Session 25

25.1

Coded Radar Signals

A coded waveform $s(t)$ is a signal of the form

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t-nT) \exp\{j2\pi d_n t/T\} \exp\{j\phi_n\},$$

Unit Energy

where

$$p(t) = 1_{[0,T]}(t),$$

$\{d_n\}_{n=0}^{N-1}$ = a sequence of integer frequency modulating indices,

$\{\phi_n\}_{n=0}^{N-1}$ = a sequence of real valued phases,

T = duration of a single waveform "chip,"

NT = total duration of the coded waveform.

We will initially take

$$\phi_0 = \phi_1 = \phi_2 = \cdots = \phi_{N-1} = 0,$$

which will give us a *frequency-coded signal*

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t-nT) \exp\left\{j2\pi\left(\frac{d_n}{T}\right)t\right\}.$$

25.2

Frequency-Coded Waveforms

A frequency coded waveform $s(t)$ is a signal of the form

$$s(t) = \sum_{l=0}^{N-1} p(t - lT) e^{-j2\pi\Omega_l t},$$

where

$$T = \textit{chip duration},$$

$$p(t) = 1_{[0,T]}(t) \qquad \text{(chip waveform)},$$

and

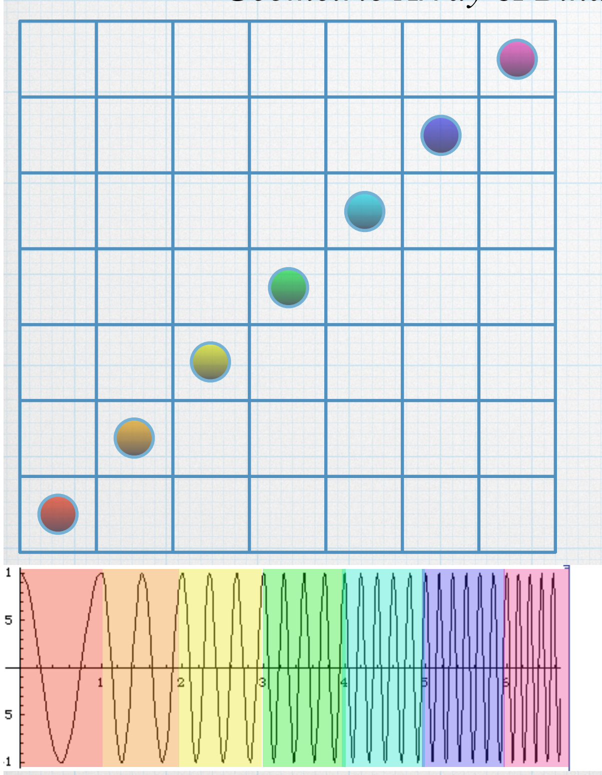
$$\Omega_l = d_l / T, \quad l = 1, 2, \ldots N,$$

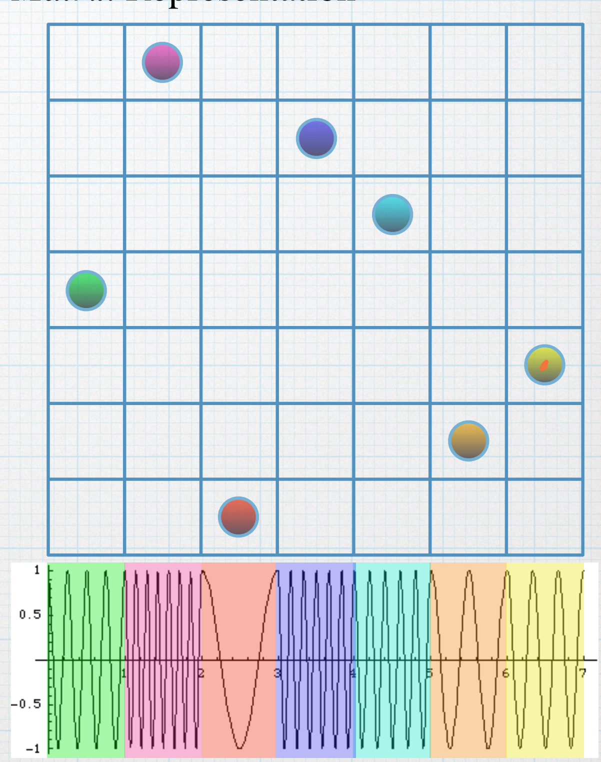
where $\{d_l\}$ is a permutation of the integers $1, 2, \ldots N$.

