Session 5

Recall...

It appears we have derived this result for square transmit apertures and arbitrary receive apertures.

Reciprocity Theorem: For linear antenna systems, P_R/P_T remains the same when the roles of the transmit and receive antennas are reversed.

$\Longrightarrow$ P_R/P_T proportional to uniformly illuminated area A_T regardless of shape.

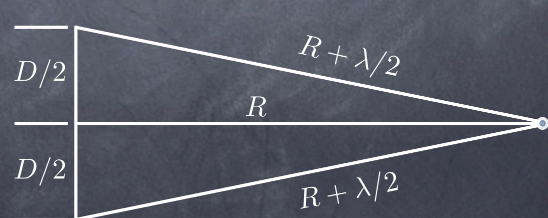
$$\therefore \quad \boxed{\frac{P_R}{P_T} = \frac{A_T A_R}{\lambda^2 R^2}}$$

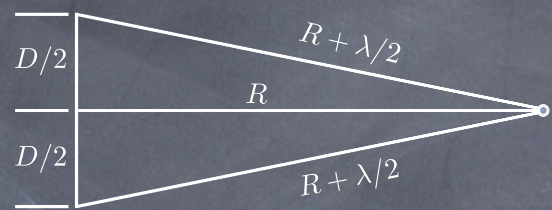
← The Friis Equation

- Friis equation derived using scalar diffraction—scalar wave equation describes propagation.
- Valid in other situations descibed by scalar wave equation:
- Acoustic radiation (in non-viscous media)
- Polarized light

- Friis equation holds when antennas are "far apart." How far?
- Uniformly illuminated aperture focuses objects at infinity.
- If we look at an antenna from close in, the edges appear farther away than the

A bad situation:





This bad situation occurs when

$$\left(R+\frac{\lambda}{2}\right)^2 = \left(\frac{D}{2}\right)^2 + R^2$$

$$\Longrightarrow \qquad R = \frac{D^2-\lambda^2}{4\lambda} \approx \frac{D^2}{4\lambda}, \qquad \text{because } D \gg \lambda$$

Thus we want

$$R \gg \frac{D^2}{4\lambda}$$

Maybe 10 times bigger—or more!

A common approach is to consider a point to be in the far-field when the center and edges of the aperture differ by no more than $\lambda/8$

If our calculations show

$$\frac{P_R}{P_T} = \frac{A_T A_R}{\lambda^2 R^2} > 1 \quad \Rightarrow \quad \text{Something is wrong!}$$

When such a situation occurs, a large fraction of the transmitted power is received by the receive aperture:

"Hertzian Cable"

This will not occur when R is such that the antennas are in each others far field.

Effective Area of an Antenna

- In deriving the Friis equation, we have assumed uniform aperture illumination.
- Practically, this is hard to achieve.
- When we do not have uniform illumination, the effective size of the aperture is smaller than the geometric area.
- How much smaller?

Suppose $E(x,y)$ is the field over the aperture (i.e., $E(x,y) \neq E_0$.)

The power P_T flowing out over the aperture is

$$P_T = \frac{1}{Z_{FS}} \int\int_{A_{TG}} |E(x,y)|^2 \, dx\, dy,$$

The electric field at the receive aperture is

$$E_R = B \int\int_{A_{TG}} E(x,y) \, dx\, dy$$

↑ constant

The total power at the receive aperture is

$$P_R = \frac{|E_R|^2}{Z_{FS}} A_R = \frac{A_R B^2}{Z_{FS}} \left| \int\int_{A_{TG}} E(x,y)\, dx\, dy \right|^2 .$$

Thus we have

$$\frac{P_R}{P_T} = B^2 A_R \frac{\left| \int\int_{A_{TG}} E(x,y)\, dx\, dy \right|^2}{\int\int_{A_{TG}} |E(x,y)|^2 \, dx\, dy}$$

