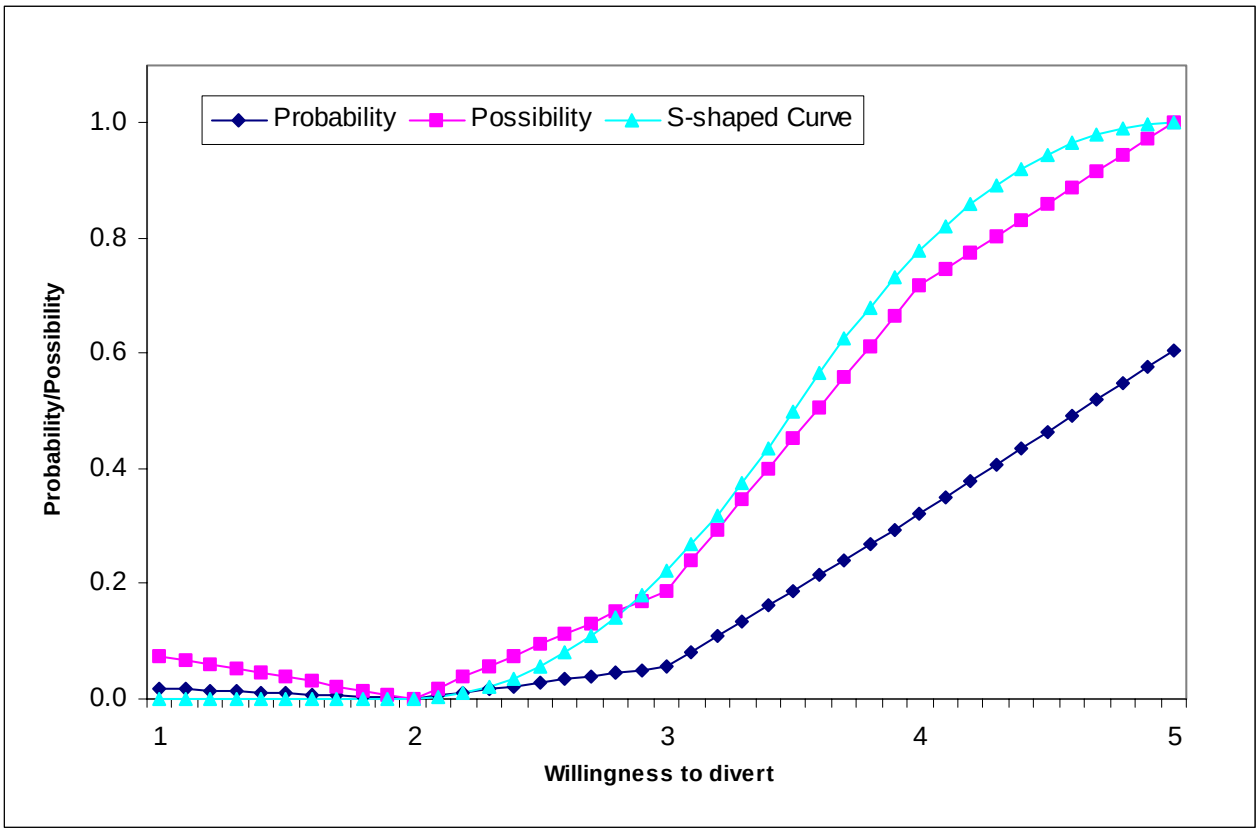

FIGURE 2 Transformation of probability to possibility for fuzzy set "driver will divert".