25.3

Frequency-Coded Waveforms

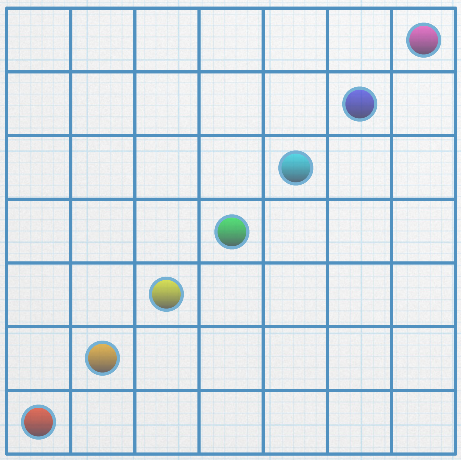
Geometric Array or *Binary Matrix* Representation



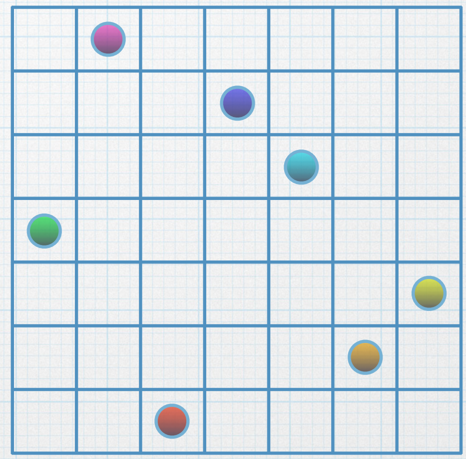


25.4

Frequency Coding Matrices for Frequency-Coded Signals



Stepped Frequency Approximation to a Chirp (LFM)



Costas Sequence

25.5

The Ambiguity Function of Frequency-Coded Waveforms

The ambiguity function of $s(t) = \sum_{l=0}^{N-1} p(t - lT)e^{-j2\pi\Omega_l t}$ is

$$\chi_s(\tau,\nu) = \chi_s^{(1)}(\tau,\nu) + \chi_s^{(2)}(\tau,\nu),$$

where

$$\chi_s(\tau,\nu) \triangleq \int_{-\infty} s(t)\, s^*(t-\tau)\, e^{+i2\pi\nu t}\, dt$$

$$\chi_s^{(1)}(\tau,\nu) = \sum_{m=0}^{N-1} e^{-j2\pi m\nu T} e^{-j2\pi\Omega_m\tau}\chi_p(\tau,\nu),$$

and

$$\chi_s^{(2)}(\tau,\nu) = \sum_{m=0}^{N-1}\ \sum_{n=0,n\neq m}^{N-1} e^{-j\pi(\Omega_m+\Omega_n)\tau} e^{-j\pi(m+n)T} \cdot\chi_p(\tau + (m-n)T, \nu + (\Omega_n - \Omega_m))$$

n.b. $\beta_s(\tau,\nu) = \chi_s(\tau,-\nu)$.

The Ambiguity Function of Frequency-Coded Waveforms

The ambiguity function of $s(t) = \sum_{l=0}^{N-1} p(t-lT)e^{-j2\pi\Omega_l t}$ is

$$\chi_s(\tau,\nu) = \chi_s^{(1)}(\tau,\nu) + \chi_s^{(2)}(\tau,\nu),$$

where $\chi_s(\tau,\nu) \triangleq \int^{\infty} s(t)\, s^*(t-\tau)\, e^{+i2\pi\nu t}\, dt$

$$\chi_s^{(1)}(\tau,\nu) = \sum_{m=0}^{N-1} e^{-j2\pi m\nu T} e^{-j2\pi\Omega_m\tau} \chi_p(\tau,\nu),$$

and

$$\chi_s^{(2)}(\tau,\nu) = \sum_{m=0}^{N-1} \sum_{n=0,n\neq m}^{N-1} e^{-j\pi(\Omega_m+\Omega_n)\tau} e^{-j\pi(m+n)T} \cdot \chi_p(\tau+(m-n)T, \nu+(\Omega_n-\Omega_m))$$

n.b. $\beta_s(\tau,\nu) = \chi_s(\tau,-\nu)$.

