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MULTIPLE USER CLASSES REAL-TIME TRAFFIC ASSIGNMENT FOR ONLINE OPERATIONS: A ROLLING HORIZON SOLUTION FRAMEWORK

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Abstract—Existing dynamic traffic assignment formulations predominantly assume the time-dependent O-D trip matrix and the time-dependent network configuration to be known a priori for the entire planning horizon. However, there is also a need to provide real-time path information to network users under ATIS/ATMS when unpredicted variations in O-D desires and/or network characteristics (e.g. capacity reduction on certain links due to incidents) occur. This paper presents a rolling horizon framework for addressing the real-time traffic assignment problem, where an ATIS/ATMS controller is assumed to have O-D desires up to the current time interval, and short-term and medium-term forecasts of future O-D desires. The assignment problem is solved in quasi-real time for a near-term future duration (or stage) to determine an optimal path assignment scheme for users entering the network in real-time for the short-term roll period. The resulting model is intricate due to the intertemporal dependencies characterizing this problem. Two formulations are discussed based on whether a capability to reroute vehicles en route exists. A rolling horizon solution procedure amenable to a quasi-real time implementation of a multiple user classes (MUC) time-dependent traffic assignment solution algorithm developed previously by the authors is described. Implementation issues are discussed from the perspective of ATIS/ATMS applications.

INTRODUCTION

Background and motivation

Operational procedures for real-time route guidance under ATIS/ATMS technologies aim at determining optimal routing policies for O-D desires entering the network. Existing dynamic traffic assignment formulations predominantly address the problem in which the O-D trips and their departure times are assumed known a priori (Carey, 1992; Friesz, Luque, Tobin, and Wie, 1989; Ghali and Smith, 1993; Janson, 1991; Ran, Boyce, and LeBlanc, 1993) for the planning horizon, or the departure times are determined simultaneously with the path choices a priori for the entire planning horizon (Friesz, Bernstein, Smith, Tobin, and Wie, 1993). In a comprehensive review of existing dynamic traffic assignment models, in view of their suitability for ATIS/ATMS applications, the following limitations have been identified (Peeta, 1994): (i) a priori solution for the entire planning horizon ignores unpredicted real-time variation in O-D desires, (ii) similarly, it cannot account for nonrecurrent variations in network characteristics (for example, due to incidents), (iii) all formulations consider a single class of users with identical characteristics in terms of information availability, information supply strategy, and response behavior to the supplied information, (iv) they lack traffic realism, and (v) they lack solution procedures for general networks. Mahmassani and Peeta (1992, 1993) and Mahmassani, Peeta, Hu and Ziliaskopoulos (1993) have developed time-dependent assignment models that address concerns (iii), (iv), and (v) and provide offline capabilities for ATIS/ATMS operations that may suffice online if the assumed network conditions prevail. However, they are inadequate operationally for providing optimal real-time path information and/or instructions to network users under ATIS/ATMS in response to unpredicted variations in network conditions. This motivates the development of models that explicitly incorporate real-time variations in network conditions into the optimal path determination process.

From the perspective of ATIS/ATMS operations, a controller (or traffic control center) seeks to specify paths to network users in real time that satisfy some systemwide or individual objectives while accounting for real-time unexpected variations in network conditions. The large-scale nature of this problem and the inherent intertemporal dependencies motivate the development of a procedure that solves this problem in quasi-real time.

The rolling horizon approach, used previously for production-inventory control (Wagner, 1977), and in transportation engineering for online demand-responsive traffic signal control (Gartner, 1982, 1983), provides a practical method for addressing the real-time traffic assignment problem. It is especially suited for problems requiring future demand information for the entire planning horizon, a defining characteristic of dynamic assignment problems (Peeta, 1994). The basic idea of the rolling horizon approach is to use currently available information and near-term forecasts with some degree of reliability to solve a problem online while preserving the effectiveness of the computational procedure in determining "good" control strategies. For the purpose of this discussion, *near-term* refers to a fraction of an hour, possibly of the order of 5 to 15 minutes; *long-term* refers to 20 to 30 minutes into the future.

The rolling horizon approach

Figure 1 illustrates the rolling horizon approach. It addresses the scenario in which demands are not known a priori for the entire planning horizon consisting of discrete time intervals, known as assignment intervals, $\tau = 1, \ldots, T$, and long-term forecasts of future demands may not be reliably available at the current time. The planning horizon is subdivided into several stages or projection horizons consisting of h assignment time intervals; in each stage, reliable short-term forecasts and not-as-reliable medium-term forecasts of demands are available when the beginning of the stage is reached. Figure 1 shows two consecutive stages to illustrate this approach. The shaded portion of stage $\sigma - 1$ represents the short-term duration for which demand information is reliably available and is referred to as the roll period of l time units. The rest of the stage (h–l) represents the medium-term duration for which forecasts are available with less reliability. The assignment problem is solved for stage $\sigma - 1$ using the available forecasts for that stage, but implemented only for the near-term roll period. The projection horizon is then rolled forward by l time units to obtain stage σ, represented by τ =

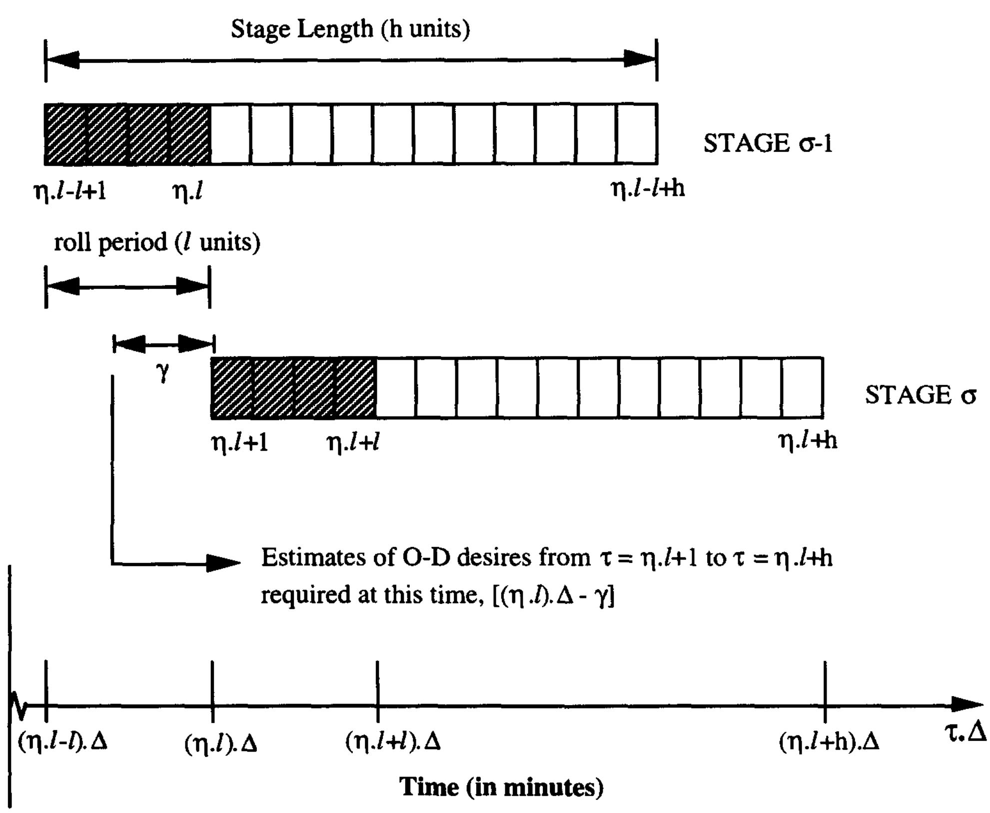

Fig. 1. The rolling horizon approach for the quasi-real time assignment problem.

$\eta.l + 1, \ldots, \eta.l + h$ (here, $\eta.l + 1$ denotes $(\eta.l) + 1$, and so forth; the parentheses are omitted for brevity). The updated short-term and medium-term forecasts are used to determine an optimal policy for that stage, which is implemented only for the corresponding roll period $\tau = \eta.l + 1, \ldots, \eta.l + l$. Here, $\eta = 0$ for stage 1, and, in general $\eta = (\sigma - 1)$, for $\sigma = 1, 2, 3, \ldots$ etc. This procedure is repeated until the end of the planning horizon.

