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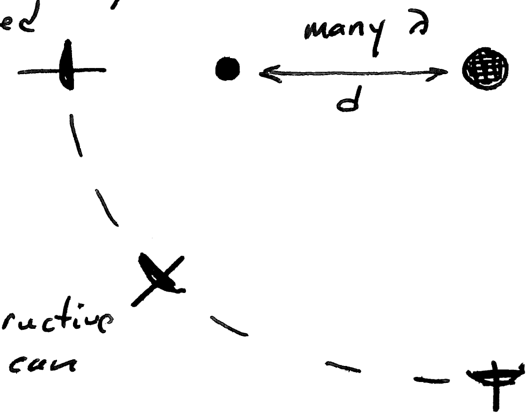
ECE 678 — Homework #2 Solutions — Radar Engineering

2.1 (Levanon 2.1) Assume that $a \gg \lambda \Rightarrow$ Both spheres are in the "optical" region

The spheres are placed many wavelengths apart, and the radar is assumed to be in the far-field.



By changing the direction of observation by the radar with respect to the target, any possible combination of constructive or destructive interference can be obtained.

Let ϕ be the phase difference between the the radar return from the first sphere and the second sphere. Then by changing the observation angle of the radar (or equivalently the orientation of the target w.r.t. the radar), ϕ can take on any angle between 0 and 2π.

Now because radar cross section σ is proportional to the receved power (see radar equation), which is proportional to the magnitude of the electric field squared, and the complex electric fields from the two target returns add, it follows that

$$\sigma(\phi) = \left|\sqrt{\sigma_1} + \sqrt{\sigma_2}\, e^{i\phi}\right|^2 = \left|\sqrt{\pi a^2} + \sqrt{\pi (2a)^2}\, e^{i\phi}\right|^2$$

$$= \left|\sqrt{\pi a^2}\left(1 + 2e^{i\phi}\right)\right|^2 = \pi a^2 \left|1 + 2e^{i\phi}\right|^2$$

When the two returns add "in-phase" ($\phi = 0$), we have

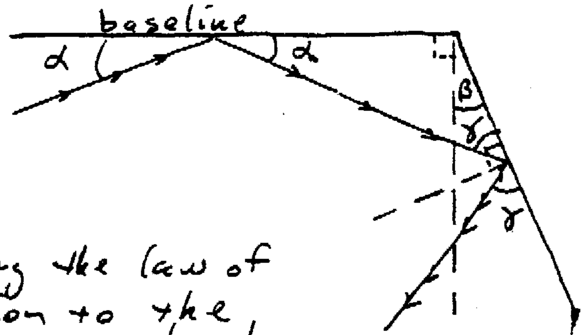
$$\sigma(0^\circ) = \pi a^2 (3)^2 = 9\pi a^2 = \sigma_{MAX}$$

When they are adding "out-of-phase" ($\phi = \pi$)

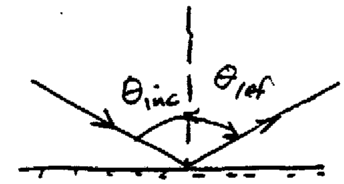
$$\sigma(\pi) = \pi a^2 |1-2|^2 = \pi a^2 = \sigma_{MIN}$$

Thus we have $\dfrac{\sigma_{MAX}}{\sigma_{MIN}} = \dfrac{9\pi a^2}{\pi a^2} = \boxed{9}$

2.2 (Levanon 2.4.) This can be solved geometrically using ray optics as follows



The law of reflection states that in specular reflection, the angle of reflection is equal to the angle of incidence



Applying the law of reflection to the reflector geometry shown above, we get

$$\gamma = \frac{\pi}{2} - (\alpha+\beta) = \pi - (\alpha+\beta+\frac{\pi}{2})$$

But the angle of reflection is

$$\theta_{ref} = \frac{\pi}{2} + \beta - \gamma \quad \text{(w.r.t baseline)}$$

$$= \frac{\pi}{2} + \beta - \left[\frac{\pi}{2} - (\alpha+\beta)\right]$$

$$= \frac{\pi}{2} + \beta - \frac{\pi}{2} + \alpha + \beta = \alpha + 2\beta$$

But the angle of incidence is

$$\theta_{inc} = \alpha \Rightarrow \theta_{ref} - \theta_{inc} = \alpha + 2\beta - \alpha = \boxed{2\beta}$$

2.3 (Levanon 2.5) What is the angle of symmetry for a trihedral corner reflector

The angle of symmetry occurs along the axis about which you could spin the corner reflector and still have the same maximum return. (This axis passing through the corner itself).