The sidelobes are given by

$$\chi_s^{(2)}(\tau,\nu) = \sum_{m=0}^{N-1} \sum_{n=0,n\neq m}^{N-1} e^{-j\pi(\Omega_m+\Omega_n)\tau} e^{-j\pi(m+n)T} \cdot \chi_p(\tau+(m-n)T, \nu+(\Omega_n-\Omega_m))$$

$$\chi_p(\tau+(m-n)T, \nu+(d_n-d_m)/T)$$

Large contribution when *these* equal zero!

$\tau = (n-m)T$ and $\nu = (d_n-d_m)/T)$

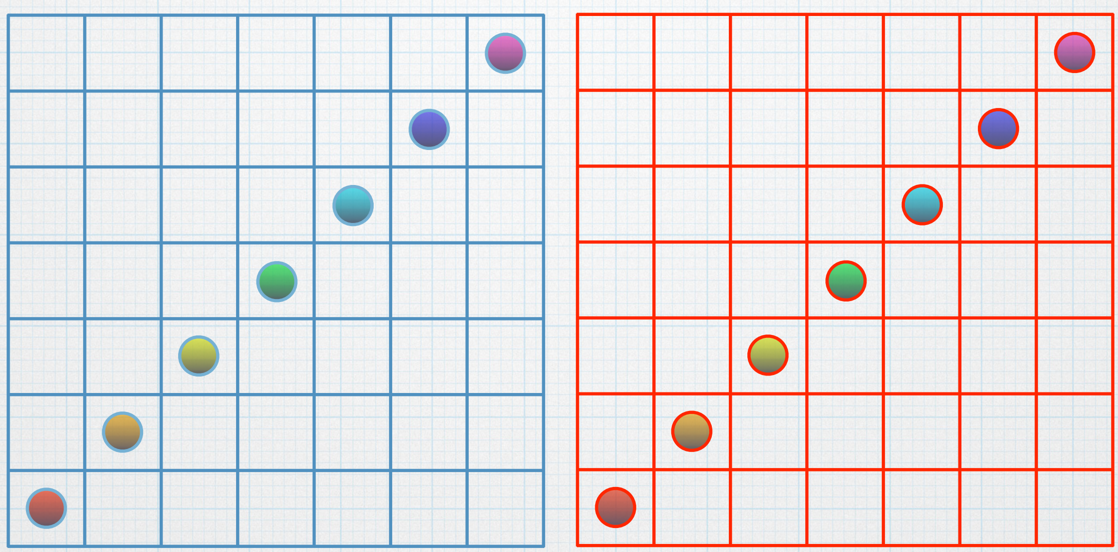
or taking $T=1$ for simplicity...

$\tau = n-m$ and $\nu = d_n - d_m$

Coincident Sidelobe Approximation

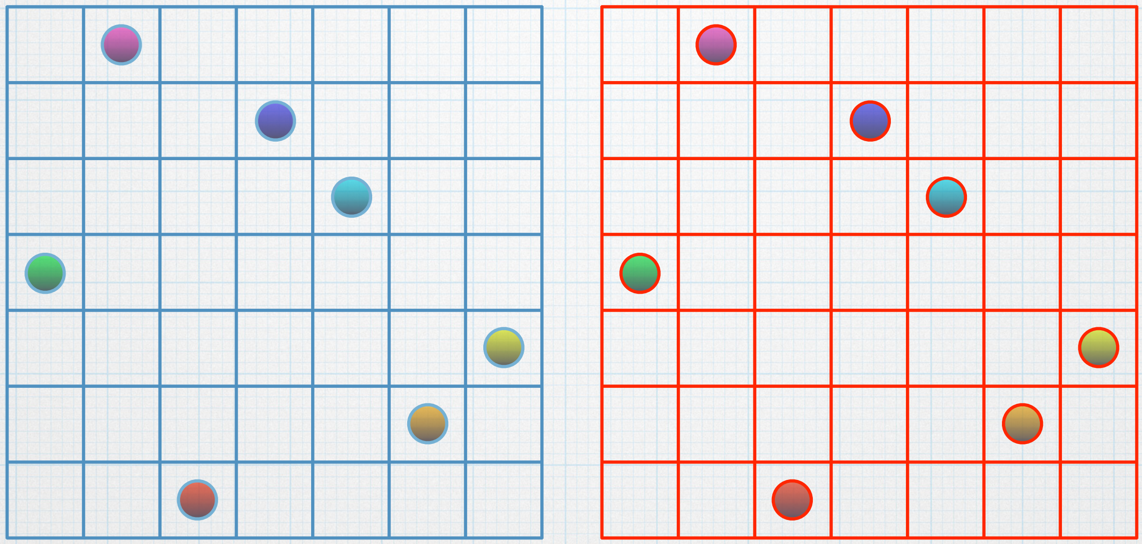
- If we consider only the sidelobe contributions due to the situations where both arguments of the ambiguity function is zero, we want to minimize the number of situations where this occurs.
- We especially want to minimize multiple "hits" for any given delay and Doppler shift.
- While this approach only minimizes an approximation of the ambiguity function sidelobes, it is surprisingly effective.
- It is, in fact, the approach John Costas used in designing Costas sequences.