The applicability and effectiveness of this approach relies on the ability to solve the embedded problem for a given stage in a sufficiently short time to allow the solution for the upcoming stage to reflect actual demand and/or network configuration. If γ represents the average amount of time necessary to solve the assignment problem for a stage spanning intervals $\tau = \eta.l + 1$ to $\eta.l + h$, the demand forecasts available for that stage at time $[(\eta.l).\Delta - \gamma]$ can be used to determine the optimal policy online. This policy is then implemented for the actual O-D desires that are realized in the near-term roll period $\tau = \eta.l + 1, \eta.l + l$. This forms the basis for the quasi-real time assignment problem.

FORMULATION OF THE MULTIPLE USER CLASSES QUASI-REAL TIME TRAFFIC ASSIGNMENT (MUCQRTA) PROBLEM

Notation

The following notation is used to represent variables in the formulations:

i = subscript for origin node, $i \in I$
j = subscript for destination node, $j \in J$
n = node in the network, $n \in N$
a = superscript for a link (or arc) in the network, $a \in A$
τ = superscript denoting departure (or assignment) time interval, $\tau = 1, \ldots, T$
t = superscript denoting current time interval, $t = 1, \ldots, T$
T' = total duration (peak period) for which assignments are to be made
Δ = length of an assignment time interval (equal to T'/T)
u = subscript for user class, $u \in U$
$k(u)$ = subscript for a path in the network, $k(u) \in K_{ij}^{u}$, for $i \in I$, $j \in J$, $u \in U$
$r_{ij}^{\tau u}$ = number of vehicles of user class u who wish to depart from i to j in period τ
$r_{ijk(u)}^{\tau u}$ = number of vehicles of user class u departing from i to j in period τ that are assigned to path k(u)
$\delta_{ijk(u)}^{\tau tau}$ = time-dependent link-path incidence indicator, equal to 1 if vehicles of class u going from i to j assigned to path k(u) at time τ are on link a in period t, i.e.:
$[\delta_{ijk(u)}^{\tau tau}]$ = 1, if the $r_{ijk(u)}^{\tau u}$ vehicles are on arc a during period t
= 0, if arc a does not belong to path k(u)
= 0, of $\tau > t$
= 0, if the $r_{ijk(u)}^{\tau u}$ vehicles are not on arc a during period t]
$T_{ijk(u)}^{\tau u}$ = experienced path travel time for vehicles of user class u going from i to j assigned to path k(u) at time τ
$x_{ijk(u)}^{\tau tau}$ = number of vehicles of user class u going from i to j assigned to path k(u) in period τ that are on link a at the beginning of period t
$d_{ijk(u)}^{\tau tau}$ = number of vehicles of user class u going from i to j assigned to path k(u) in period τ which enter link a in period t
$m_{ijk(u)}^{\tau tau}$ = number of vehicles of user class u going from i to j assigned to path k(u) in period τ which exit link a in period t

Problem definition

Consider a traffic network represented by a directed graph G(N, A), with multiple origins and multiple destinations. Given a set of short-term and medium-term forecasts of time-dependent O-D vehicle trip desires for stage σ expressed as the number of vehicle trips $\hat{r}_{ij}^{\tau u}$ of user class u leaving node i for node j in time slice τ, $\forall\, i \in I, j \in J, \tau = \eta.l + 1, \ldots, \eta.l$

+ h, and u = 1, . . . , U, the actual path assignments up to the current stage, $r^{\tau u}_{ijk(u)}$, $\forall$ i $\in$ I, j $\in$ J, τ = 1, . . . , $\eta.l$, u = 1, . . . , U, and the current network conditions, determine a set of paths to be assigned to the actual O-D desires $r^{\tau u}_{ij}$ arriving in the roll period, $\tau = \eta.l +$ 1, . . . , $\eta.l + l$, of stage σ (see Fig. 1). In other words, find the number of vehicles $\hat{r}^{\tau u}_{ijk(u)}$ of class u that follow path k(u) = 1, . . . , K^u_{ij} between i and j at time τ, $\forall$ i $\in$ N, j $\in$ J, $\tau = \eta.l$ + 1, . . . , $\eta.l$ + h, and u = 1, . . . , U, so as satisfy some systemwide objectives and/or certain conditions specific to each user class. The actual O-D desires $r^{\tau u}_{ij}$ that arrive in that roll period are then assigned these time-dependent "optimal" paths in real time. However, the assignment decisions themselves are made in quasi-real time.

Four user classes are incorporated in the formulation: (i) equipped drivers who follow prescribed system optimal paths (called user class 1 or SO class); (ii) equipped drivers who follow user optimum routes (called user class 2 or UE class); (iii) equipped drivers who follow a boundedly rational switching rule in response to descriptive information on prevailing conditions (called user class 3 or BR class); and (iv) nonequipped drivers who follow externally specified paths, which may be historically known or solved for exogenously (called user class 4 or PS class).

The boundedly rational path switching rule states that users switch from the current path at a decision point (typically a node on the network) if travel times savings on an alternative route exceed a threshold value. Experimental evidence by Mahmassani and Stephan (1988) suggests that commuter route choice behavior exhibits a boundedly rational character whereby drivers look for gains that exceed a particular threshold, within which results are satisfying and sufficing; as represented mathematically in the following route switching model (Mahmassani and Jayakrishnan, 1991):

$$\beta_p(m) = \begin{cases} 1 & \text{if } \hat{T}^{\tau u}_{mjk(u)} - \hat{T}^{\tau u}_{mjk(u)*} > \max(\zeta_p.\hat{T}^{\tau u}_{mjk(u)}, \xi_p) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $\beta_p(m)$ is a binary indicator variable equal to 1 when user p switches from the current path to the best alternate path (from node m to the destination), and 0 if the current path is maintained; and $\hat{T}^{\tau u}_{mjk(u)}$ and $\hat{T}^{\tau u}_{mjk(u)*}$ are respectively the trip times on the current and best paths from node m to the destination j based on current link travel times (and hence distinguished from the experienced travel time T(.) using $\hat{T}(.)$); ζ_p is the relative indifference threshold (or band), and ξ_p is an absolute minimum travel time improvement needed for a switch.

Two scenarios based on rerouting of vehicles

The multiple user classes (MUC) quasi-real time assignment problem is highly complicated due to the interactions among vehicle classes and the interdependencies across stages. Vehicle conservation constraints must be satisfied from one stage to the next, in addition to within the stage. Vehicles assigned in a roll period may not all reach their respective destinations by the end of the roll period. At any stage σ, there may be vehicles in the network assigned in some previous stages that have not reached their destinations. There are two ways of treating vehicles already in the network at the beginning of the current stage: (a) they follow previously prescribed paths and thus represent background vehicles for the current stage, or (b) they may be rerouted in the current stage. The BR class vehicles are not affected by this differentiation, as they are governed by user behavioral rules. The paths of the PS class vehicles are fixed from the perspective of the formulation (as they are obtained exogenously) and hence are unaffected by this differentiation.

MUC rolling horizon formulation without vehicle rerouting

This formulation assumes that vehicles assigned in previous stages that have not yet reached their destinations are not rerouted in the current stage. It is less involved computationally than the scenario that assumes rerouting of remaining previously assigned vehicles. It is primarily suitable for situations in which the reliability of the forecast future O-D desires is high, thereby the paths prescribed for O-D desires in previous stages continue to be near optimal in the current stage. However, it may not be applicable even if the reliability of future forecasts is high, when unaccounted for stochastic events such as incidents occur leading to rapid changes in network conditions over short durations of time.

