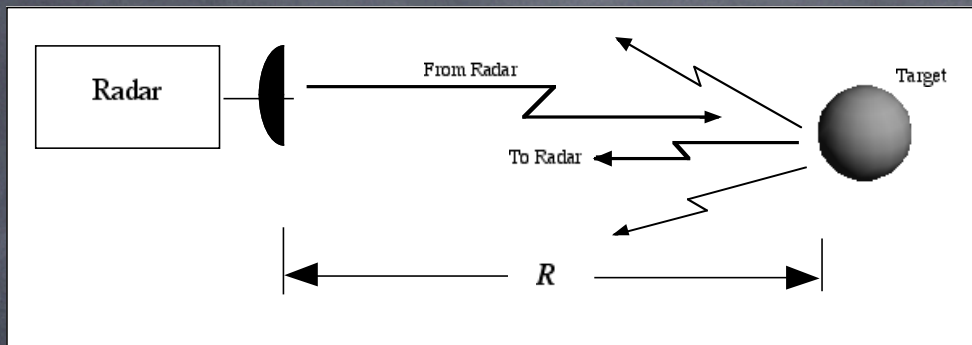


## Session 6

Recall...

6.1

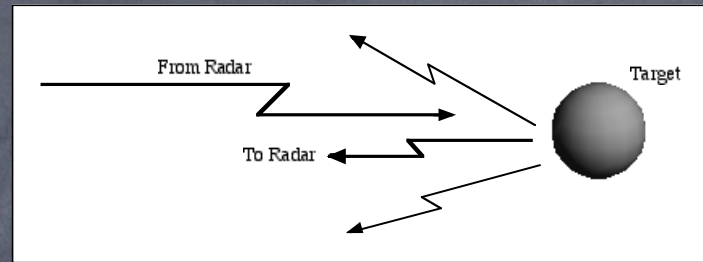
# The Radar Equation



Given that power  $P_T$  is transmitted, what is the received power  $P_R$ ?

To answer this, we must understand the target's behavior.

## Target Scattering Characteristics

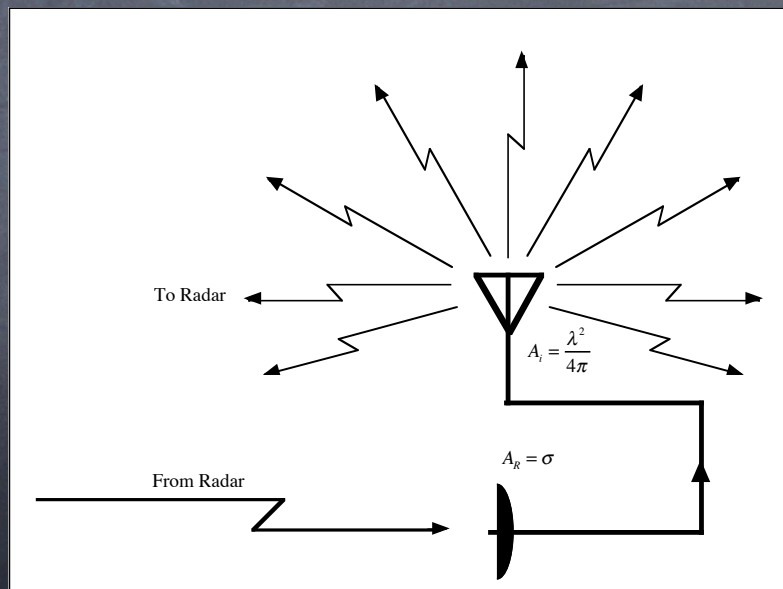


Assume target has following characteristics:

- (i) As a receive aperture, it has  $A_R = \sigma$  ( $\text{m}^2$ );
- (ii) It reradiates all of this received energy isotropically.

## Assumed Target Characteristics

- (i) As a receive aperture, it has  $A_R = \sigma$  ( $\text{m}^2$ );
- (ii) It reradiates all of this received energy isotropically.