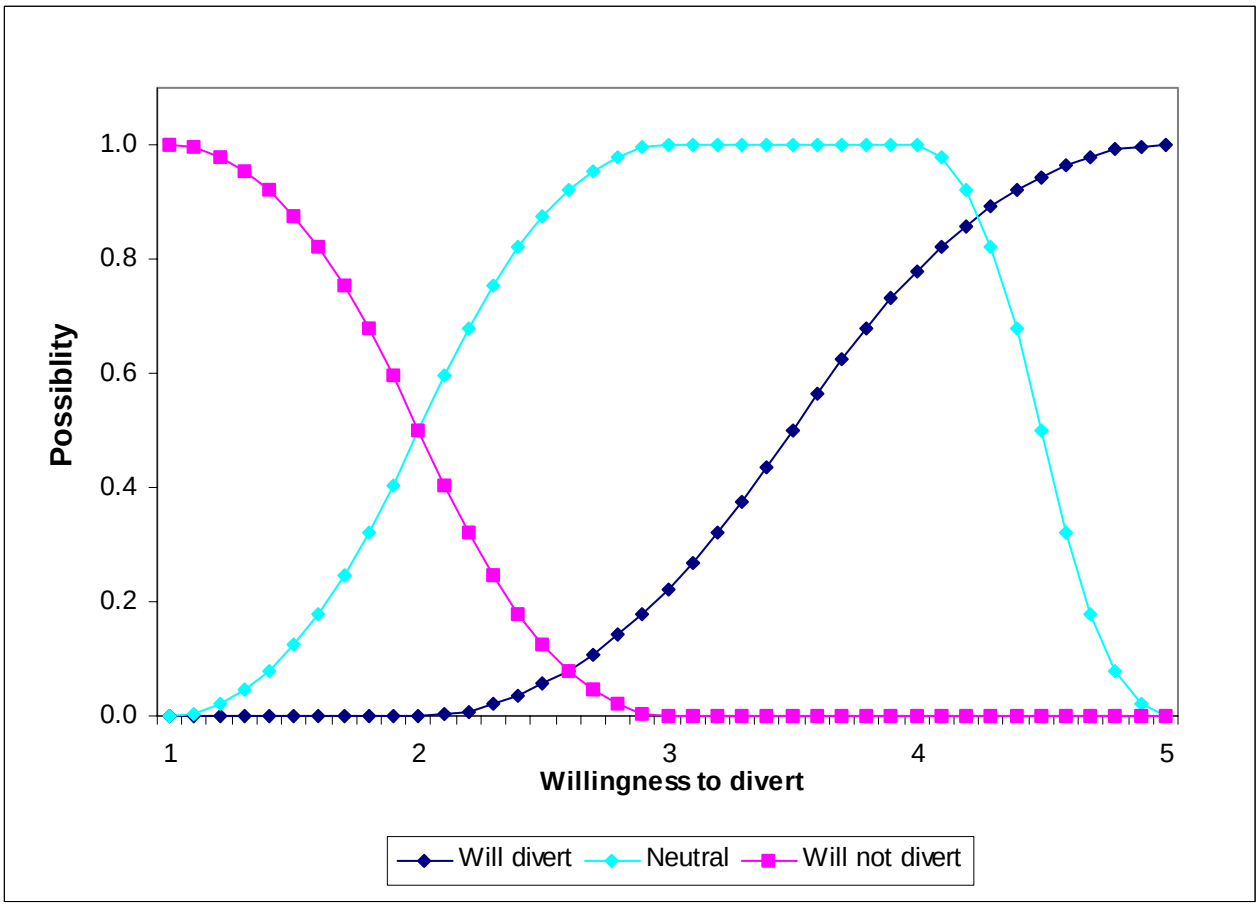

FIGURE 3 Membership functions for driver willingness to divert.

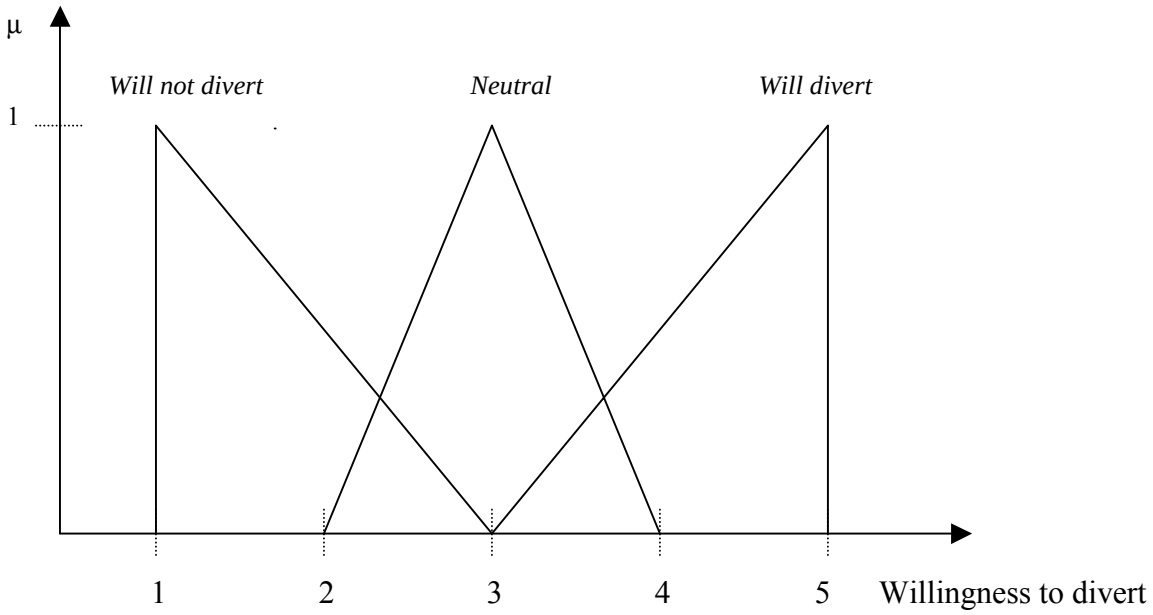

FIGURE 4 Triangular membership functions for driver willingness to divert.

