Session 9

Recall...

RCS of Corner Reflectors

- A corner reflector is a radar target constructed by letting a number of planar surfaces come together, forming a corner.
- Corner reflectors tend to have the property that they reflect strongly back in the direction of the incident wave.
- For this reason, they are also called retro-reflectors.
- One common corner reflector is the "corner cube," used to construct optical bicycle reflectors.

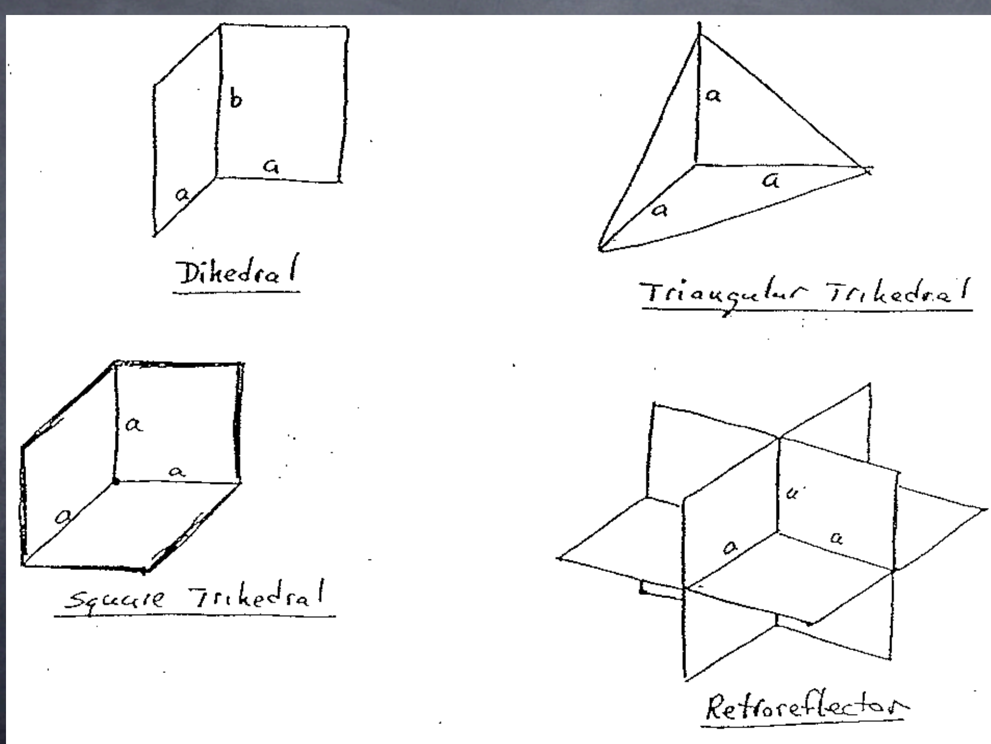
Recall ...

Another Recreational Use...

A typical sailboat corner reflector
(hung from mast)

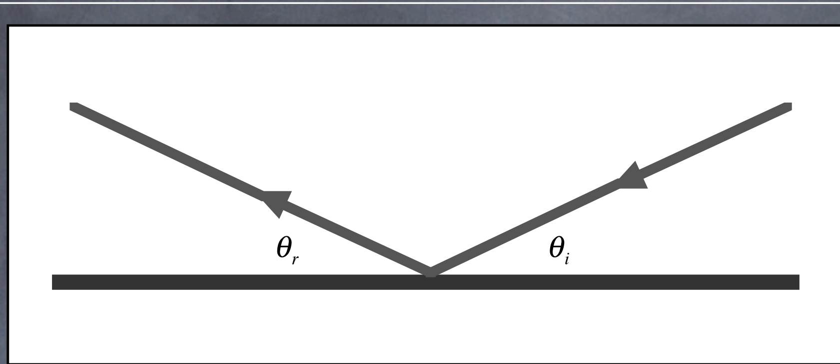
A typical situation where they are useful!

Common Corner Reflector Geometries



9.4

Fresnel's Law of Reflection



"The angle of incidence equals the angle of reflection."