The problem here aims at minimizing the total time spent by all vehicles in the network during the current stage $\tau = \eta.l + 1, \ldots, \eta.l + h$, while satisfying applicable conditions for each class. Let $s^{\tau u}_{ijk(u)}$ denote the number of vehicles assigned from origin i to destination j in period τ, $\tau = 1, \ldots, \eta.l$, to path k that have not reached j before the interval $\tau = \eta.l + 1$ of the current stage σ. The vehicles included in the current stage include the forecasts of O-D desires $\hat{f}^{\tau u}_{ij}$ for $\tau = \eta.l + 1, \ldots, \eta.l + h$, and the previously assigned vehicles that have not yet reached their destinations, $s^{\tau u}_{ijk(u)}$. The formulation is as follows:

Given:

(i) $s^{\tau u}_{ijk(u)}$, $\forall$ i, j, k(u), $\tau = 1, \ldots. \eta.l$, and $u = 1, \ldots, 4$, all O-D desires assigned to paths up to the beginning of the current stage σ that have not yet reached their destinations.
(ii) $\hat{f}^{\tau u}_{ij}$, $\forall$ i, j, $\tau = \eta.l + 1, \ldots, \eta.l + l$, and $u = 1, 2, 3$, the short-term forecast O-D desires for the current roll period for which an SO assignment policy is to be determined.
(iii) $\hat{f}^{\tau 4}_{ijk(4)}$, $\forall$ i, j, $k(4) \in K^4_{ij}$, $\tau = \eta.l + 1, \ldots, \eta.l + h$, the paths for the forecast unequipped class vehicles.
(iv) $\hat{f}^{\tau u}_{ij}$, $\forall$ i, j, $\tau = \eta.l + l + 1, \ldots, \eta.l + h$, and $u = 1, 2, 3$, the medium-term forecast O-D desires for the rest of current stage σ.

Objective function:

$$\text{Min.} \left\{ \sum_{\tau=\eta.l+1}^{\eta.l+h} \sum_i \sum_j \sum_u \sum_{k(u)} (\hat{f}^{\tau u}_{ijk(u)} \cdot T^{\tau u}_{ijk(u)}) \right.$$

$$+ \sum_{\tau=1}^{\eta.l} \sum_i \sum_j \sum_{u=1,2,4} \sum_{k(u)} [s^{\tau u}_{ijk(u)} \cdot (T^{\tau u}_{ijk(u)} + \tau.\Delta - \eta.l.\Delta)]$$

$$\left. + \sum_{\tau=1}^{\eta.l} \sum_i \sum_j \sum_{\tilde{k}(3)} [(s^{\tau 3}_{ij\tilde{k}(3)}).(T^{\tau 3}_{ij\tilde{k}(3)} + \tau.\Delta - \eta.l.\Delta)] \right\} \tag{2a}$$

or

$$\text{Min. } \mathbf{T}[(\hat{f}^{\tau u}_{ijk(u)}, s^{\kappa \upsilon}_{ijk(\upsilon)}, s^{\kappa 3}_{ij\tilde{k}(3)}), \quad \forall \text{ i, j, u, k(u)}, \upsilon = 1, 2 \text{ and } 4, k(\upsilon),$$

$$\tilde{k}(3), \tau = \eta.l + 1, \ldots, \eta.l + h, \text{ and } \kappa = 1, \ldots, \eta.l] \tag{2a'}$$

Subject to:

$$\hat{f}^{\tau u}_{ij} = \sum_{k(u)} \hat{f}^{\tau u}_{ijk(u)}, \quad \forall \text{ i, j, u}, \tau = \eta.l + 1, \ldots, \eta.l + h \tag{2b}$$

$$\sum_b d^{tb} = \sum_c m^{tc} + I^t_n - O^t_n, \forall\, t = \eta.l + 1, \ldots, \eta.l + h, n, b \in B(n), c \in C(n) \tag{2c}$$

$$x^{ta} = x^{t-1a} + d^{t-1a} - m^{t-1a}, \quad \forall\, t = \eta.l + 1, \ldots, \eta.l + h, a \tag{2d}$$

$$x^{ta} = \left\{ \sum_{\tau=\eta.l+1}^{\eta.l+h} \sum_{i} \sum_{j} \sum_{u} \sum_{k(u)} (\hat{r}^{\tau u}_{ijk(u)} \cdot \delta^{\tau tau}_{ijk(u)}) + \sum_{\tau=1}^{\eta.l} \sum_{i} \sum_{j} \sum_{u=1,2,4} \sum_{k(u)} (s^{\tau u}_{ijk(u)} \cdot \delta^{\tau tau}_{ijk(u)}) \right.$$

$$\left. + \sum_{\tau=1}^{\eta.l} \sum_{i} \sum_{j} \sum_{\tilde{k}(3)} (s^{\tau 3}_{ij\tilde{k}(3)} \cdot \delta^{\tau ta3}_{ij\tilde{k}(3)}) \right\}, \quad \forall\, t = \eta.l + 1, \ldots, \eta.l + h, a \tag{2e}$$

$$T^{\tau u}_{ijk(u)} = \sum_{t=1}^{\eta.l+h} \sum_{a} [\delta^{\tau tau}_{ijk(u)} \cdot \Delta], \quad \forall\, i, j, u, k(u), \tau = 1, \ldots, \eta.l + h \tag{2f}$$

$$\delta^{\tau tau}_{ijk(u)} = F[(\hat{r}^{\tau u}_{ijk(u)}, s^{\kappa \upsilon}_{ijk(\upsilon)}, s^{\kappa 3}_{ij\tilde{k}(3)}), \quad \forall\, i, j, u, k(u), \upsilon = 1, 2 \text{ and } 4, k(\upsilon), \tilde{k}(3), \tau$$

$$= \eta.l + 1, \ldots, \eta.l + h, \text{ and } \kappa = 1, \ldots, \eta.l]$$

$$\forall\, i, j, u, k(u), \tau = 1, \ldots, \eta.\, l + h, t = \eta.l + 1, \ldots, \eta.l + h, a \tag{2g}$$

$$d^{ta} = \sum_{\tau=1}^{\eta.l+h} \sum_{i} \sum_{j} \sum_{u} \sum_{k(u)} d^{\tau tau}_{ijk(u)}, \quad \forall\, t = \eta.l + 1, \ldots, \eta.l + h, a \tag{2h}$$

$$m^{ta} = \sum_{\tau=1}^{\eta.l+h} \sum_{i} \sum_{j} \sum_{j} \sum_{k(u)} m^{\tau tau}_{ijk(u)}, \quad \forall\, t = \eta.l + 1, \ldots, \eta.l + h, a \tag{2i}$$

$$I^{t}_{n} = \sum_{j} \sum_{u} \hat{r}^{\tau u}_{nj}, \quad \forall\, t = \eta.l + 1, \ldots, \eta.l + h, n \in I \tag{2j}$$

$$O^{t}_{n} = \sum_{\tau=1}^{\eta.l+h} \sum_{u} \sum_{k(u)} \sum_{i} \sum_{c} m^{\tau tcu}_{ink(u)}, \quad \forall\, t = \eta.l + 1, \ldots, \eta.l + h, n \in J, c \in C(n) \tag{2k}$$

$$\tau \leqslant t \tag{2l}$$

$$\delta^{\tau tau}_{ijk(u)} = 0 \text{ or } 1, \quad \forall\, i, j, u, k(u), \tau = 1, \ldots, \eta.\, l + h, t = \eta.l + 1, \ldots, \eta.l + h, a \tag{2m}$$

$$(T^{\tau 2}_{ijk(2)} - \theta^{\tau}_{ij}) \cdot \hat{r}^{\tau 2}_{ijk(2)} = 0, \quad \forall\, i, j, \tau = \eta.l + 1, \ldots, \eta.l + h, k(2) \tag{2n}$$

$$(T^{\tau 2}_{ijk(2)} - \theta^{\tau}_{ij} \geqslant 0, \quad \forall\, i, j, \tau = \eta.l + 1, \ldots, \eta.l + h, k(2) \tag{2o}$$

$$\beta_p(v) = \begin{cases} 1 & \text{if } \hat{T}^{\tau 3}_{vjk''(3)} - \hat{T}^{\tau 3}_{vjk(3)*} > \max(\zeta_v . \hat{T}^{\tau 3}_{vjk''(3)}, \xi_v) \\ 0 & \text{otherwise} \end{cases}$$

$$\forall \quad v \in N, j, u = 3, k''(3), \tau = \eta.l + 1, \ldots, \eta.l + h, a \tag{2p}$$

$$\text{All variables} \geqslant 0 \tag{2q}$$

There are two functional forms for the objective function. Both represent the minimization of the total system travel time for the current stage for all vehicles present in the network at any time during $\tau = \eta.l + 1, \ldots, \eta.l + h$. The expression (2a) consists of three terms. The first term represents the total travel time for the O-D forecasts of the current stage, $\tau = \eta.l + 1, \ldots, \eta.l + h$. Here, $T^{\tau u}_{ijk(u)}$ refers to the travel time until $\tau = \eta.l + h$, and includes the time spent by vehicles either until they reach their destination or until the end of the current stage (whichever comes first). The calculation of $T^{\tau u}_{ijk(u)}$ is discussed further in conjunction with constraint (2f). The second term of (2a) represents the travel time in this stage for all previously assigned vehicles of user classes 1, 2, and 4 still in the network, $s^{\tau u}_{ijk(u)}$. As illustrated in Fig. 2, this travel time is $(T^{\tau u}_{ijk(u)} + \tau.\Delta - \eta.l.\Delta)$ for a vehicle assigned in any time interval τ before the current stage. Though this formulation assumes that the paths of previously assigned vehicles $s^{\tau u}_{ijk(u)}$ are fixed, the travel times of these vehicles in the current stage are dependent on the assignment decisions $\hat{r}^{\tau u}_{ijk(u)}$ made in this stage. The third term distinguishes between past assignments that are currently background vehicles ($s^{\tau u}_{ijk(u)}$ for user classes 1, 2, and 4) and vehicles of user class 3 whose paths may be modified in the current stage depending on prevailing conditions (giving actual path assignments $s^{\tau 3}_{ijk(3)}$ at the end of the stage). Hence, the objective is to determine path assignments for O-D desires generated in stage σ that minimize the total travel time for the duration $t = \eta.l + 1, \ldots, \eta.l + h$ of all vehicles present in the network at any time during stage σ.