This is the axis along which the unit vector

$$\hat{r} \quad \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{3}} \quad \text{lies}$$

We want the azimuth angle θ and elevation angle ϕ. From spherical to rectangular conversion, we have

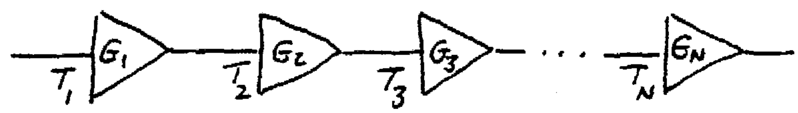
$$x = \sin\phi\cos\theta \Rightarrow 1/\sqrt{3} = \sin\phi\cos\theta$$

$$y = \sin\phi\sin\theta \Rightarrow 1/\sqrt{3} = \sin\phi\sin\theta$$

$$z = \cos\phi \Rightarrow \frac{1}{\sqrt{3}} = \cos\phi$$

$\therefore \quad \phi = 54.74^\circ$ and $\theta = 45^\circ$

2.4. The amplifier cascade appears as follows:



Noise adds as it goes through amplifier chain (input noise added to internal noise). Hence the noise power at the output is

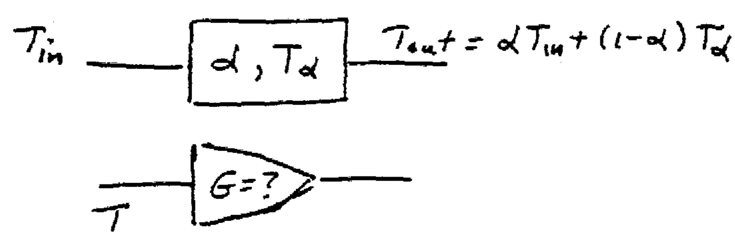
$$P_n = kT_N G_N B + (kT_{N-1} G_{N-1} B)\cdot G_N + (kT_{N-2} G_{N-2} B)\cdot G_{N-1} G_N + \cdots + (kT_1 G_1 B)\cdot G_2 G_3 \cdots G_N$$

Refering this noise level as a noise temperature at the first stage of the chain, we have

$$T_{eq} = \frac{P_n}{kBG_1\cdots G_N} = \boxed{T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots + \frac{T_N}{G_1 G_2 \cdots G_{N-1}}}$$

From this expression for the equivalent noise temperature, it is obvious why an emphasis is put on a high-gain, low noise front-end (first stage). The noise temp of the first stage enters into T_{eq} directly, whereas the noise temp of successive stages is multiplied by a factor of $1/G_1$ or smaller. A large G_1 makes the value of $T_{eq} \simeq T_1$. It is thon obviouse why in addition to high-gain, we would want a low noise (low T_1) front end.

2.5



Clearly the power passing through the attenuator is reduced by a factor of α, that is $P_0 = \alpha P_{in}$, $0<\alpha<1$.

The one-sided PSD of the noise at the output is

$P_n(f) = k\left[\alpha T_{in} + (1-\alpha)T_\alpha\right]$. Refering this to the input, we have $\dfrac{k\left[(1-\alpha)T_\alpha\right]}{\alpha}$ when we look at a source of noise temp 0 K.

Thus the ~~receiver~~ amplifier equivalent noise temp is $\dfrac{1-\alpha}{\alpha}T_\alpha = \left(\dfrac{1}{\alpha}-1\right)T_\alpha$

Thus we have that for this "amplifier"

$$\boxed{\begin{aligned} G &= \alpha \\ T &= (1/\alpha - 1)T_\alpha \end{aligned}}$$

2.6 Both receiver #1 and receiver #2 have identical input waveguide sections, which are attenuators at ambient temperature 290°K. The total attenuation is

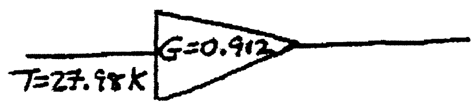
$$\alpha_{dB} = (0.01\ dB/ft)(40\ ft) = 0.4\ dB$$

$$\alpha = 10^{-0.4/10} = 0.9120$$

The equivalent noise temperature of the waveguide referred to its input is

$$T = \left(\frac{1}{\alpha} - 1\right) T_\alpha = \left[\frac{1}{0.9120} - 1\right] 290°K = 27.98°K$$

Thus the waveguide can be viewed as an "amplifier" with gain $G = 0.9120$ and $T = 27.98°K$

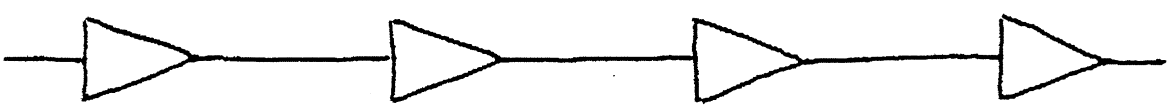


Expressing the receiver chain in terms of absolute gains and noise temperatures, we have

Rcvr #1:

T = 27.98 K	T = 8 K	T = 1163 K	T = 5365 K
G = 0.9120	G = 10	G = 1000	G = 1000

Rcvr #2:



T = 27.98 K	T = 106 K	T = 1000 K	T = 5365 K
G = 0.9120	G = 100	G = 100	G = 1000

Calculating the equivalent noise temperatures of the two receivers, we have

$$T_{E,1} = 27.98°K + \frac{8°K}{0.9120} + \frac{1163°K}{9120} + \frac{5365°K}{9120} = \boxed{164.86°K}$$

$$T_{E,2} = 27.98°K + \frac{106°K}{0.9120} + \frac{1000°K}{91.2} + \frac{5365°K}{9120} = \boxed{155.76°K}$$

∴ Receiver #2 is better.

