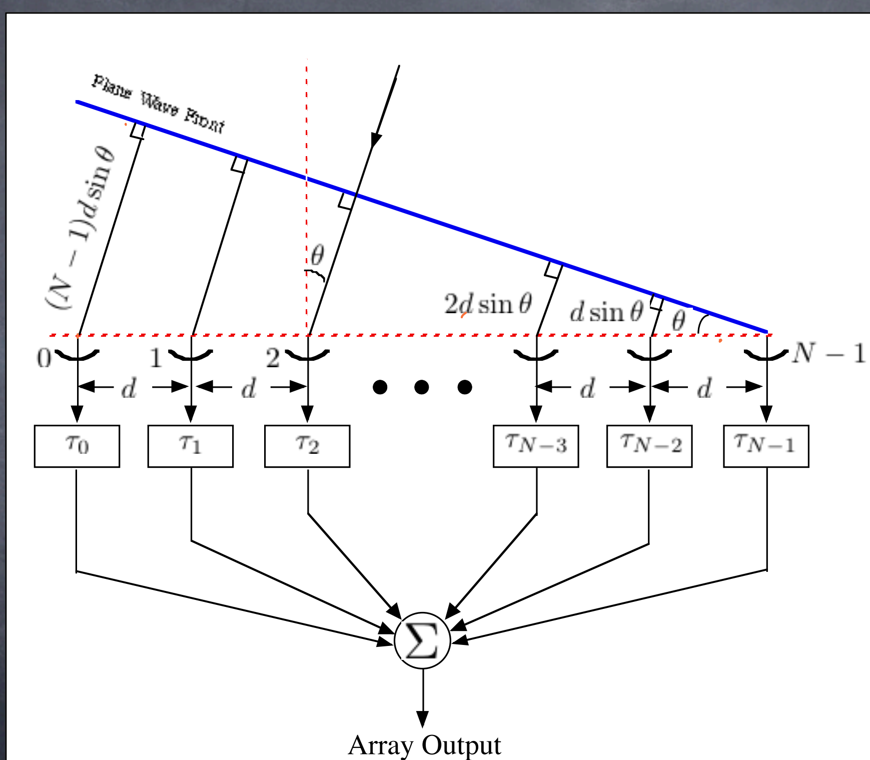
Session 30

Recall...

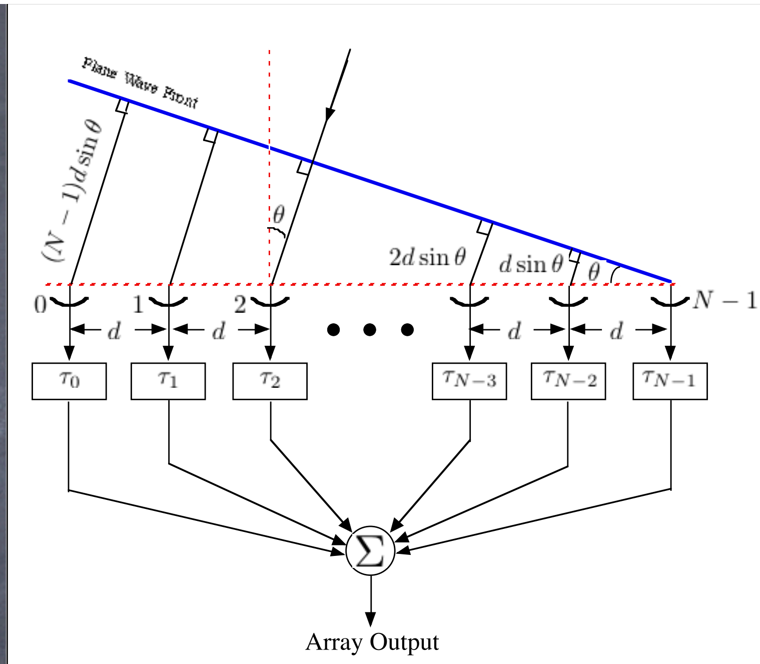
One-Dimensional Uniform Array

30.1



Recall...