The second form of the objective function (2a′) also expresses the above objective, and is motivated by the fact that path travel times $T^{\tau u}_{ijk(u)}$ are themselves a complicated nonexplicit function of the assignment decisions $\hat{r}^{\tau u}_{ijk(u)}$, and hence, computationally intractable (Mahmassani and Peeta, 1992; Peeta, 1994). Here, T(.) is a function of both $\hat{r}^{\tau u}_{ijk(u)}$ and $s^{\tau u}_{ijk(u)}$, and represents the total travel time in stage σ of all vehicles in the network from $\tau = \eta.l + 1, \ldots, \eta.l + h$. The solution algorithm discussed in the next section uses a simulation model to evaluate (2a′) by simulating the assignment decisions for stage σ. A significant challenge for the solution algorithm and its implementation is the ability to keep track of vehicle assignments $s^{\tau u}_{ijk(u)}$ in future stages to ensure conservation of vehicles.

Constraints (2b) represent the conservation of O-D desires (vehicles) at the origin nodes. Constraints (2c) denote the conservation of vehicles at nodes. They imply that vehicles cannot be stored at nodes and state that at any time t in stage σ on a node n, the number of vehicles entering all links incident from the node should equal the sum of the number of vehicles exiting from all links incident to that node and the net generation. Constraints (2d) represent the

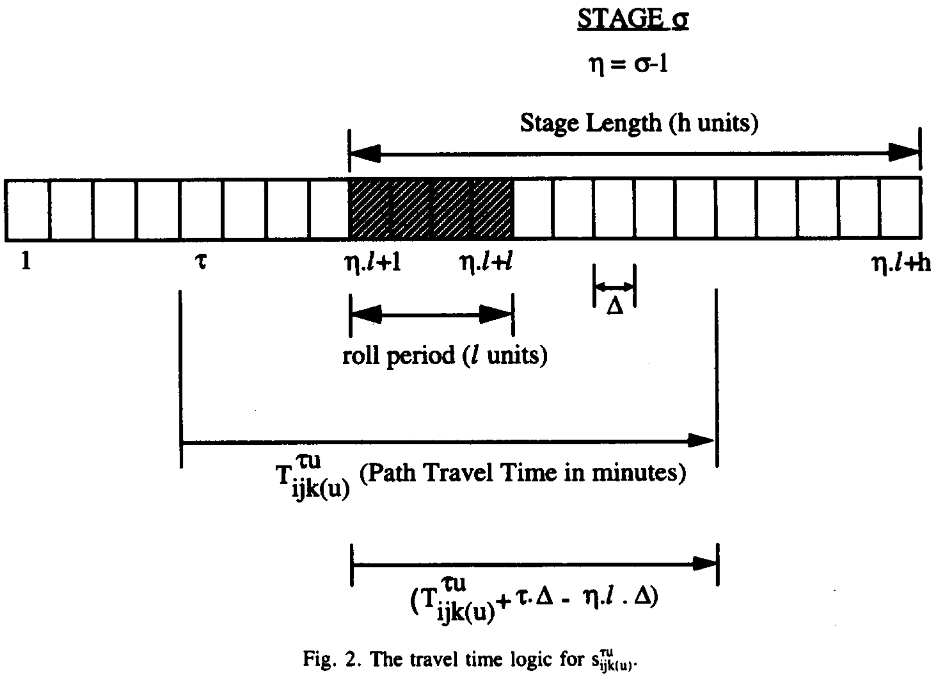

Fig. 2. The travel time logic for $s^{\tau u}_{ijk(u)}$.

conservation of vehicles on links and state that the number of vehicles on any link a at the beginning of time interval t is the net algebraic sum of the number of vehicles on the link at the beginning of the previous time interval (t − 1), vehicles entering the link in interval (t − 1), and vehicles exiting the link in interval (t − 1).

Constraints (2e) provide interstage conservation of vehicles and represent a key difference of this formulation from the a priori full information assignment models. They state that the number of vehicles present on a link at time t is the sum of three components, those due to current and past assignments, expressed using the link-path incidence variables $\delta^{\tau tau}_{ijk(u)}$, for $t = \eta.l + 1, \ldots, \eta.l + h$, and $\tau = 1, \ldots, \eta.l$, which capture the experience of previous assignments $s^{\tau u}_{ijk(u)}$ in the current stage. The variables $\delta^{\tau tau}_{ijk(u)}$ for $t = 1, \ldots, \eta.l + h$, determine the path travel times $T^{\tau u}_{ijk(u)}$ of vehicles up to the end of stage σ in equations (2f); for vehicles in the network beyond the current stage, $T^{\tau u}_{ijk(u)}$ is the travel time on that portion of the path traversed until $t = \eta.l + h$.

Perhaps the most critical and difficult to characterize constraints in dynamic traffic assignment problems are (2g) that represent physical traffic flow. Constraints (2g) state that the time-dependent link-path incidence variables are a function F(.) of all the assignment decisions $\hat{f}^{\tau u}_{ijk(u)}$ made in stage σ and the previous vehicle assignments still in the network $s^{\tau u}_{ijk(u)}$. Implicit in these constraints is the satisfaction of the first-in, first-out (FIFO) property of traffic flow, preclusion of holding of vehicles, and the representation of link interactions and other dynamic traffic phenomena. However, there are no known analytical functions that can adequately represent F(.). The simulation-based solution algorithm discussed in the next section uses a traffic simulator to evaluate F(.). While the properties of F(.) are not well understood, the ability of simulation to adequately represent it obviates the need for unrealistic link performance and/or link exit functions. Equations (2g) prove to be most troublesome for formulating and solving dynamic traffic assignment problems. The complexity of the traffic flow interactions represented by these constraints precludes the guarantee of well-behaved properties like convexity for these constraints and, in turn, for constraints (2e) and (2f). In addition, their implication of the dependence of current decisions on future travel conditions lead to a complicated fixed-point problem.

The definitional constraints (2h) and (2i), respectively, define the number of vehicles entering and exiting link a in any time interval t in the current stage, and consist of vehicles generated at any time $\tau = 1, \ldots, \eta.l + h$ that are on link a at time t. Constraints (2j) and (2k) are, respectively, the definitional constraints for the number of vehicles entering and exiting the network at node n in any time interval t in the current stage. As expressed, vehicles generated in the current stage belong only to $\tau = \eta.l + 1, \ldots, \eta.l + h$, whereas vehicles exiting the network at any node n in the current stage may be generated at any time $\tau = 1, \ldots, \eta.l + h$. Constraints (2l) are the temporal correctness constraints that restrict the start (or departure) time interval τ of assigned vehicles to be at most the current time interval t. Constraints (2m) restrict the time-dependent incidence variables to take values of 0 or 1 for the current stage.

As indicated in (2n)–(2o), only those UE class vehicles forecast for the current stage, $\hat{f}^{\tau 2}_{ijk(2)}$, are subjected to the UE conditions in the current stage. Previously generated UE class vehicles that are in the network in the current stage, $s^{\tau 2}_{ijk(2)}$, are not subject to the UE conditions in the current stage, as they are viewed as background vehicles (their routes are fixed in some previous stage).

