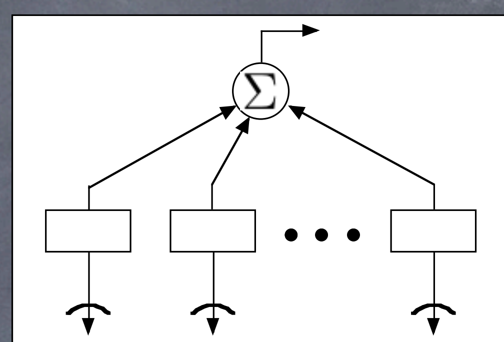
Session 39

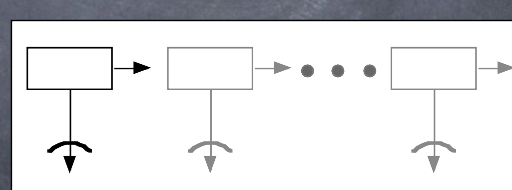
Synthetic Arrays

A real array:

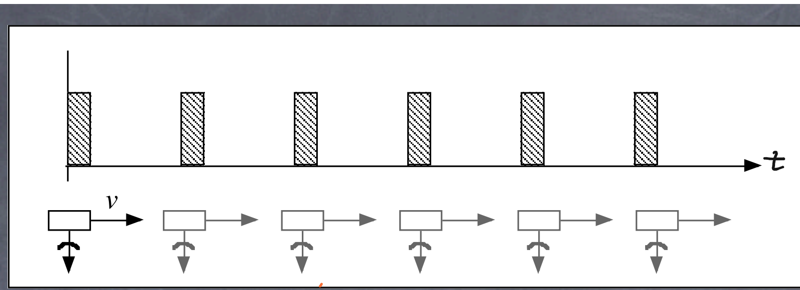


A synthetic array:

Another approach is to use a single element and move it between observations



Signal processing is used to synthesize an "equivalent" array.



The received signal is recorded with phase information.

We collect data at each position.

We apply proper phase shifts to the received data.

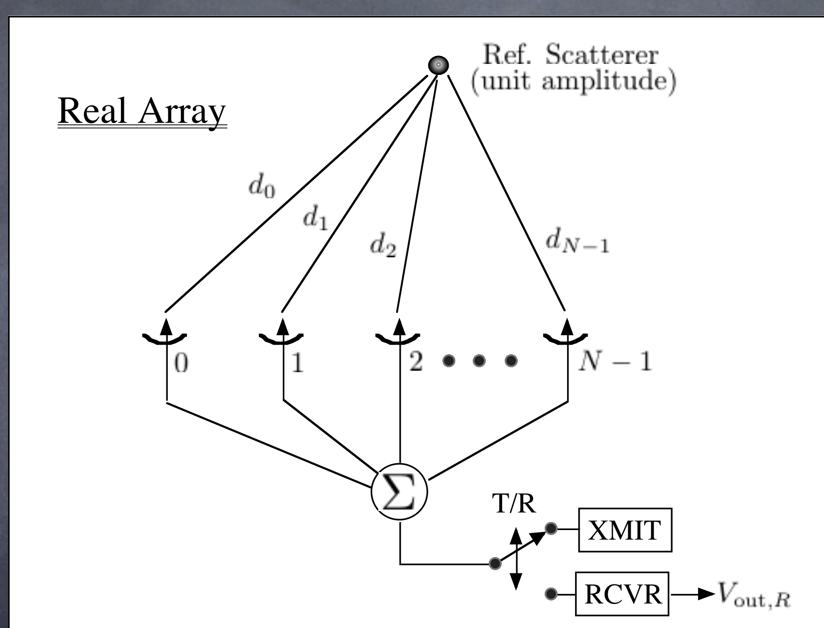
We sum to synthesize an array antenna.

This arrray—being sequentially generated—is a little different than a real array.

It does have high angular resolution like a real array.

This is the approach used in Synthetic Aperture Radar (SAR).