LFM Chirp Sidelobe Overlay Demo



25.10

Costas Sidelobe Overlay Demo



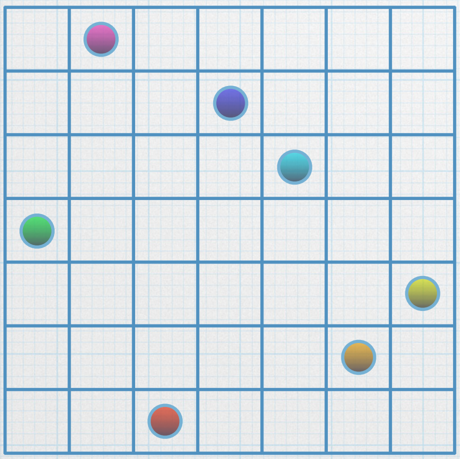
25.11

Characteristics of Stepped-Frequency Waveforms

- A wide variety of waveforms with different ambiguity functions can be generated.
- These waveforms can be easily generated and amplified for transmission.
- The ambiguity characteristics of these waveforms can be easily visualized because of their localization in time and frequency.
- Provides a straightforward approach to characterizing "ambiguity state" of a target environment.
- These characteristics make them ideal for adaptive waveform radar.

The Sidelobe Array

- If we count up the number of sidelobe coincidences for each combination of integer delay-Doppler shifts, we can tabulate the coincidences in an array called the *sidelobe array*.



Doppler ↑

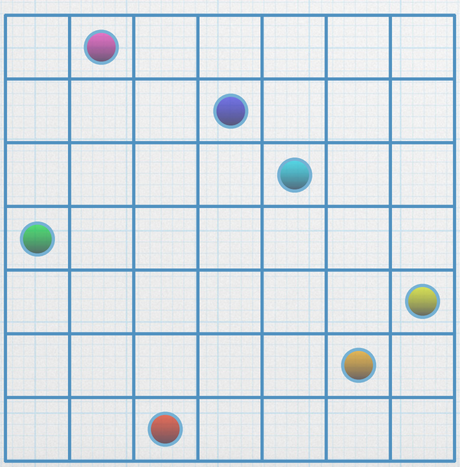
					1							
		1					1					
	1			1				1				
			1	1	1		1					
	1		1	1					1	1		
1				1	1		1		1	1		
						7						
		1	1		1		1	1				1
		1	1					1	1		1	
					1		1	1	1			
				1				1			1	
					1					1		
							1					

Delay →

Costas Sequences

Definition: A **Costas sequence** of length N is a integer frequency firing sequence $\{d_1, \ldots, d_N\}$ (or $\{d_0, \ldots, d_{N-1}\}$ that is a permutation of the integers $1, \ldots, N$ (or $0, \ldots, N-1$) such that the maximum sidelobe height orcoincidence number in the sidelobe array is 1 for any nonzero integer delay-Doppler shift.

An Example ...