30.2



Setting the delays to

$$\tau_m = \frac{[(N-1)-m]d\sin\theta}{c}, \quad m = 0, \ldots, N-1$$

causes the received signals to add up "in-sync."

30.3

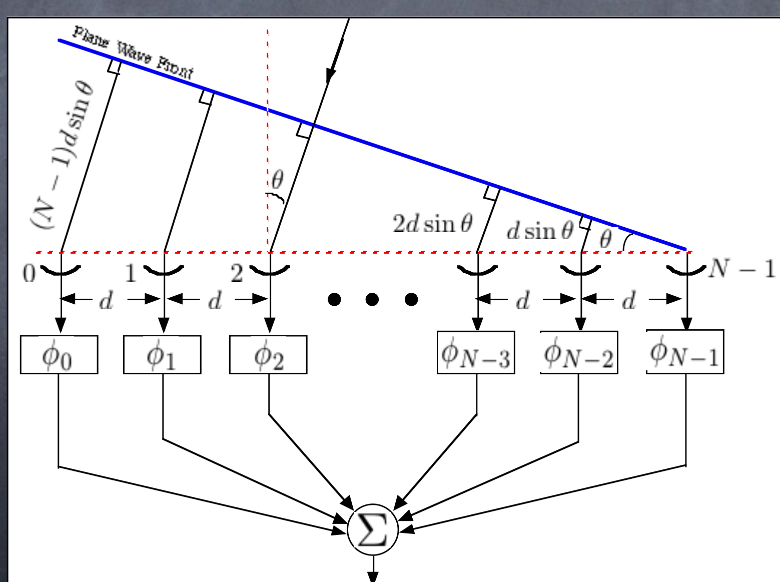
By the time shift theorem of Fourier transforms, we have

$$s(t) \overset{\mathcal{F}}{\leftrightarrow} S(f) \quad \Rightarrow \quad s(t-\tau) \overset{\mathcal{F}}{\leftrightarrow} S(f)\, e^{-i2\pi f\tau}.$$

So for a sinusoidal or narrowband signal at frequency f_0, we can replace the delay τ_m by phase shift

Recall...

$$\phi_m = 2\pi f_0 \tau_m.$$



30.4

Assuming narrowband waves and phase shifters with

$$\phi_0 = \phi_1 = \phi_2 = \cdots = \phi_{N-1} = 0$$

and N identical elements with effective area $A_e(\theta)$ (gain $G_e(\theta)$) for a wave from direction θ, it can be shown the effective area of the array is

$$A(\theta) = A_e(\theta) \cdot \frac{1}{N} \left| \sum_{n=0}^{N-1} e^{i2\pi n(d/\lambda)\sin\theta} \right|^2$$

or equivalently

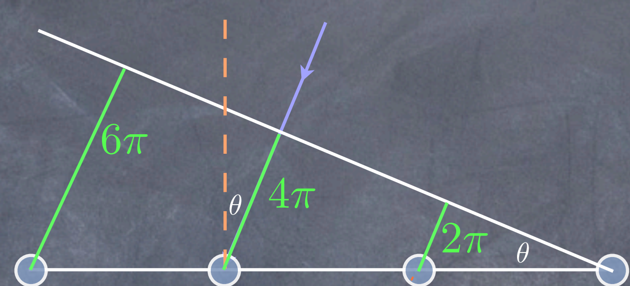
$$G(\theta) = G_e(\theta) \cdot \frac{1}{N} \left| \frac{\sin\left[N\pi(d/\lambda)\sin\theta\right]}{\sin\left[\pi(d/\lambda)\sin\theta\right]} \right|^2$$

$$\text{Array Length} = (N-1)d$$

Larger d implies higer resolution, but there is a price to pay.

30.5

If $d > \lambda/2$, we get grating lobes due to constructive interference at Bragg angles:



30.6

In order to reduce grating lobes, you must have $d \lesssim \lambda/2$.

You can also

1. Use nonuniform spacing of elements;
2. Use an $A_e(\theta)$ that reduces the most problematic grating lobes. (elements may be large)

Radio astronomy arrays often have severe grating lobes.