In comparison, all BR class vehicles in the network in the current stage, including those generated in previous stages that are still in the network, $s^{\tau 3}_{ijk(3)}$, and those forecast for the current stage, $\hat{f}^{\tau 3}_{ijk(3)}$, are subject to constraints (2p). Here, k″(3) represent the paths of user class 3, and the particular notation indicates that these paths can change for vehicles at any time during the current stage. The rest of the formulation depicts the paths for this user class as $\tilde{k}(3)$, which represent the paths of these vehicles at the end of the current stage; or if they leave the network during the current stage, their actual travel paths from the origin to the destination. The paths of user class 3 vehicles ($s^{\tau 3}_{ijk(3)}$) at the end of the previous stage (beginning of the current stage), k(3), serve as initial paths for these vehicles for the current stage. Constraints (2q) are the nonnegativity constraints.

MUC rolling horizon formulation with vehicle rerouting

Unlike the previous formulation, some/all vehicles assigned in previous stages may be rerouted in the current stage to the constraints on individual user classes. The rerouting capability affects only the equipped user classes 1 and 2, whose vehicular paths are assumed fixed in future stages in the previous formulation (i.e., $s^{\tau u}_{ijk(u)}$, for u = 1, 2). Vehicles of user class 3 may change paths during any stage based on boundedly rational switching rules independent of any capability for rerouting. The paths of unequipped vehicles (user class 4) are taken as "initial conditions" and remain unchanged even under the rerouting scenario.

Let $\mu^{\tau\kappa u}_{i^*jk(u)}$ represent those vehicle assignments that are assigned in any previous stage, $\kappa = 1, \ldots, \eta.l$, from origin $i^* \in N$ to destination j on path k(u) that are to be reassigned from the first intermediate node i reached at time τ in the current stage, for user classes u = 1, 2. The nodes i represent the new origins for these vehicles, and their new start times τ are based on when they reach those nodes. Let $\Psi^{\tau u}_{ij}$ denote such O-D desires in the current stage, so that:

$$\Psi^{\tau u}_{ij} = \sum_{\kappa=1}^{\eta.l} \sum_{i^*} \sum_{k(u)} \mu^{\tau\kappa u}_{i^*ijk(u)}, \quad \forall\, i, j, u = 1, 2, \tau = \eta.l + 1, \ldots, \eta.l + h \tag{3}$$

Let $s^{\tau u}_{i^*jk(u)}$ denote those vehicles assigned from origin i* to destination j in any period $\tau = 1, \ldots, \eta.l$, to path k(u) that have not reached j before the current stage σ, which are not rerouted in the current stage; such vehicles belong to any of the user classes 1, 2, and 4. An assumption for user classes 1 and 2 is that $s^{\tau u}_{i^*jk(u)} = 0$ for $\mu^{\tau\kappa u}_{i^*ijk(u)} > 0$, and $\mu^{\tau\kappa u}_{i^*ijk(u)} = 0$ for $s^{\tau u}_{i^*jk(u)} > 0$, which implies that vehicles belonging to a previously assigned "packet" $\gamma^{\kappa u}_{i^*jk(u)}$ are either all rerouted or not rerouted in the current stage. It follows that $\delta^{\kappa tau}_{i^*jk(u)} = 0$ for the assignments $\mu^{\tau\kappa u}_{i^*ijk(u)}$ for $t > \tau$, implying that the packet $r^{\kappa u}_{i^*jk(u)}$ does not exist beyond τ, the interval in which the first intermediate node is reached in the current stage. The formulation is as follows:

Given:

(i) $s^{\tau u}_{ijk(u)}$, $\forall$ i, j, u = 1, 2, 4, k(u), and $\tau = 1, \ldots \eta.l$; those O-D desires assigned to paths up to the current stage σ that have not yet reached their destinations, which are not rerouted in the current stage; they follow previously prescribed paths k(u) in the current stage.

(ii) $s^{\tau 3}_{ijk(3)}$, $\forall$ i, j, k(3), and $\tau = 1, \ldots \eta.l$; all O-D desires of user class 3 assigned to paths up to the beginning of the current stage σ that have not yet reached their destinations; these vehicles make real-time path switching decisions in the current stage based on the boundedly rational behavioral rule.

(iii) $\mu^{\tau\kappa u}_{i^*ijk(u)}$, $\forall$ i*, i, j, u = 1, 2, k(u), $\tau = \eta.l + 1, \ldots, \eta.l + h$, and $\kappa = 1, \ldots \eta.l$; those vehicle assignments that are assigned in any previous stage from origin $i^* \in N$ to destination j on path k(u) that are to be rerouted from the first intermediate node i reached at time τ in the current stage.

(iv) $\Psi^{\tau u}_{ij}$, $\forall$ i, j, u = 1, 2, and $\tau = \eta.l + 1, \ldots, \eta.l + h$; the O-D desires in the current stage based on those previous assignments in the network, $\mu^{\tau\kappa u}_{i^*ijk(u)}$, that seek to be rerouted in the current stage, as indicated in eqn (3).

(v) $\phi^{\tau u}_{ij}$, $\forall$ i, j, u = 1, 2, 3, and $\tau = \eta.l + 1, \ldots, \eta.l + l$; the short-term forecast O-D desires for the current roll period for which an SO assignment policy is to be determined.

(vi) $\phi^{\tau u}_{ij}$, $\forall$ i, j, u = 1, 2, 3, and $\tau = \eta.l + l + 1, \ldots, \eta.l + h$; the medium-term forecast O-D desires for the rest of current stage based on currently obtained information and historical data.

(vii) $\phi^{\tau 4}_{ijk(4)}$, $\forall$ i, j, $k(4) \in K^4_{ij}$, $\tau = \eta.l + 1, \ldots, \eta.l + h$; the paths for the forecast unequipped class vehicles.

Objective function:

$$\text{Min.}\left\{\sum_{\tau=\eta.l+1}^{\eta.l+h}\sum_{i}\sum_{j}\sum_{u}\sum_{k(u)}(\hat{f}^{\tau u}_{ijk(u)}\cdot T^{\tau u}_{ijk(u)})\right.$$

$$+\sum_{\tau=1}^{\eta.l}\sum_{i}\sum_{j}\sum_{u=1,2,4}\sum_{k(u)}[s^{\tau u}_{ijk(u)}\cdot(T^{\tau u}_{ijk(u)}+\tau.\Delta-\eta.l.\Delta)]$$

$$+\sum_{\tau=1}^{\eta.l}\sum_{i}\sum_{j}\sum_{\tilde{k}(3)}[(s^{\tau 3}_{ij\tilde{k}(3)})\cdot(T^{\tau 3}_{ij\tilde{k}(3)}+\tau.\Delta-\eta.l.\Delta)]$$

$$\left.+\sum_{\kappa=1}^{\eta.l}\sum_{\tau=\eta.l+1}^{\eta.l+h}\sum_{i^*}\sum_{i}\sum_{j}\sum_{u=1}^{2}\sum_{k(u)}\mu^{\tau\kappa u}_{i^*ijk(u)}\cdot(\tau.\Delta-\eta.l.\Delta)\right\} \tag{4a}$$

or

$$\text{Min. } \mathbf{T}[(\hat{f}^{\tau u}_{ijk(u)}, s^{\kappa \upsilon}_{ijk(\upsilon)}, s^{\kappa 3}_{ij\tilde{k}(3)}, \mu^{\tau\kappa\phi}_{i^*ijk(\phi)}),\quad \forall\, i, i^*, j, u, k(u), \varphi = 1, 2, \upsilon = 1, 2 \,\&\, 4, k(\upsilon), \tilde{k}(3), \tau = \eta.l+1, \ldots, \eta.l+h, \text{ and } \kappa = 1, \ldots, \eta.l] \tag{4a'}$$