Now define the *spatial average field* across A_{TG}

$$\overline{E} = \frac{1}{A_{TG}} \int\int_{A_{TG}} E(x,y)\, dx\, dy$$

and the *spatial mean-square field* across the aperture as

$$\overline{|E|^2} = \frac{1}{A_{TG}} \int\int_{A_{TG}} |E(x,y)|^2 \, dx\, dy$$

Hence

$$\frac{P_R}{P_T} = B^2 A_R A_{TG} \left[\frac{|\overline{E}|^2}{\overline{|E|^2}} \right]$$

To find B in

$$\frac{P_R}{P_T} = B^2 A_R A_{TG} \left[\frac{|\overline{E}|^2}{\overline{|E|^2}} \right]$$

we note that when $E(x,y) = E_0$ (uniform illum. $\Rightarrow A_{TG} = A_T$)

$$|\overline{E}|^2 = \overline{|E|^2} \qquad \Rightarrow \qquad B^2 = \frac{P_R}{P_T} \cdot \frac{1}{A_T A_R}$$

Thus A_T in the Friis equation is

$$A_T = A_{TG} \left[\frac{|\overline{E}|^2}{\overline{|E|^2}} \right] = \eta_T \cdot A_{TG}$$

and by reciprocity

$$A_R = A_{RG} \left[\frac{|\overline{E}|^2}{\overline{|E|^2}} \right] = \eta_R \cdot A_{RG}$$

Aperture Efficiency (η_T, η_R)

Effective Area

- Even antennas that don't have well defined apertures can be assigned effective areas for use in the Friis equation.
- Here we can use EM field theory to compute the field strength due to the transmit antenna at the receive aperture:

$$A_T = \frac{\lambda^2 R^2}{A_R} \cdot \frac{P_R}{P_T}$$

For example, for a half-wave dipole

$$A_T \approx 0.130\lambda^2$$

Antenna Gain over an Isotropic Radiator

- Isotropic Radiator: An antenna that radiates energy uniformly in all directions (transmit).
- On receive, it is equally sensitive to energy from all directions (by reciprocity).

If at a distance R from an isotropic radiator, we place a receive aperture A_R

$$\frac{P_R}{P_T} = \frac{A_R}{4\pi R^2}$$

surface area of sphere of radius R

$$\frac{P_R}{P_T} = \frac{A_R}{4\pi R^2} = \frac{A_i A_R}{\lambda^2 R^2} \Rightarrow \boxed{A_i = \frac{\lambda^2}{4\pi}}$$

$$\boxed{A_i = \frac{\lambda^2}{4\pi}}$$

- Note that the usual requirements for aperture size are not met.
- However, it works in the Friis equation.
- For this reason we will find it useful.

The *gain* of A_T over A_i is

$$G = \frac{(P_R/P_T)_T}{(P_R/P_T)_i} = \frac{(P_R)_T}{(P_R)_i} = \frac{4\pi A_T}{\lambda^2}$$

So in general, the relation between the effective area A and gain G of antenna is

$$\boxed{G = \frac{4\pi A}{\lambda^2}} \qquad \boxed{A = \frac{\lambda^2 G}{4\pi}}$$

Gain is often expressed in dB:

$$G(\text{dB}) = 10 \log_{10} G \quad (\text{dB})$$

Antenna Gain...

- By reciprocity, the gain of an antenna on transmit is equal to the gain of an antenna on receive.
- The Friis Equation can be written in terms of antenna gains:

$$\frac{P_R}{P_T} = \frac{G_T G_R \lambda^2}{16\pi^2 R^2}$$

Antenna Directivity and Beam Pattern

For a uniformly illuminated aperture

$$G_T = \frac{4\pi A_{TG}}{\lambda^2}$$

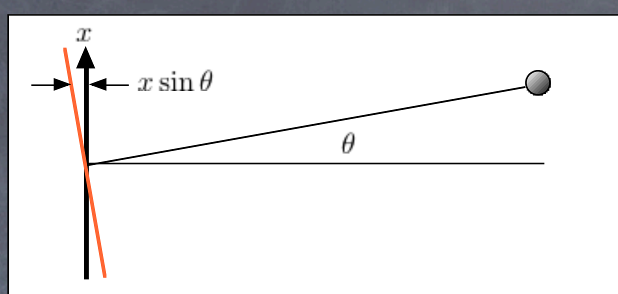
If A_{TG} is not uniformly illuminated

$$G_T = \frac{4\pi A_T}{\lambda^2} = \frac{4\pi A_{TG}}{\lambda^2}\eta$$

If we go off axis by θ (azimuth) and ϕ (elevation) the gain is not as large as in direction $(\theta, \phi) = (0, 0)$. It can be written as