					1							
		1					1					
	1			1				1				
			1	1	1		1					
	1		1	1					1	1		
1				1	1		1		1	1		
						7						
		1	1		1		1	1				1
		1	1					1	1		1	
					1		1	1	1			
				1				1			1	
					1					1		
							1					

Reference: N. Levanon and E. Mozeson, ***Radar Signals***, Wiley, 2004 (ISBN 0-471-47378-2)

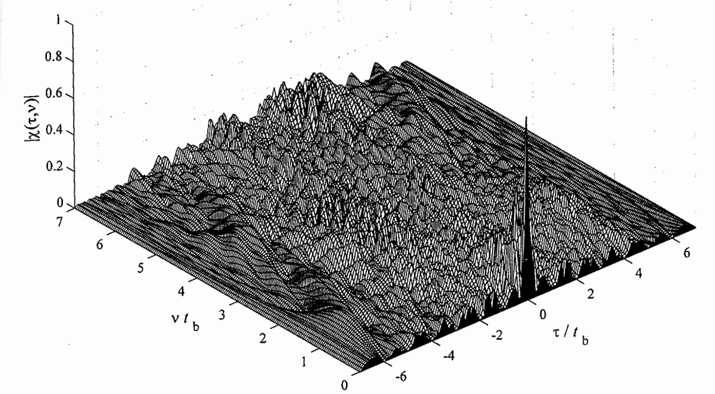
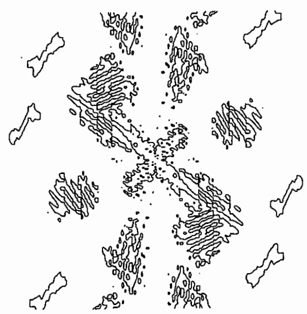


FIGURE 5.4 Partial ambiguity function of a Costas signal with code sequence {4 7 1 6 5 2 3}.



Sidelobe Matrix

Doppler \ Delay	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
6						1							
5			1					1					
4		1			1				1				
3				1	1	1		1					
2		1		1	1					1	1		
1	1				1	1		1		1	1		
0							7						
-1			1	1		1		1	1				1
-2			1	1					1	1		1	
-3						1		1	1	1			
-4					1				1			1	
-5						1					1		
-6								1					

FIGURE 5.5 Ambiguity function contour at 0.125 (left) compared with the sidelobe matrix (right).

- <u>A Length 40 Costas Sequence:</u>

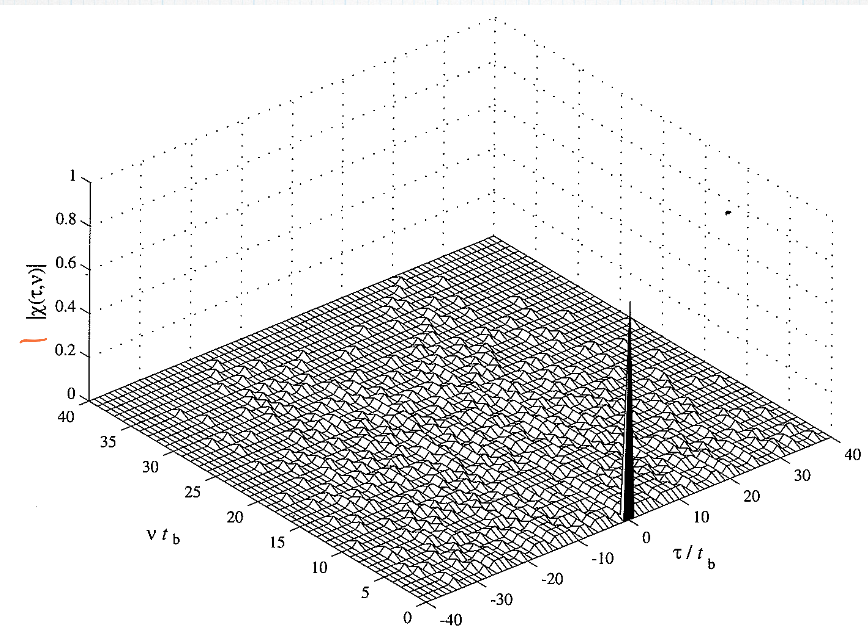


FIGURE 5.9 Ambiguity function of a Costas signal (length $M = 40$) at all relevant grid points.

