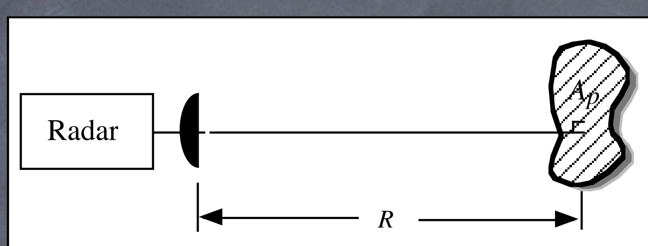
Session 7

7.1

RCS of a Perfectly Conducting Plate

Consider a perfectly conducting plate with dimensions. much greater than λ. Assume area A_p.



Assume perpendicular orientation to incident wave.

In the far field, the plate is uniformly illuminated.

The plate reflects or radiates the wave as if it were a uniformly illuminated aperture of area A_p. (It is!)

7.2

Applying the Friis equation twice—once for each trip—we get

$$\frac{P_R}{P_T} = \frac{P_\sigma}{P_T} \cdot \frac{P_R}{P_\sigma} = \frac{A A_p}{\lambda^2 R^2} \cdot \frac{A_p A}{\lambda^2 R^2} = \frac{A^2 A_p^2}{\lambda^4 R^4}$$

But

$$\frac{P_R}{P_T} = \frac{A^2 \sigma}{4\pi \lambda^2 R^4}$$

Equating these expressions, we have

$$\frac{A^2 A_p^2}{\lambda^4 R^4} = \frac{A^2 \sigma}{4\pi \lambda^2 R^4} \quad \Longrightarrow \quad \boxed{\sigma = \frac{4\pi A_p^2}{\lambda^2}.}$$

7.3

$$\boxed{\sigma = \frac{4\pi A_p^2}{\lambda^2}.}$$

Applies only to a plate perpendicular to the direction of propagation.

If the plate is tilted, it will not be uniformly illuminated. There will be a phase shift across it.

This nonuniformly illuminator then transmits the wave back to the radar, which is off axis.

7.4

Using this approach, we can find that

For a square plate with dimensions $W \times W$ tilted by θ along an axis parallel to a side, we have

$$\sigma = \frac{4\pi W^4}{\lambda^2}\left[\frac{\sin(kW\sin\theta)}{kW\sin\theta}\right]^2$$

and for a circular plate of radius a

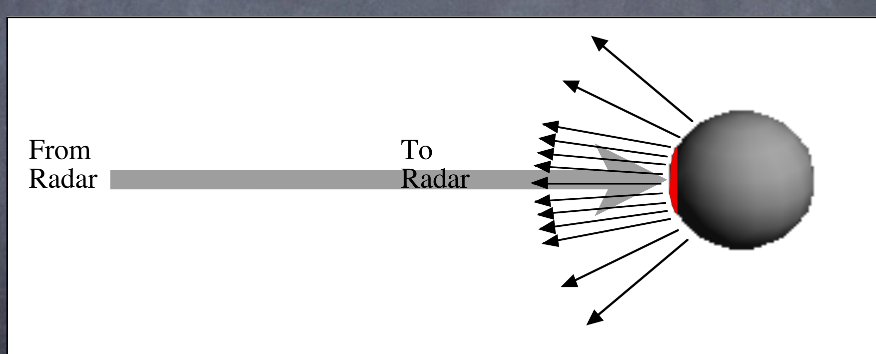
$$\sigma = \frac{\pi a^2}{\tan^2\theta}\left[J_1(2ka\sin\theta)\right]^2$$

where $k = 2\pi/\lambda$ and $J_1(\cdot)$ is the *first-order Bessel function of the first kind.*

7.5

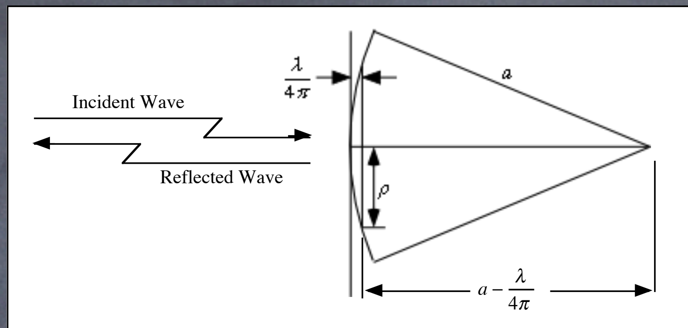
Back to the Sphere

In the optical region, most of the return comes from the "cap" of the sphere perpendicular to the direction of propagation.