Subject to:

$$\hat{f}^{\tau u}_{ij} = \Psi^{\tau u}_{ij} + \phi^{\tau u}_{ij},\quad \forall\, i, j, u, \tau = \eta.l+1, \ldots, \eta.l+h \tag{4b}$$

$$\hat{f}^{\tau u}_{ij} = \sum_{k(u)}\hat{f}^{\tau u}_{ijk(u)},\quad \forall\, i, j, u, \tau = \eta.l+1, \ldots, \eta.l+h \tag{4c}$$

$$\sum_{b} d^{tb} = \sum_{c} m^{tc} + I^{t}_{n} - O^{t}_{n},\quad \forall\, t = \eta.l+1, \ldots, \eta.l+h, n, b \in B(n), c \in C(n) \tag{4d}$$

$$x^{ta} = x^{t-1a} + d^{t-1a} - m^{t-1a},\quad \forall\, t = \eta.l+1, \ldots, \eta.l+h, a \tag{4e}$$

$$x^{ta} = \left\{\sum_{\tau=\eta.l+1}^{\eta.l+h}\sum_{i}\sum_{j}\sum_{u}\sum_{k(u)}(\hat{f}^{\tau u}_{ijk(u)}\cdot\delta^{\tau tau}_{ijk(u)})\right.$$

$$+\sum_{\tau=1}^{\eta.l}\sum_{i}\sum_{j}\sum_{u=1,2,4}\sum_{k(u)}(s^{\tau u}_{ijk(u)}\cdot\delta^{\tau tau}_{ijk(u)}) + \sum_{\tau=1}^{\eta.l}\sum_{i}\sum_{j}\sum_{\tilde{k}(3)}(s^{\tau 3}_{ij\tilde{k}(3)}\cdot\delta^{\tau ta3}_{ij\tilde{k}(3)})$$

$$\left.+\sum_{\kappa=1}^{\eta.l}\sum_{\tau=\eta.l}^{\eta.l+h}\sum_{i^*}\sum_{i}\sum_{j}\sum_{u=1}^{2}\sum_{k(u)}\mu^{\tau\kappa u}_{i^*ijk(u)}\cdot\delta^{\kappa tau}_{i^*jk(u)}\right\},$$

$$\forall\, t = \eta.l+1, \ldots, \eta.l+h, a \tag{4f}$$

$$T^{\tau u}_{ijk(u)} = \sum_{t=1}^{\eta.l+h} \sum_{a} [\delta^{\tau tau}_{ijk(u)} \cdot \Delta], \quad \forall\ i, j, u, k(u), \tau = 1, \ldots, \eta.l + h \tag{4g}$$

$$\delta^{\tau tau}_{ijk(u)} = F[(\hat{f}^{\tau u}_{ijk(u)}, s^{\kappa \upsilon}_{ijk(\upsilon)}, s^{\kappa 3}_{ij\bar{k}(3)}, \mu^{\tau\kappa\varphi}_{i*ijk(\varphi)}), \quad \forall\ i, i*, j, u, k(u), \varphi = 1, 2,$$

$$\upsilon = 1, 2 \text{ and } 4, k(\upsilon), \bar{k}(3), \tau = \eta.l + 1, \ldots, \eta.l + h, \text{ and } \kappa = 1, \ldots, \eta.l]$$

$$\forall\ i, j, u, k(u), \tau = 1, \ldots, \eta.l + h, t = \eta.l + 1, \ldots, \eta.l + h, a \tag{4h}$$

Equations (2h) through (2q), relabeled here as (4i) through (4r).

The two forms for the objective function (4a) and (4a′) are similar to (2a) and (2a′), and differ primarily by the presence of $\mu^{\tau\kappa u}_{i*ijk(u)}$. The first three terms of (4a) are identical to the those of (2a). However, as illustrated by constraints (4b), the definitions of $\hat{f}^{\tau u}_{ij}$ here is slightly different than in the formulation (2a)–(2q). It includes both forecasts of O-D trips for current stage, $\phi^{\tau u}_{ij}$, and previous assignments (of user classes 1 and 2) that seek to be rerouted in the current stage, $\Psi^{\tau u}_{ij}$. Consequently, the fourth term of the objective function computes the total travel time of all such vehicles in the current stage, $\mu^{\tau\kappa u}_{i*ijk(u)}$, before they reach their first intermediate node in the current stage. Unlike the previous formulation, $s^{\tau u}_{ijk(u)}$ here represent only part of the previously assigned vehicles. The fourth term represents the travel time of vehicles to be reassigned in the current stage, $\mu^{\upsilon\kappa u}_{i*ijk(u)}$, until the time interval υ when the first intermediate node i in the current stage is reached. Hence, it denotes the travel time until node i for these vehicles on the previously prescribed path k in the current stage. This component of the system travel time is $(\upsilon.\Delta - \eta.l.\Delta)$ for the vehicles $\mu^{\upsilon\kappa u}_{i*ijk(u)}$. Hence, υ itself is dependent on all assignment decisions $\hat{f}^{\tau u}_{ijk(u)}$ made in the current stage. The second form of the objective function (4a′) states that the total system travel time T(.) under a given assignment strategy for this stage depends additionally on $\mu^{\tau\kappa u}_{i*ijk(u)}$, compared to (2a′).

Constraints (4b) state that the O-D desires in any stage consist of newly generated vehicles and previously assigned vehicles that are candidates for reassignment. Constraints (4c), (4d), and (4e) are as defined in the previous formulation, though some variables take slightly different meanings in this formulation. The redefinition of $\hat{f}^{\tau u}_{ij}$, and the additional variables, $\mu^{\tau\kappa u}_{i*ijk(u)}$, modify constraints (4f) and (4h). The other constraints are identical to the corresponding constraints in the formulation (2a)–(2q), except that in addition to (2l), $\kappa \leq t$ must also be satisfied.

MUCQRTA SOLUTION ALGORITHM: ROLLING HORIZON IMPLEMENTATION

Figure 3 illustrates the overall solution framework for the MUCQRTA problem. For any stage σ, the assignment problem to be solved is analogous to a time-dependent assignment problem for a horizon $\tau = \eta.l + 1, \ldots, \eta.l + h$, as O-D forecasts are available for the duration of that stage. Hence, the solution algorithm for a Multiple User Classes Time-Dependent Traffic Assignment (MUCTDTA) problem developed previously by the authors forms the basis for the rolling horizon solution procedure. As illustrated in the figure, an additional feature in the MUCQRTA problem is the existence of interstage vehicular conservation constraints.

MUCTDTA solution algorithm

Given a set of time-dependent O-D vehicle trip desires for the entire peak period duration (or planning horizon), expressed as the number of vehicle trips $r^{\tau u}_{ij}$ of user class u leaving node i for node j in time slice τ, $\forall\ i \in I, j \in J, \tau = 1, \ldots, T$, and $u = 1, \ldots, 4$, the MUCTDTA solution algorithm (Mahmassani et al. 1993; Peeta, 1994) determines a time-dependent assignment of vehicles to network paths and corresponding arcs so as to achieve some systemwide objectives and/or satisfy certain conditions for the user classes. In the current context, given the short-term and medium-term O-D forecasts, $\hat{f}^{\tau u}_{ij}$, for the planning horizon $\tau = \eta.l + 1, \ldots,$