$P_R/P_N = 16.5\ dB = 44.668 \Rightarrow P_R = 44.668\ P_N$ $(P_N = 4.5\times10^{-14} W)$ $= 2.010\times10^{-12} W$

$$T_N = \frac{P_N}{KB} = \frac{4.5\times10^{-14} W}{(1.38\times10^{-23} J/K)(40\ MHz)} = 81.52°K$$

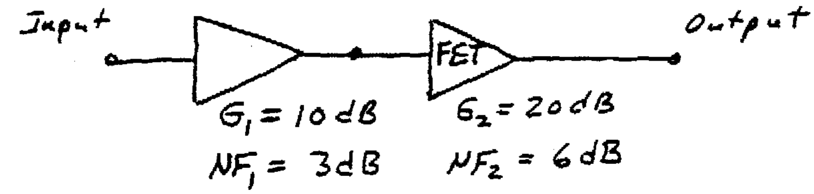
$$SNR_1 = \frac{P_R}{K(T_{E,1}+T_N)B} = 15.44 = \boxed{11.89\ dB}$$

$$SNR_2 = \frac{P_R}{K(T_{E,2}+T_N)B} = 15.35 = \boxed{11.86\ dB}$$

So the signal-to-noise ratios are very close in both cases.

A diagram of this receiver appears as follows:

2.7



In order to find the overall noise figure, we will refer everything to the front-end.

$$G_1 = 10\ ,\ NF_1 = 1.9953 \Rightarrow T_1 = 290°K\,(0.9953) = 289°K$$

$$G_2 = 100\ ,\ NF_2 = 3.9811 \Rightarrow T_2 = 290°K\,(2.9811) = 865°K$$

Thus the equivalent receiver noise temperature is

(a) $T_{eq} = T_1 + \dfrac{T_2}{G_1} = 289°K + \dfrac{865°K}{10} = \boxed{375.5°K}$

(b) The overall noise figure is $NF = \dfrac{T_{eq}}{290°K} + 1 = \dfrac{375.5}{290} + 1 = 2.2948$
In dB this is $\boxed{NF_{Eq} = 3.61\ dB}$

(c) This does not depend on the gain of the FET in any way, since $T_{EQ} = T_1 + \dfrac{T_2}{G_1}$, which does not involve G_2.

2.8 The received power is given by

$P_R = \dfrac{A_T A_R}{\lambda^2 R^2} P_T$, Here $A_T = A_R = \pi\ m^2$, $\lambda = \dfrac{c}{f} = \dfrac{3\times10^8\ m/s}{3\times10^9\ 1/s}$

Thus we have $= 0.1m$

$$P_R = \frac{\pi^2 m^4 \quad P_T}{(0.01\,m^2)(42{,}180{,}000m)^2} = 5.5474\times10^{-13} P_T$$

The noise power spectral density is given by kT where T is the sum of T_{EQ} from the above problem (2.7) and the 300°K background provided by the earth

$$\therefore\ T = 375.5°K + 300°K = 675.5°K$$

The noise power over the ~~1 MHz~~ signal bandwidth is

$$P_n = kTB = (1.380\times10^{-23}\,J/°K)(675.5°K)(10^6 Hz) = 9.3219\times10^{-15} W$$

$SNR = \dfrac{P_R}{P_N} = \dfrac{5.5474\times10^{-13}\ P_T}{9.3219\times10^{-15} W} = 59.509\ P_T$. For $SNR = 3dB = 2$, this implies

$$P_T = \frac{SNR}{59.509\ 1/W} = \frac{2}{59.509} W = 0.03361\ W = \boxed{33.6\ mW}$$

2.9 First, lets consider the RCS of a single bee. For wavelengths λ much larger than the "radius" of a bee, we can assume a bee is a a sphere scattering in the Rayleigh region. We can use as an estimate of the RCS of a single bee, σ_B, the number given by Levanon on page 6.

$$\sigma_B = 10^{-5}\,m^2$$

Under the assumptions given, it is fair to assume that we have 5000 constant scatterers with cross-section $10^{-5}m^2$ with randomly distributed phases on $[0, 2\pi]$ (constant amplitude mean and zero-var. with random phase). [constant amplitude = $\sqrt{\sigma_B}$ normalized].

Thus we have that the total swarm will have an RCS σ_S that is an exponentially distributed R.V. with mean $2 \cdot \frac{n \cdot \sigma_S}{2} = 5000 \times 10^{-5}\,m^2 = 0.05\,m^2$

Thus σ_S has pdf

$$f_{\sigma_S}(\sigma) = \frac{1}{0.05\,m^2} \exp\left\{\frac{-\sigma}{0.05\,m^2}\right\} \mathbb{1}_{[0,\infty)}(\sigma)$$
$$= 20 \exp\{-20\,\sigma\} \mathbb{1}_{[0,\infty)}(\sigma)$$