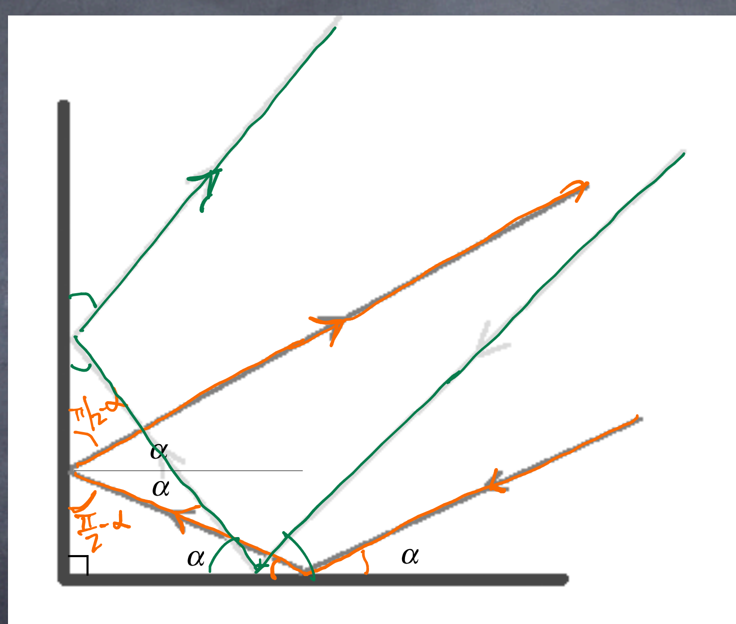
$$\theta_i = \text{Angle of Incidence}$$

$$\theta_r = \text{Angle of Reflection}$$

n.b., We assume surface is large compared to a wavelength (Ray Optics Approximation)

9.5

Consider a Dihedral Corner Reflector



From the Law of Reflection, simple geometry indicates that a ray is reflected back in the direction from which it came.

Consider a Trihedral Corner Reflector

Thought Experiment: Can you convince yourself that if the walls of this room were mirrors, that a laser bean shining into a corner would be reflected back in the direction from which it came.

Geometry Exercise: Can you show this mathematically in 3-D space using vectors to represent the rays.

See Ruck et al., Chapter 8, for more on corner reflectors

Thermal Noise in Microwave Receivers

- Thermal motion of charges in any conducting or lossy body produces fluctuating currents and voltages.
- Nyquist (1927) by considering the average energy in a resonator in thermal equlibrium with its environment.
- For a derivation, see any good book on Radio Astronomy (e.g., Krauss, Radio Astronomy, 1986) or for a brief outline, see Minkoff, Signals, Noise and Active Sensors, 1992.

Using Nyquist's approach, it can be shown that if an antenna is pointing at a black body at absolute temperature T, the power in a band of width Δf centered about frequency f, the power out of the antenna is

$$\Delta P_n = \frac{hf}{e^{hf/kT} - 1}\Delta f$$

Equivalently, the one-sided power spectral density (PSD) of the noise is

$$P_n(f) = \frac{hf}{e^{hf/kT} - 1}$$

$k =$ Boltzmann's constant $= 1.38 \times 10^{-23}$(J/K)
$h =$ Planck's constant $= 6.62 \times 10^{-34}$ (J-sec)

$$\boxed{\Delta P_n = \frac{hf}{e^{hf/kT} - 1}\Delta f}$$

- This is independent of antenna gain, as long as the antenna sees a source of constant temperature.
- This would occur if the antenna was in a box with walls that were a black body at constant temperature T.
- This is well approximated if an object of temperature T fills the main beam.
- Not all sources encountered satisfy this (e.g. Radio Astronomy)

9.10

$$P_n(f) = \frac{hf}{e^{hf/kT} - 1}$$

When $hf/kT \ll 1$

$$P_n(f) = \frac{hf}{1 + \frac{hf}{kT} + o(\frac{hf}{kT}) - 1}$$

$$\approx \frac{hf}{hf/kT}$$

$$= kT$$

9.11

At microwave frequencies $hf \ll kT \Rightarrow hf/kT \ll 1$, so

$$P_n(f) = kT$$

Not a function of frequency

White Noise Approximation

- This is where the white noise assumption in microwave communications comes from.
- This is only a low frequency

Clearly not true when $hf \approx kT$

(Average energy per mode approaches energy per photon)

9.12

At microwave frequencies...

$hf/kT \ll 1$ for $T = 300°$K

$$hf = \text{energy per photon at frequency } f$$

$$\begin{aligned} kT &= \text{energy per second per hertz} \\ &= \text{energy (averge energy per mode)} \end{aligned}$$

$$kT/hf = \text{average number of photons per mode}$$

$$kT/hf \gg 1 \;\Rightarrow\; \text{Central Limit Theorem holds}$$

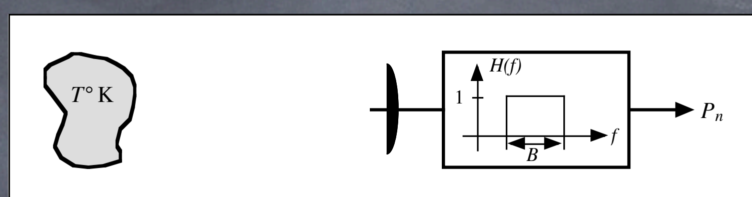
$$\Rightarrow\; \text{Gaussian noise}$$

So we can see where the white Gaussian Noise model comes from.