In radar they can be more problematic. Usually take $d \approx \frac{\lambda}{2}$

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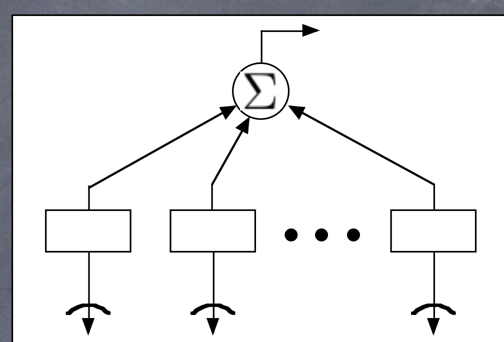
Usually take $d \approx \lambda/2$.

30.7

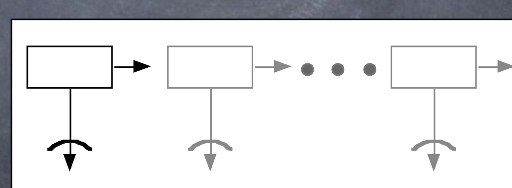
Synthetic Arrays

A real array:



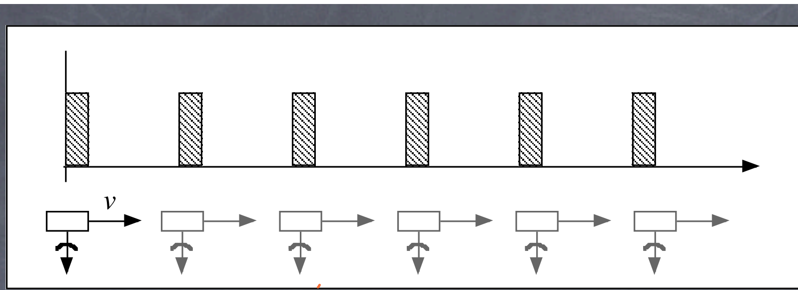
A synthetic array:

Another approach is to use a single element and move it between observations



Signal processing is used to synthesize an "equivalent" array.

30.8



The received signal is recorded with phase information.

We collect data at each position.

We apply proper phase shifts to the received data.

We sum to synthesize an array antenna.

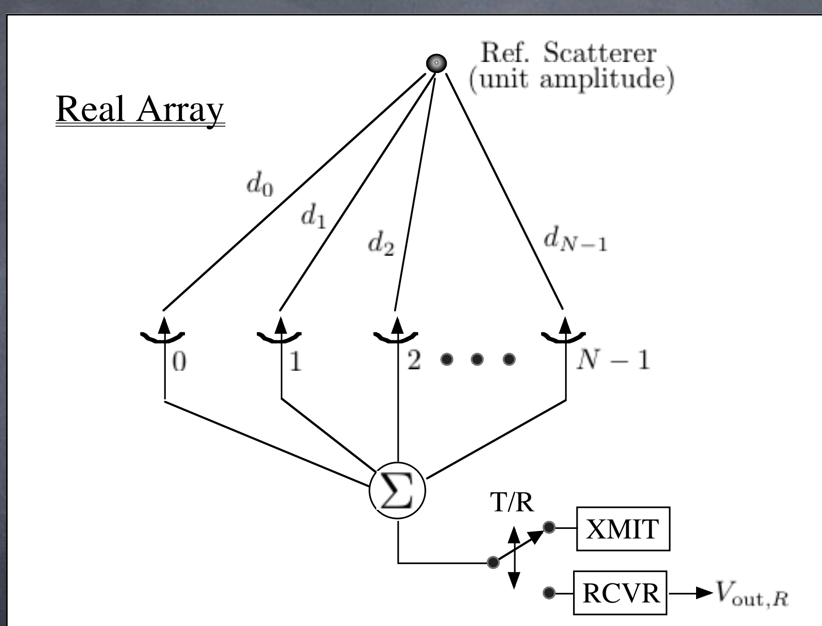
This arrray—being sequentially generated—is a little different than a real array.