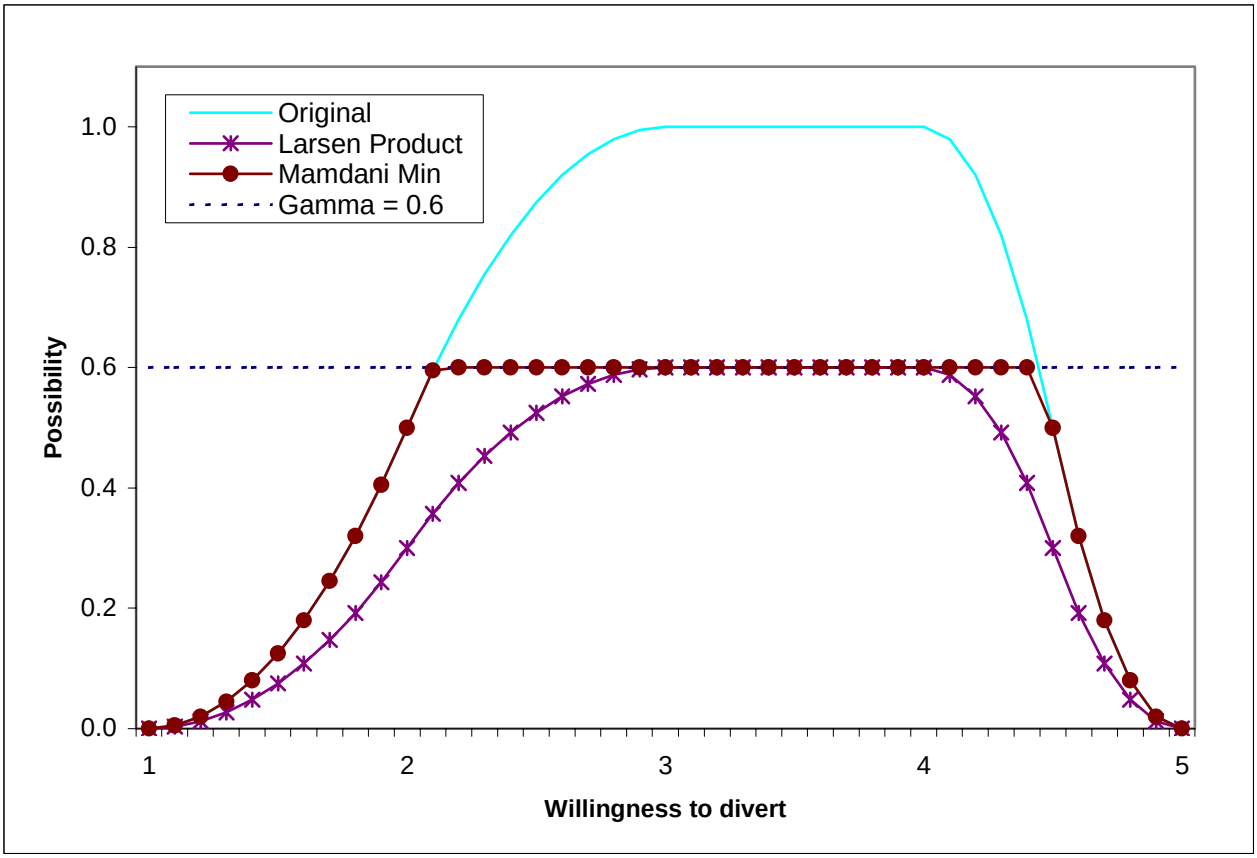

FIGURE 5 Implication operators.

TABLE 1 *If-then* Rules for Driver Route Diversion Decisions

Category	LHS	RHS
Gender	If driver is male	He/she will divert
	If driver is female	He/she will not divert
Age	If driver is young	He/she will divert
	If driver is middle-aged	He/she will be neutral
	If driver is old	He/she will not divert
Education	If driver is well-educated	He/she will divert
	If driver is educated	He/she will be neutral
	If driver is less educated	He/she will not divert
Household Size	If driver has a small family	He/she will divert
	If driver has a regular-sized family	He/she will be neutral
	If driver has a large family	He/she will not divert
Familiarity	If driver is familiar with the area	He/she will divert
	If driver is not familiar with the area	He/she will not divert
Trust on Information	If driver trusts information	He/she will divert
	If driver is neutral to information	He/she will be neutral
	If driver does not trust information	He/she will not divert
Sensitivity to Delay	If driver is very sensitive	He/she will divert
	If driver is a little sensitive	He/she will be neutral
	If driver is not sensitive	He/she will not divert
Freeway Preference	If driver prefers freeway travel	He/she will not divert
	If driver does not prefer freeway travel	He/she will divert
Consideration of Neighborhood	If driver considers the neighborhood	He/she will not divert
	If driver does not consider the neighborhood	He/she will divert
Content of VMS Message	If information is very detailed	He/she will divert
	If information is a little detailed	He/she will be neutral
	If information is not detailed	He/she will not divert

TABLE 2 Prediction Rates for Different Models

Model		General	Truck Drivers	Non-truck Drivers
Sample Size		1984	928	1056
Model Component		Correct Prediction (%)		
Determination of Membership function	Triangular (heuristic)	54.10	60.87	58.18
	Transformation	58.52	65.45	62.28
Implication Operator	Mamdani Min	58.52	65.45	62.28
	Larsen Product	59.14	65.85	63.36
Method of Defuzzification	COA	59.14	65.85	63.36
	COS	59.66	67.02	64.02
Weights of *if-then* rules	Uniform	59.66	67.02	64.02
	Calibrated	62.77	70.59	67.36
Logit Model		56.85	63.25	60.51