$$G(\theta, \phi) = \frac{4\pi A_{\mathrm{TG}}}{\lambda^2} \cdot f(\theta, \phi)$$

Aperture Efficiency?



Looking back at the aperture off-axis we see a "virtual aperture" with a phase shift across it.

In general, at angle (θ, ϕ) we have

$$G(\theta, \phi) = \frac{4\pi A_{\mathrm{TG}}}{\lambda^2} \cdot f(\theta, \phi)$$

$$f(\theta, \phi) = \left(\frac{1}{A_{\mathrm{TG}}}\right) \frac{\left| \int\int_{A_{\mathrm{TG}}} \exp\left\{ i\frac{2\pi}{\lambda}[x\sin\theta + y\sin\phi]\right\} E(x,y)\, dx\, dy \right|^2}{\int\int_{A_{\mathrm{TG}}} |E(x,y)|^2 \, dx\, dy}.$$

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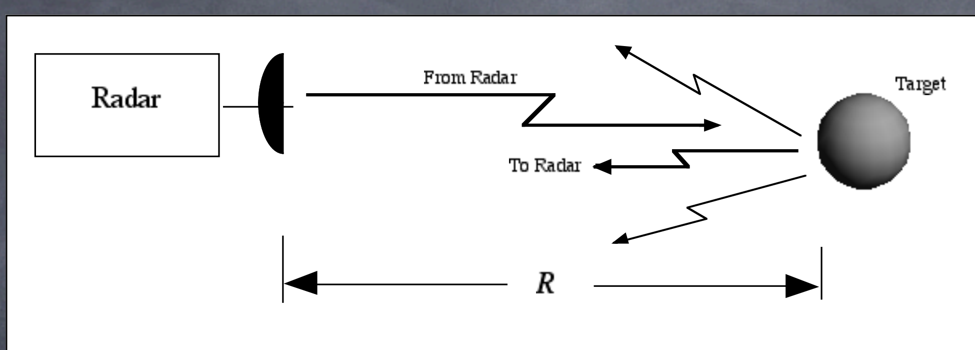
This expression acts like an efficiency, taking on values between 0 and 1.

It provides a measure of the directivity of the antenna for transmitting and receiving power.

We call this quantity the beam pattern of the antenna.

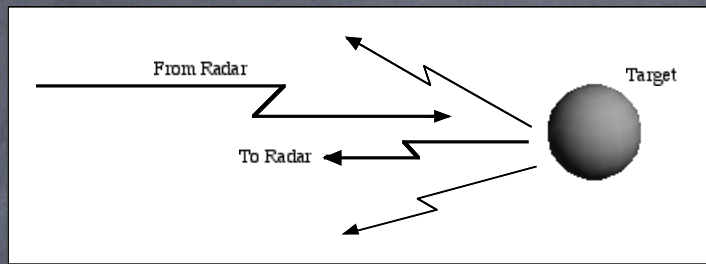
The Radar Equation



Given that power P_T is transmitted, what is the received power P_R?

To answer this, we must understand the target's behavior.

Target Scattering Characteristics



Assume target has following characteristics:

(i) As a receive aperture, it has $A_R = \sigma \, (\text{m}^2)$;

(ii) It reradiates all of this received energy isotropically.

Assumed Target Characteristics

(i) As a receive aperture, it has $A_R = \sigma \, (\text{m}^2)$;

(ii) It reradiates all of this received energy isotropically.