It does have high angular resolution like a real array.

This is the approach used in Synthetic Aperture Radar (SAR).

30.9

Comparison of Real and Synthetic Arrays



30.10

Real Array

The complex field at the n-th element is

$$v_n = \sum_{m=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}(d_m + d_n)\right\}$$

$$= \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\sum_{m=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_m\right\},$$

$$m = 0, 1, \ldots, N-1.$$

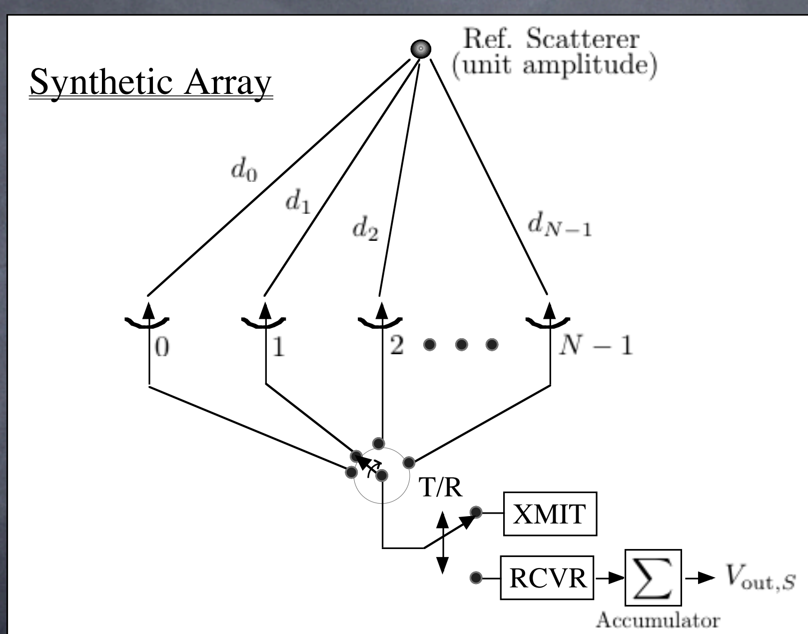
The output of the entire array is the sum

$$V_{\text{out},R} = \sum_{n=0}^{N-1} v_n = \sum_{n=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\sum_{m=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_m\right\}$$

$$= \left[\sum_{n=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

30.11

Synthetic Array



The synthetic array is sequentially built, one element at a time.

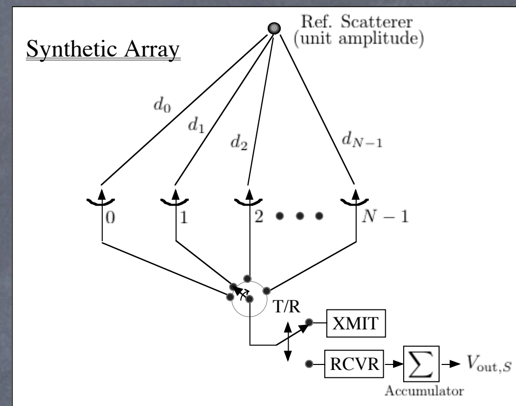
Synthetic Array Response

30.12

The response of the n-th array element is

$$v_n = \exp\left\{-i\frac{2\pi}{\lambda}(d_n + d_n)\right\}$$

$$= \exp\left\{-i\frac{4\pi}{\lambda}d_n\right\}$$



The synthetic array output response is

$$V_{\mathrm{out},S} = \sum_{n=0}^{N-1} v_n = \sum_{n=0}^{N-1} \exp\left\{-i\frac{4\pi}{\lambda}d_n\right\} = \sum_{n=0}^{N-1}\left[\exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

30.1

Real Aperture:

$$V_{\mathrm{out},R} = \left[\sum_{n=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

(Square of Sum)

cross terms!

Synthetic Aperture:

$$V_{\mathrm{out},S} = \sum_{n=0}^{N-1}\left[\exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

(Sum of Squares)

no cross terms!