Comparison of Real and Synthetic Arrays



Real Array

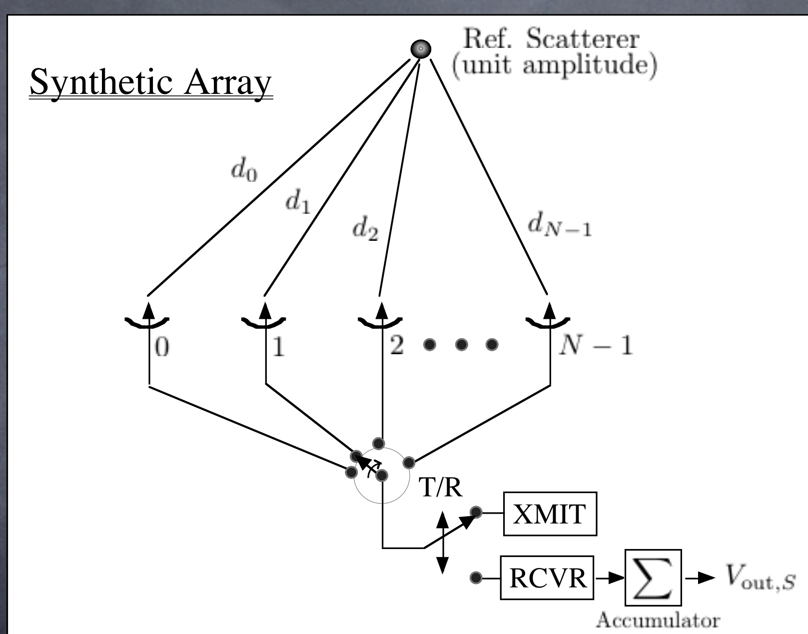
The complex field at the n-th element is

$$v_n = \sum_{m=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}(d_m + d_n)\right\}$$
$$= \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\} \sum_{m=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_m\right\},$$
$$m = 0, 1, \ldots, N-1.$$

The output of the entire array is the sum

$$V_{\text{out},R} = \sum_{n=0}^{N-1} v_n = \sum_{n=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\} \sum_{m=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_m\right\}$$
$$= \left[\sum_{n=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

Synthetic Array

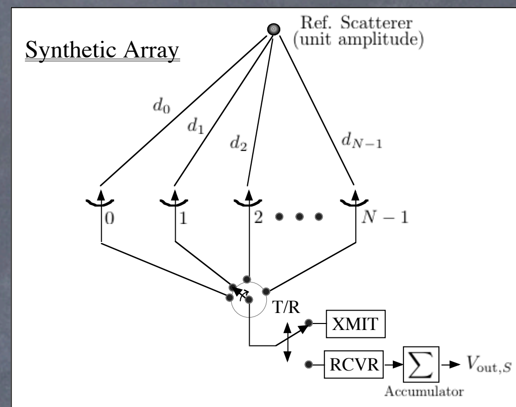


The synthetic array is sequentially built, one element at a time.

Synthetic Array Response

The response of the n-th array element is

$$v_n = \exp\left\{-i\frac{2\pi}{\lambda}(d_n + d_n)\right\}$$
$$= \exp\left\{-i\frac{4\pi}{\lambda}d_n\right\}$$



The synthetic array output response is

$$V_{\text{out},S} = \sum_{n=0}^{N-1} v_n = \sum_{n=0}^{N-1} \exp\left\{-i\frac{4\pi}{\lambda}d_n\right\} = \sum_{n=0}^{N-1}\left[\exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

Real Aperture:

$$V_{\text{out},R} = \left[\sum_{n=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

(Square of Sum)

cross terms!

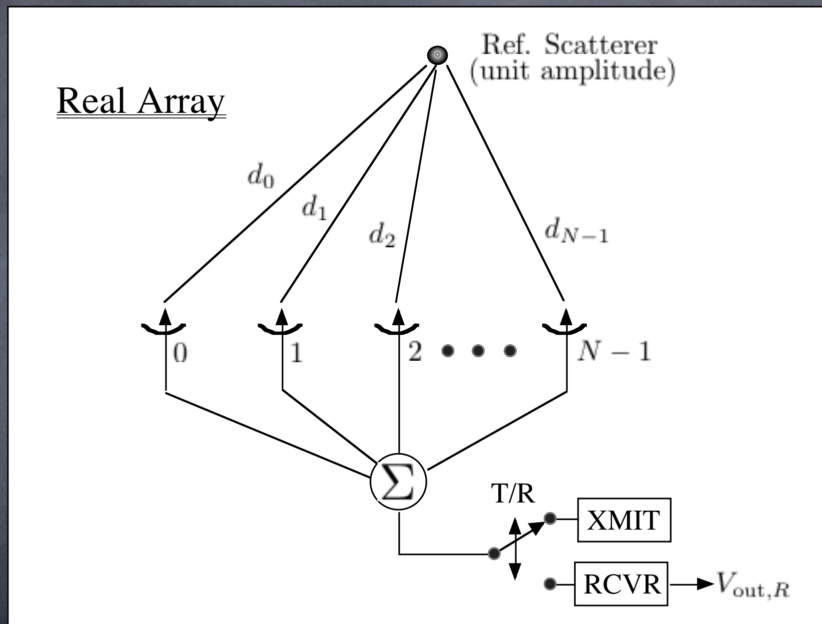
Synthetic Aperture:

$$V_{\text{out},S} = \sum_{n=0}^{N-1}\left[\exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