9.13

Noise Temperature and Noise Figure

Recall:



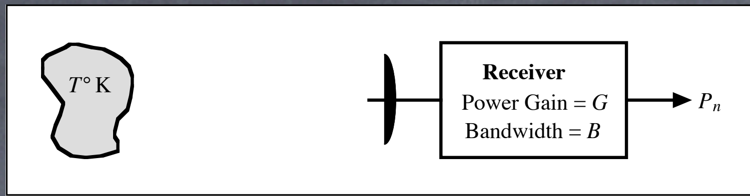
$$P_n(f) = \frac{hf}{e^{hf/kT} - 1}$$

$$\Delta P_n = \frac{hf}{e^{hf/kT} - 1}\Delta f$$

When $hf/kT \ll 1$ (true at microwave for $T = 300°$ K)

$$P_n(f) = \frac{hf}{1 + \frac{hf}{kT} + o(\frac{hf}{kT}) - 1} \approx \frac{hf}{hf/kT} = kT$$

9.14



Assume antenna pointing at object of temperature T.

Assume receiver has bandwidth B and power gain G.

The noise power at the output of the receiver is

$$P_n = kTGB$$

where

GB = *Gain-Bandwidth Product* of the receiver.

9.15

Gain-Bandwidth Product

Generally, the power gain of a receiver is a function of frequency.

We write the power gain as $G(f)$.

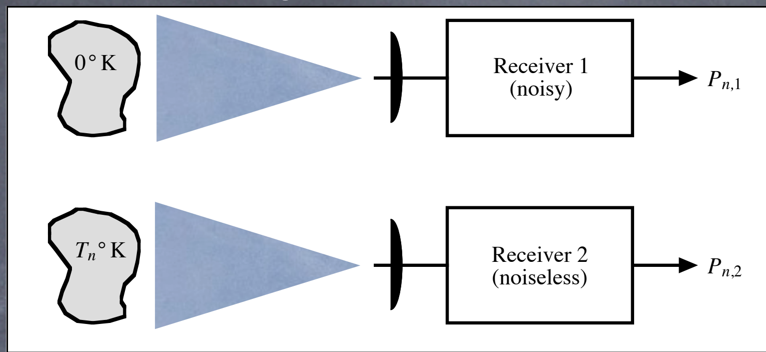
Then the *gain-bandwidth product* is

$$GB = \int_0^\infty G(f)\,df \qquad \text{(Gain-Bandwidth Product)}$$

9.16

Noise Temperature

(Characterizing Microwave System Noise)



Receiver 1 is an *actual* noisy receiver. Its antenna pointed at a black body of temperature $T = 0°$ K.

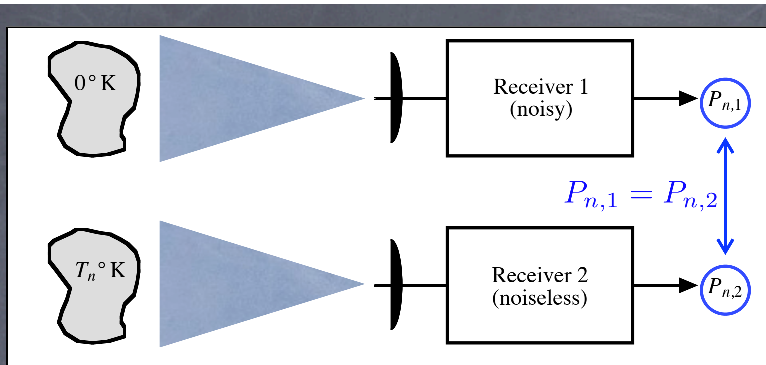
Receiver 2 is a *hypothetical* noiseless receiver. Its antenna is pointing at a black body of temperature T_n.

Assume we can adjust T_n in second scenario until

$$P_{n,1} = P_{n,2}$$

We call the T_N achieving this the *noise temperature* of Receiver 1.

9.17



- When we quote a noise temperature for a real receiver, we are referring internal receiver noise to a hypothetical external noise source.
- This is a convenient accounting trick, as it allows us to look at all noise contributions at the same location—the input to the