Why is the RCS given by the geometric cross section? The "active area" is smaller.

Suppose we approximate the "cap" as a flat plate



How big is the cap?

Assume maximum deviation in "cap" is $\delta x = \lambda/4\pi$.

Then the radius ρ of the "cap" satisfies:

$$\rho^2 + \left(a - \frac{\lambda}{4\pi}\right)^2 = a^2$$

$$\rho^2 + \left(a - \frac{\lambda}{4\pi}\right)^2 = a^2$$

Solving for ρ yields

$$\rho = \sqrt{a^2 - \left(a - \frac{\lambda}{4\pi}\right)^2}$$

$$= \sqrt{a^2 - \left(a^2 - \frac{a\lambda}{2\pi} + \frac{\lambda^2}{16\pi^2}\right)}$$

$$= \sqrt{\frac{a\lambda}{2\pi} - \frac{\lambda^2}{16\pi^2}}$$

$$\approx \sqrt{\frac{a\lambda}{2\pi}}, \qquad \text{for } a \gg \lambda.$$

7.8

Thus the area A_p of the "cap" is

$$A_p = \pi\rho^2 = \pi\left(\frac{a\lambda}{2\pi}\right) = \frac{a\lambda}{2}$$

The resulting RCS of this circular "plate" is

$$\sigma = \frac{4\pi A_p^2}{\lambda^2} = \frac{4\pi(a\lambda/2)^2}{\lambda^2} = \pi a^2.$$

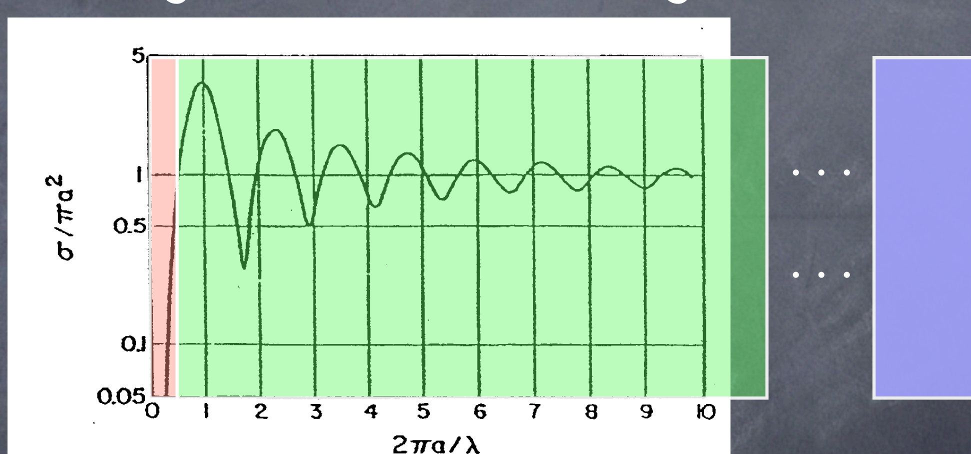
Cheating? For general δx:

$$\sigma = 16\pi^3\left(\frac{\delta x}{\lambda}\right)^2 a^2$$

7.9

RCS of a (Perfectly Conducting) Sphere

A strong function of wavelength



The *Rayleigh Region*, where $2\pi a/\lambda < 0.4$

The *Mie Region*, where $0.4 < 2\pi a/\lambda < 20$

The *Optical Region*, where $2\pi a/\lambda > 20$

In the Rayleigh Region

($\lambda \gg$ maximum dimension)

Assume a smooth (small) object has volume V:

$$\sigma \approx \frac{4V^2}{\pi}\left(\frac{2\pi}{\lambda}\right)^4 = \frac{4k^4V^2}{\pi}$$

This is for an arbitrary object—not just sphere.

This is not quite right for a sphere:

$$\text{coefficient } \frac{64}{9} \approx 7 \neq 9$$

Recall for sphere: $\sigma \approx 9\pi a^2 \left(\frac{2\pi a}{\lambda}\right)^4 = \pi a^2 \left[9(ka)^4\right]$

Ruck et. al, Radar Cross Section Handbook, Plenum 1970.