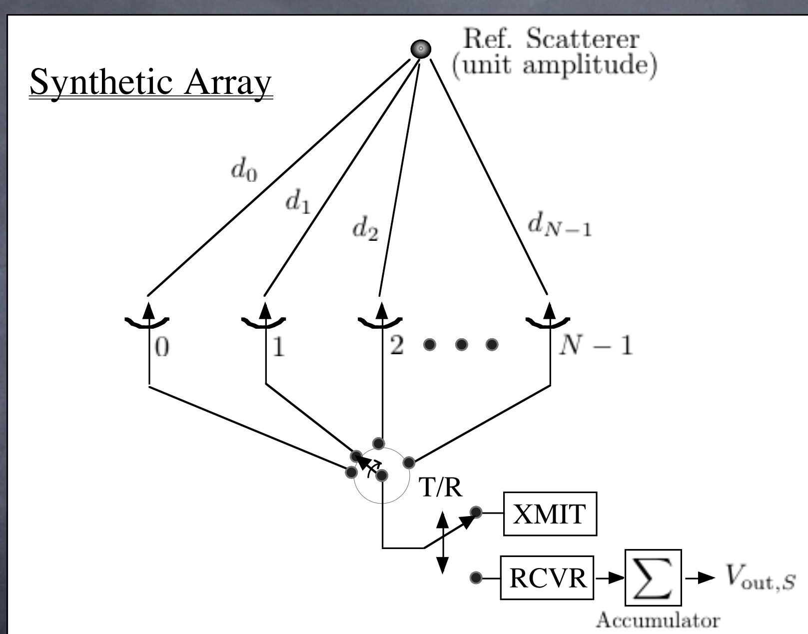
(Sum of Squares)

no cross terms!

Comparison of Real and Synthetic Arrays

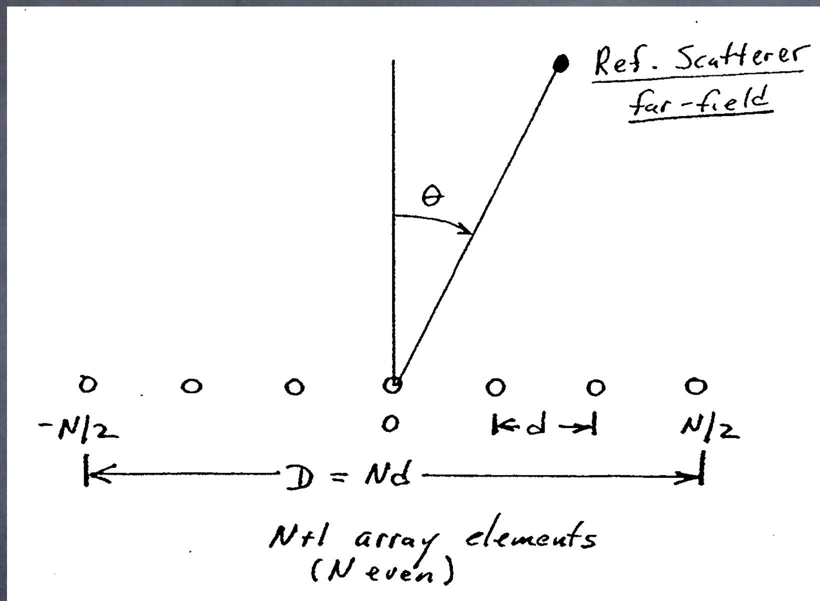


Synthetic Array



The synthetic array is sequentially built, one element at a time.

Let's Compare with Identical Array Geometries



Both arrays have $N+1$ identical isotropic elements.

Elements distributed along line with separation d.

Reference scatterer in far-field.