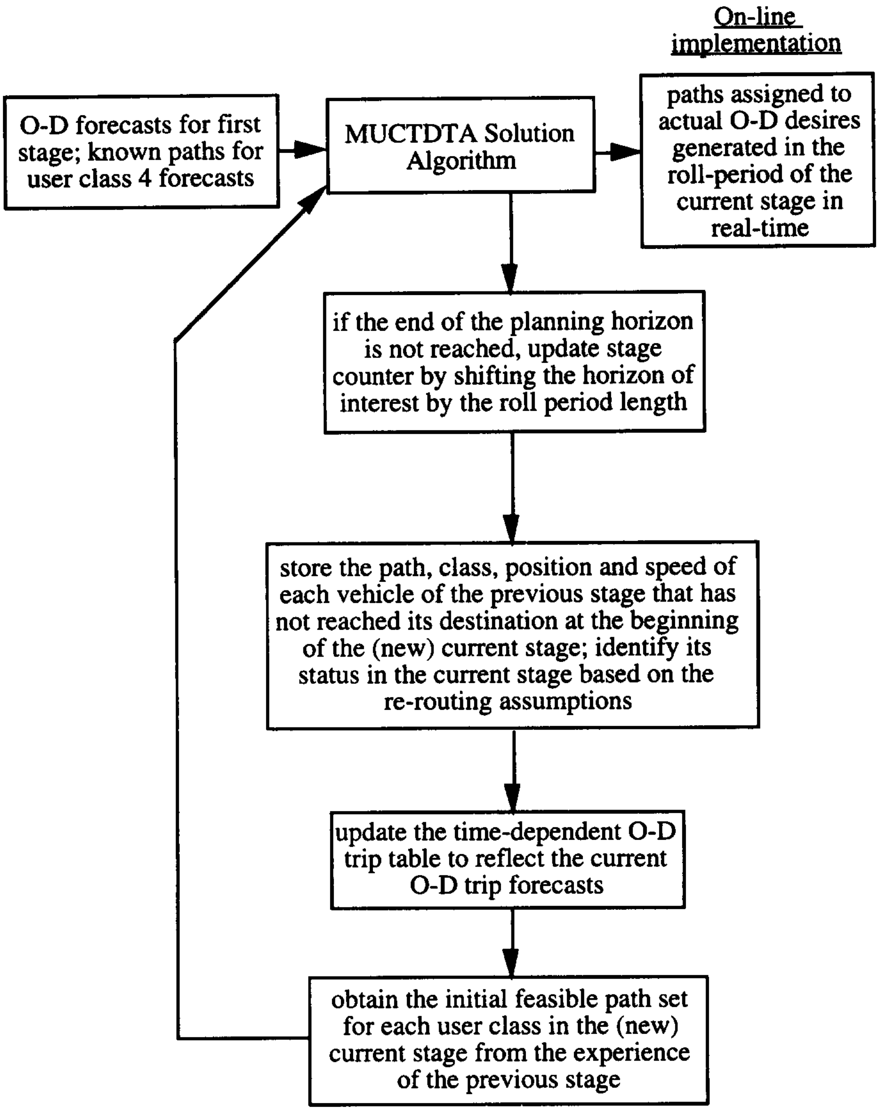

Fig. 3. Rolling horizon solution framework.

$\eta.l + h$, the MUCTDTA algorithm determines the number of vehicles $\hat{f}^{\tau u}_{ijk(u)}$ of user class u that follow path $k(u) = 1, \ldots, K^u_{ij}$ between i and j at time τ, $\forall\, i \in I, j \in J, \tau = 1, \ldots, T$, and $u = 1, \ldots, 4$.

Figure 4 illustrates the MUCTDTA algorithm, which is a conceptual extension of the simulation-based solution algorithm for the single-class problem previously developed by the authors (Mahmassani and Peeta, 1992; Peeta, 1994). The simulation results (from the DYNASMART simulation-assignment model [Mahmassani et al., 1993]) in the current iteration provide the basis for a direction-finding mechanism for the search process embedded in the algorithm, through the experienced vehicular trip times and the associated marginal trip times.

The algorithmic framework for the search process consists of an inner loop that incorporates a direction-finding mechanism for the search process for the SO and UE user classes based on the simulation results of the current iteration. The complexity of the interactions captured by the simulator preclude the kind of well-behaved properties required to guarantee a descent direction in every iteration. The class of equipped users that follow behavioral rules in response to descriptive information on current traffic conditions (user class 3 or BR class) is not directly involved in the search process. The paths of this user class are obtained based on the traffic pattern that evolves in the network for the current assignment strategy (and the behavioral rules assumed), unlike classes 1 and 2, which obtain their paths based on search directions from the

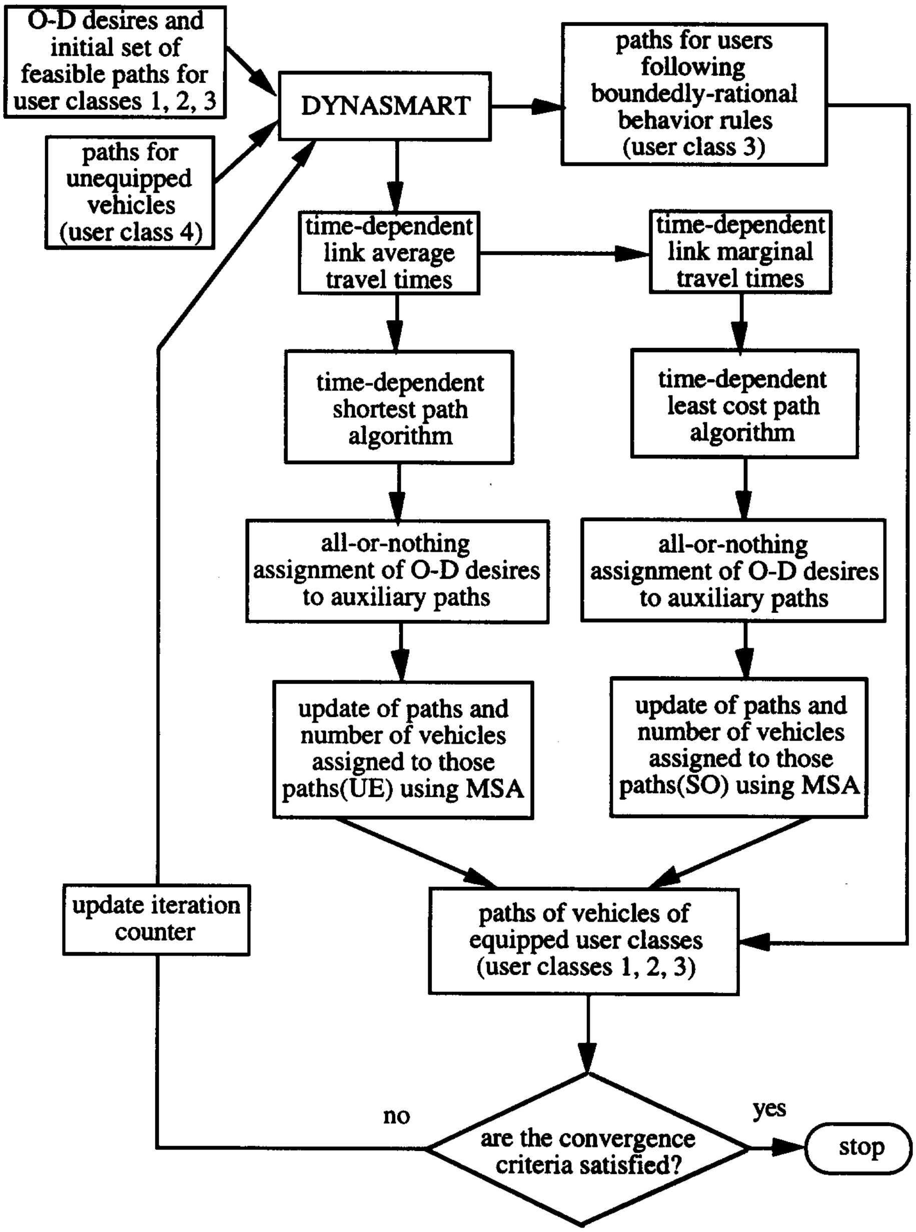

Fig. 4. MUCTDTA solution algorithm.

experience of previous iterations and an update mechanism based on the method of successive averages (Sheffi and Powell, 1982). Hence, from an algorithmic standpoint there is no direct guiding mechanism involved in obtaining the paths for user class 3, other than their being predicated on the assignment strategy for the SO and UE class vehicles. As illustrated in Fig. 4, they form the outer loop of the iterative procedure. The unequipped users (user class 4) are exogenous to the iterative loop of the search process and represent constant background information (for each iteration) as their paths remain unchanged.

Solution procedure for the MUCQRTA problem

If the characteristics of all previously assigned vehicles that are in the network in the current stage σ $s^{\tau u}_{ijk(u)}$, are known (as well as $\mu^{\tau \kappa u}_{i^*ijk(u)}$ if rerouting is performed), and if the O-D forecasts for the current stage, $\phi^{\tau u}_{ij}$, are available, the MUCQRTA problem reduces to the MUCTDTA problem for the current stage, $t = \eta.l + 1, \ldots, \eta.l + h$. This forms the essence of the MUCQRTA solution procedure. As indicated in Fig. 3, the components of the solution procedure other than the MUCTDTA algorithm perform exactly this task of determining $^{\tau u}_{ijk(u)}$ (and $\mu^{\tau\kappa u}_{i*ijk(u)}$ if rerouting is performed) and O-D forecasts $\phi^{\tau u}_{ij}$ for stage σ. However, the task of determining $s^{\tau u}_{ijk(u)}$ (and $\mu^{\tau\kappa u}_{i*ijk(u)}$) is significantly complicated by the intertemporal dependencies.