The power received by the target is given by

$$\frac{P_\sigma}{P_T} = \frac{A\sigma}{\lambda^2 R^2}$$

The fraction of the reradiated power received is

$$\frac{P_R}{P_\sigma} = \frac{A_i A}{\lambda^2 R^2} = \frac{(\lambda^2/4\pi)A}{\lambda^2 R^2} = \frac{A}{4\pi R^2}$$

It follows that

$$\frac{P_R}{P_T} = \frac{P_R}{P_\sigma} \cdot \frac{P_\sigma}{P_T} = \frac{A^2 \sigma}{4\pi \lambda^2 R^4}$$

## The Radar Equation

$$\frac{P_R}{P_T} = \frac{A^2 \sigma}{4\pi \lambda^2 R^4}$$

(i)  $\frac{P_R}{P_T}$  proportional to  $\frac{1}{R^4}$

(ii)  $\frac{P_R}{P_T}$  proportional to  $\sigma$

## Notes on Radar Equation

- Alternative form using antenna gain(s) instead of effective area can be derived.
- Bistatic version with different transmit and receive ranges and effective areas can be derived (requires generalization of radar cross section.)
- As we will see, radar cross section and geometric cross section can be quite different—don't let this throw you for now.

## Radar Targets

$$\frac{P_R}{P_T} = \frac{A^2 \sigma}{4\pi\lambda^2 R^4}$$

← Radar Cross Section

- RCS is used to characterize the scattering characteristics of target.
- Defined in terms of hypothetical target—defines an equivalence class of targets.
- Is used to describe physical targets that behave nothing like the hypothetical target that defines it. This is OK!

# RCS Contributing Factors

- ⑥ Size of Object
- ⑥ Shape of Object
- ⑥ Wavelength of Radiation
- ⑥ Material(s) Object is Made of
- ⑥ Orientation w.r.t. Radar

Table 4.1: Typical Values for the Radar Cross Section of some Common Objects.

Object	RCS (m <sup>2</sup> )
Small Insect (fly)	10 <sup>-5</sup>
Large Insect (locust)	10 <sup>-4</sup>
Medium-Sized Bird	0.001
Large Bird	0.01
Small Open Boat	0.02
Small Missile	0.1
Man	1
Small Single-engine Airplane	1
Small Fighter or Four-Passenger Jet	2
Helicopter	2
Bicycle	2
Small Pleasure Boat (20–30 ft.)	2
Large Tactical Fighter Airplane	6
Cabin Cruiser (40–50 ft.)	10
Large Bomber or Commercial Airliner	40
Jumbo Jet	100
Automobile	100
Pickup Truck	200
Ship	3000–1000000

For real targets, we almost never know the value of the RCS a priori.

We may know a range of values that  $\sigma$  may lie in:

$$\sigma_L \leq \sigma \leq \sigma_U$$

Sometimes it makes sense to treat  $\sigma$  as a random variable:

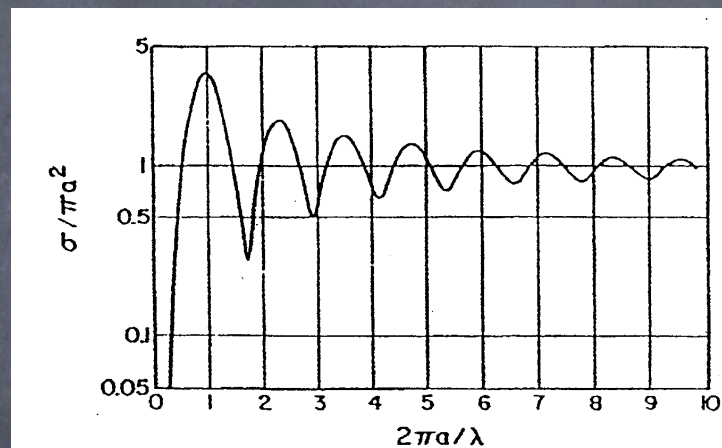
$$\sigma(\omega) \text{ defined on } (\mathcal{S}, \mathcal{F}, P)$$