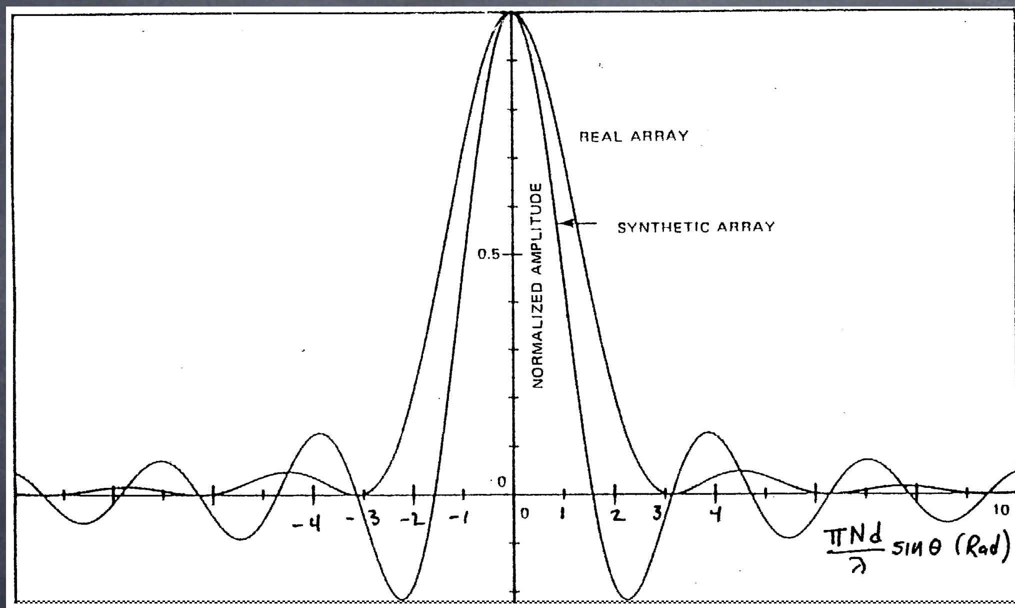
$$V_{\text{out},R}(\theta) = \left[\sum_{n=-N/2}^{N/2} \exp\left\{ +i\frac{2\pi}{\lambda} nd\sin\theta \right\} \right]^2$$

$$= \boxed{\left[\frac{\sin\left[\frac{\pi d}{\lambda}(N+1)\sin\theta\right]}{\sin\left[\frac{\pi d}{\lambda}\sin\theta\right]} \right]^2}$$

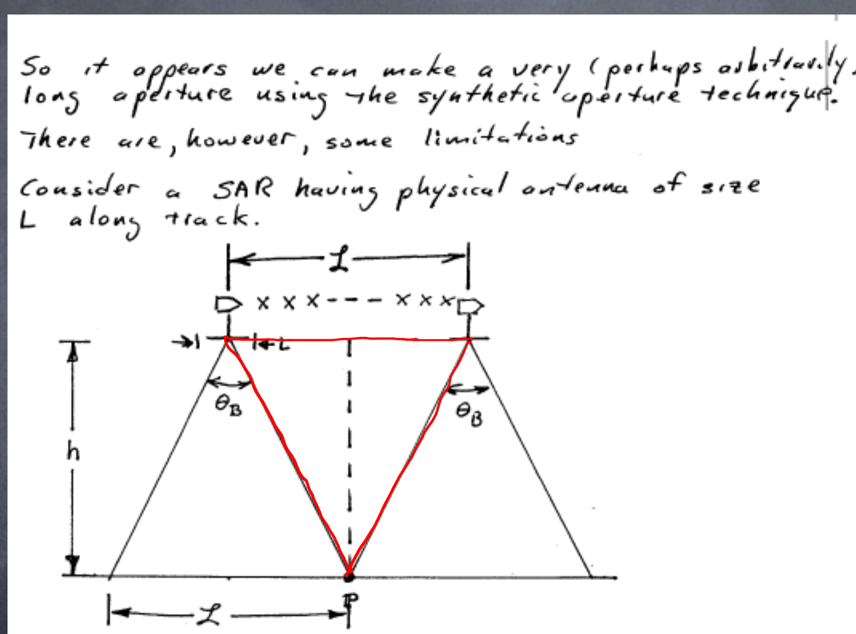
$$V_{\text{out},s}(\theta) = \sum_{n=-N/2}^{N/2} \exp\left\{ i\frac{4\pi}{\lambda} nd\sin\theta \right\}$$

$$= \boxed{\frac{\sin\left[(N+1)\frac{2\pi d\sin\theta}{\lambda}\right]}{\sin\left[\frac{2\pi d\sin\theta}{\lambda}\right]}}$$

A Plot of the Responses



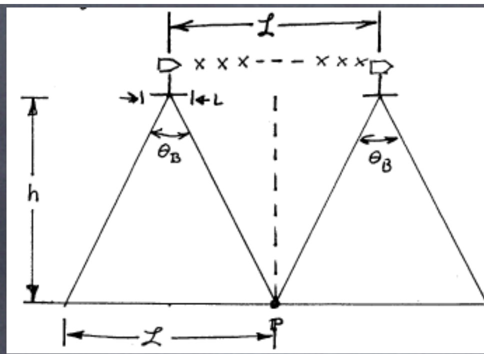
The synthetic array has higher resolution, but also higher sidelobes.