Reference: N. Levanon and E. Mozeson, ***Radar Signals***, Wiley, 2004 (ISBN 0-471-47378-2)

- A Length 40 Costas Sequence:

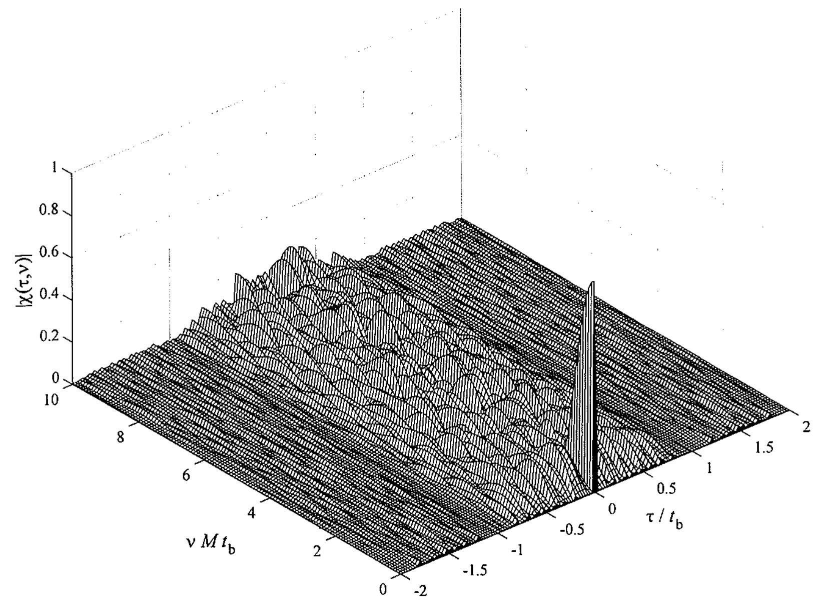


FIGURE 5.10 Ambiguity function of a Costas signal (length $M = 40$) zoom near the origin.

Reference: N. Levanon and E. Mozeson, ***Radar Signals***, Wiley, 2004 (ISBN 0-471-47378-2)

- A Length 40 Costas Sequence:

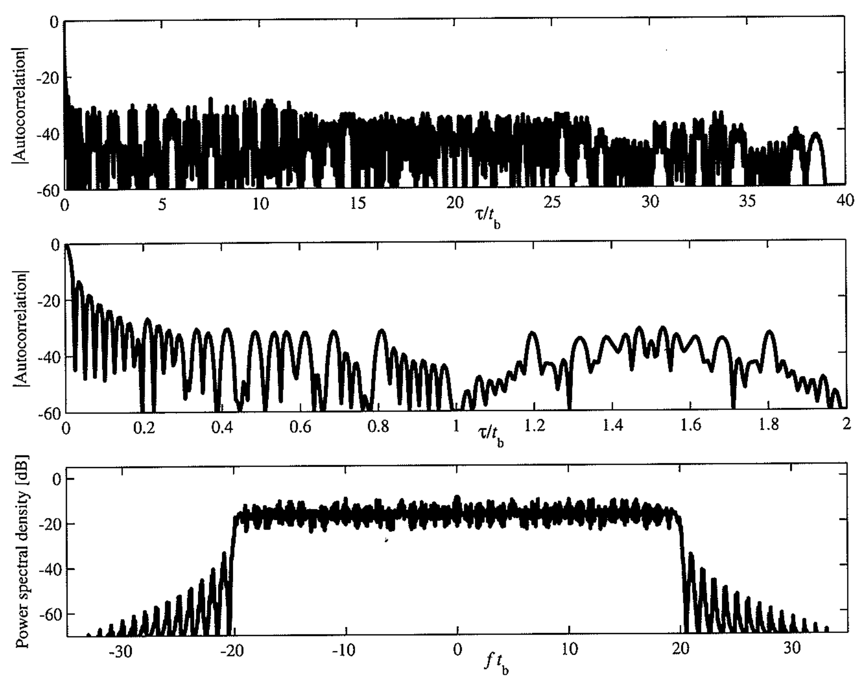


FIGURE 5.11 ACF (top and middle) and the spectrum (bottom) of a Costas signal (length 40).

Reference: N. Levanon and E. Mozeson, ***Radar Signals***, Wiley, 2004 (ISBN 0-471-47378-2)