Sometimes it makes sense to treat  $\sigma$  as a random process:

$$\sigma(t, \omega) \text{ defined on } (\mathcal{S}, \mathcal{F}, P)$$

## RCS of a Sphere

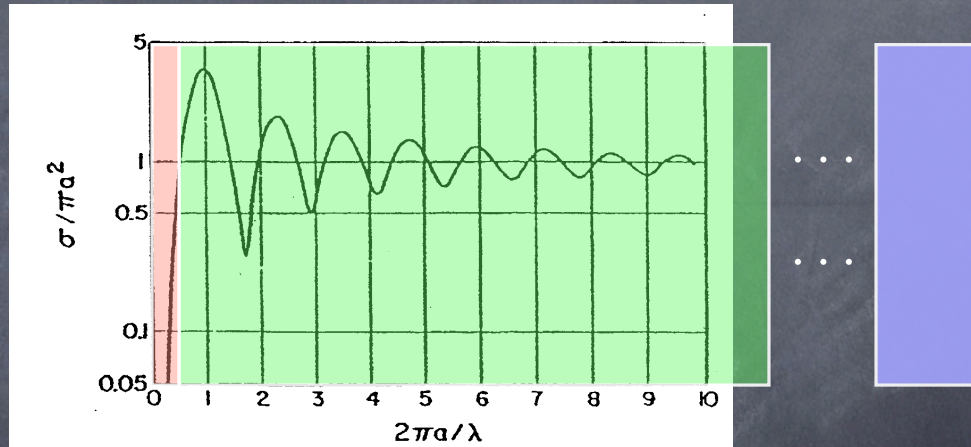
A strong function of wavelength



The *Optical Region*, where  $2\pi a / \lambda > 20$

## RCS of a (Perfectly Conducting) Sphere

A strong function of wavelength



The *Rayleigh Region*, where  $2\pi a/\lambda < 0.4$

The *Mie Region*, where  $0.4 < 2\pi a/\lambda < 20$

The *Optical Region*, where  $2\pi a/\lambda > 20$

## RCS of a (Perfectly Conducting) Sphere

In the *Optical Region*,  $\sigma \approx \pi a^2$ . This is the geometric cross section of a sphere.

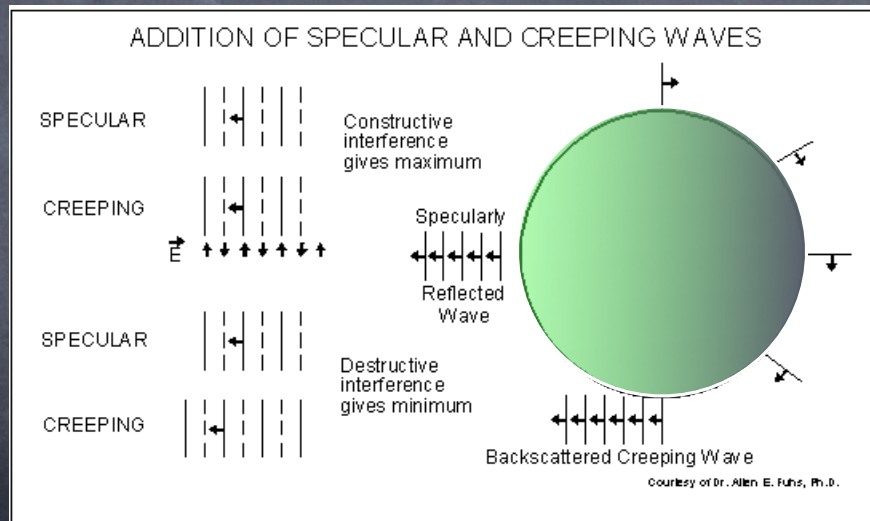
Because a sphere is invariant to changes in its orientation, it makes a convenient calibration target.

In the *Rayleigh Region*, where  $\lambda \gg a$ ,

$$\sigma \approx 9\pi a^2 \left( \frac{2\pi a}{\lambda} \right)^4 = \pi a^2 [9(ka)^4]$$

# RCS of a (Perfectly Conducting) Sphere

In the *Mie Region*, where  $\lambda \approx a$ , “creeping waves” travel around the sphere and interfere with the specular reflection:



This gives rise to the “resonance” seen in this region.