Figure 5 shows two consecutive stages of the rolling horizon procedure, $\sigma - 1$ and σ. As illustrated in Fig. 3, the solution procedure is as follows:

1. Obtain the O-D trip forecasts for the first stage at some time γ before the start of the planning horizon. The paths of the unequipped user class forecasts are assumed known exogenously.
2. The information from Step 1 forms the input to the first iteration of the MUCTDTA solution algorithm. Execute this algorithm for the current stage $\sigma - 1$. Following its termination, perform an on-line assignment of the actual O-D desires (arriving in real time) for the current roll period to paths obtained at the convergence of the MUCTDTA algorithm.
3. If the end of the planning horizon is reached, stop the procedure. Otherwise, shift the current stage σ by a time duration equal to the roll period to obtain the next stage σ. Hence, σ becomes the (new) current stage.
4. Store the paths, identification numbers, speeds, and positions of all assigned vehicles in the network in stage $\sigma - 1$ that did not reach their destinations by the beginning of stage σ. The rerouting capability is introduced here. As discussed earlier, this capability affects only user classes 1 and 2:
   (i) *No rerouting capability:* change the user class index of previously assigned vehicles belonging to user classes 1 and 2 to 4, meaning that their paths are fixed (from the current stage σ) until they reach their destinations (user class 4 vehicular paths are unchanged in the search process).

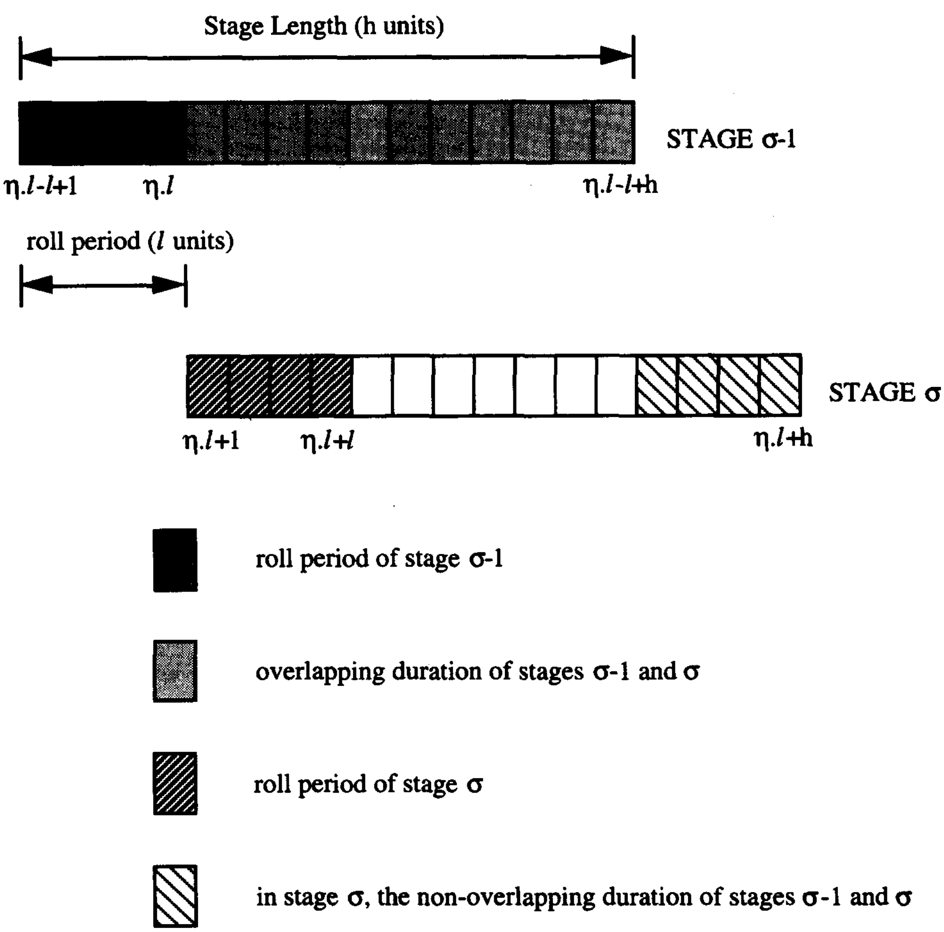

Fig. 5. Successive stages of the solution procedure.

(ii) *Rerouting capability:* retain the original user class indices of those vehicles for whom rerouting is performed. Change the user class indices of all vehicles (of user classes 1 and 2) for whom rerouting is not performed to 4.

5. Update the O-D matrix to reflect the current time-dependent forecasts of O-D trips for the current stage σ. If rerouting is performed, it includes the actual previously assigned O-D trips that are rerouted in the current stage (the updated O-D matrix is available at some time γ before the beginning of stage σ).
6. Obtain the initial feasible path set for the current stage σ based on historical data and the link travel time experience from the previous stage. The path sets for user classes 1 and 2 in stage $\sigma - 1$ serve as the initial path sets for these classes for the current stage σ for the overlapping duration of stages $\sigma - 1$ and σ. The initial paths for user class 3 in the current stage are also assumed to be the time-dependent paths available for the overlapping duration for these vehicles. For these three classes, the initial feasible path set for the nonoverlapping duration is obtained by using the current best path scheme. As before, the path assignments of user class 4 forecasts for the current stage are assumed to be known exogenously. Go to Step 2.

The rolling horizon solution procedure has been implemented on a CRAY Y-MP supercomputer (Mahmassani et al., 1994; Peeta, 1994). Details of the implementation are outside the scope of this paper.

ATIS IMPLEMENTATION ISSUES

To the extent that the actual O-D desires $r_{ij}^{\tau u}$ may differ from the forecasts, $\hat{r}_{ij}^{\tau u}$, used in determining the path assignments for a given roll period, two issues may be encountered in the implementation of the rolling horizon procedure for ATIS applications. First, if the actual O-D desires in a stage are significantly different from the short-term and medium-term forecasts for that stage, the "optimal" solution obtained using the rolling horizon procedure for the roll period may be suboptimal. This emphasizes the need for "good" O-D forecast models, and/or robust assignment procedures vis-à-vis errors in the O-D forecasts.

Second, in the online implementation of the procedure, there are three possible scenarios regarding O-D trips. The ideal case occurs when the actual trips $r_{ij}^{\tau u}$ for the entire roll period are identical to the forecasts $\hat{r}_{ij}^{\tau u}$ used in determining the "optimal" path set. Then, the paths determined in quasi-real time provide the "best" solution for that stage, assuming all other characteristics remain unchanged (for example, no unaccounted for incident occurs in the network in real time for that stage), and are assigned to the O-D desires in real time. The second and third scenarios, respectively, represent the situations in which $r_{ij}^{\tau u}$ is less than or greater than $\hat{r}_{ij}^{\tau u}$ for some intervals in the roll period. These do not affect the online implementation because the "optimal" path sets for user classes 1 and 2 contain paths for every i, j, and τ. If there are "additional" vehicles in real time (that is, $r_{ij}^{\tau u}$ is greater than $\hat{r}_{ij}^{\tau u}$) for a given i, j, and τ, they are assigned paths selected randomly from the optimal path set for that class. If vehicles are "missing" in real time (that is, $r_{ij}^{\tau u}$ is less than $\hat{r}_{ij}^{\tau u}$), this does not affect the online implementation, as all actual O-D desires have paths to their destinations. Since vehicles of user class 3 make decisions in real time, the rolling horizon procedure serves only the purpose of determining "good" initial paths for them. If there are "additional" vehicles in real time for user class 3, their initial paths are determined by some initial path schemes. The three scenarios are not an issue for user class 4, as their paths are determined exogenous to the online implementation.

CONCLUDING COMMENTS

This paper discusses two formulations and a solution procedure for the MUCQRTA problem, which addresses the real-time implementation needs of ATIS. As illustrated, real-time assignment problems are considerably more complicated than the time-dependent assignment problems that assume complete a priori O-D information, due to the intertemporal dependencies. The consideration of multiple user classes complicates an already formidable problem due to interactions among the various user classes. Quasi-real time implementation of solution algorithms is much more intricate than a priori implementation as conservation equations must be satisfied across stages in addition to within a stage (which corresponds to the time-dependent assignment problem).

The rolling horizon solution procedure was implemented on a CRAY Y-MP supercomputer. While the implementation aspects represent the major challenge, sensitivity analyses are necessary to address questions regarding the choice of parameter values (for example, roll period, stage length) and their influence on the effectiveness of the solution procedure. These issues and issues pertaining to the computational efficiency of the solution procedure (in light of its online applicability) are the focus of current research efforts on this topic.

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