So it appears we can make a very (perhaps arbitrarily) long aperture using the synthetic aperture technique.

There are, however, some limitations

Consider a SAR having physical antenna of size L along track.



The beamwidth of the physical antenna is

$$\theta_B \simeq \frac{\lambda}{L}$$

If the radar is at height h, what is the path length $\mathcal{L}$ illuminated on the ground?

$$\mathcal{L} = h\,\theta_B = \frac{h\lambda}{L}$$

Now if we image a point P starting when it enters the beam and ending when it leaves the beam, we get a synthetic aperture of size $\underline{\mathcal{L}}$.

This synthetic aperture has has a beamwidth

$$\theta_S = \frac{\lambda}{2\mathcal{L}} = \frac{\lambda}{2\frac{h\lambda}{L}} = \frac{L}{2h}$$

Thus the azimuth resolution on the ground is

$$X_a = h\,\theta_S = h\left(\frac{L}{2h}\right) = \frac{L}{2}$$ } Half the antenna length.

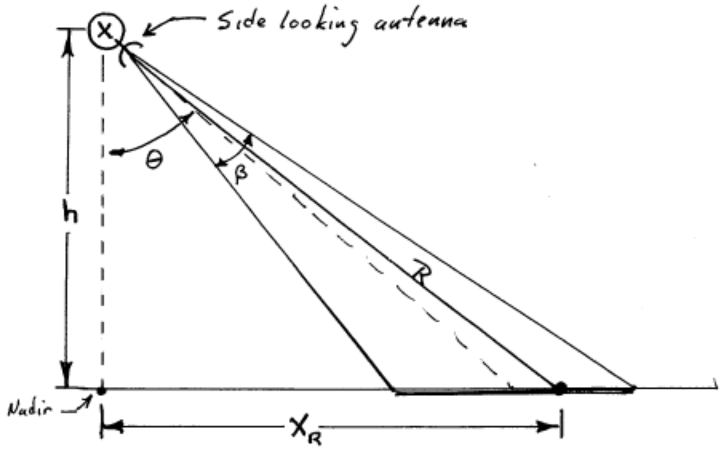
⇒ Small physical antenna gives high resolution.

⇒ Ultimate resolution X_a is <u>not</u> a function of distance from surface.

<u>Bottom Line</u>: In order to generate a large synthetic aperture, you must have a broad illumination pattern.

- The farther radar is from surface, the larger the footprint on the ground.
  - ⇒ Larger synthetic aperture
  - ⇒ finer synthetic beamwidth

  This exactly counterbalances the increase in distance h, giving X_a independent of h.



The distance X_R can be determined by noting that the range R to the point can be related to X_r by

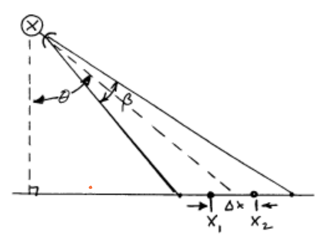
$$\frac{X_R}{R} \simeq \sin\theta \quad (\beta \ll \theta)$$

Then R can be determined by the time delay required to receive the signal

$$2R = c\tau \Rightarrow R = \frac{c\tau}{2}$$

$$\therefore X_R \simeq R\sin\theta = \frac{c\tau}{2}\sin\theta \quad (\beta \ll \theta).$$

What about the ability to resolve two targets closely spaced in range?



If two points are separated by ΔX in "cross track" dimension, then there echos will be separated by time difference

$$\Delta t \cong \frac{2\Delta X}{c}\sin\theta \Rightarrow \Delta X \simeq \frac{c\Delta t}{2\sin\theta}$$

If $\Delta\tau$ is the smallest time delay difference that can be resolved (can be determined from ambiguity function), then

$$\Delta\tau \cong \frac{1}{B} \qquad B = \text{signal bandwidth (one-sided)}$$

Thus if two points are resolved, $\Delta t \geq \Delta\tau$, and hence

$$\Delta X_{min} = \frac{c\Delta\tau}{2\sin\theta} = \frac{c}{2B\sin\theta}$$

e.g. B = 20 MHz and θ = 20°

$$\Rightarrow \Delta X_{min} = \frac{3\times10^8}{2(20\,MHz)\sin 20^\circ} \simeq 22\,m$$

at $\theta = 45^\circ \Rightarrow \Delta X_{min} \cong 10\,m$.