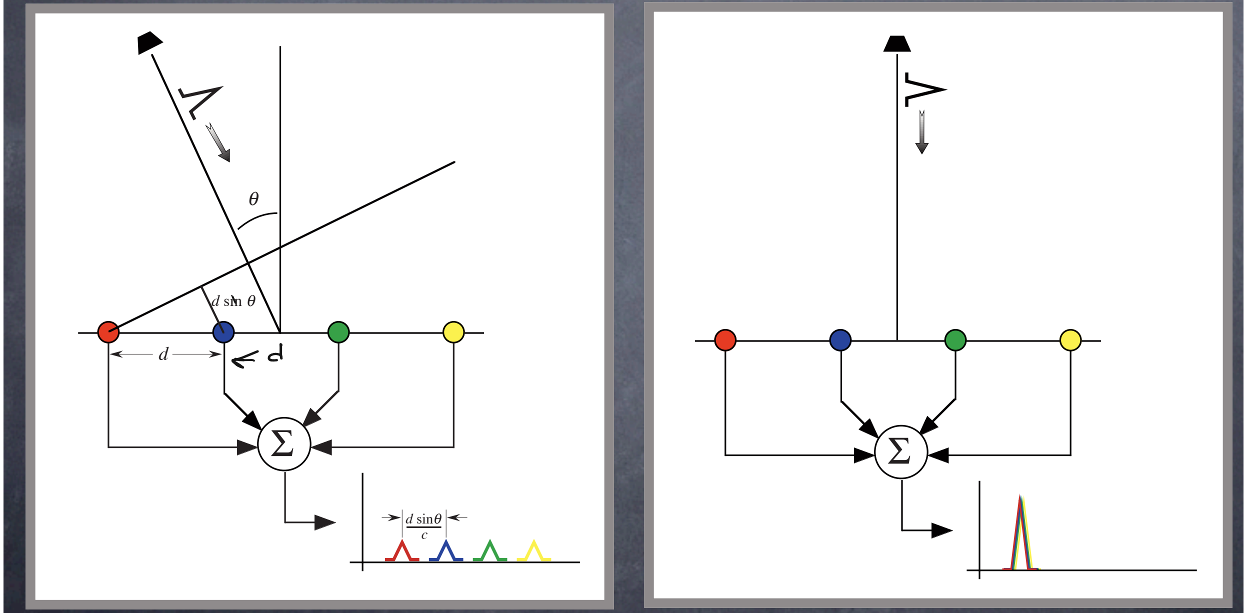
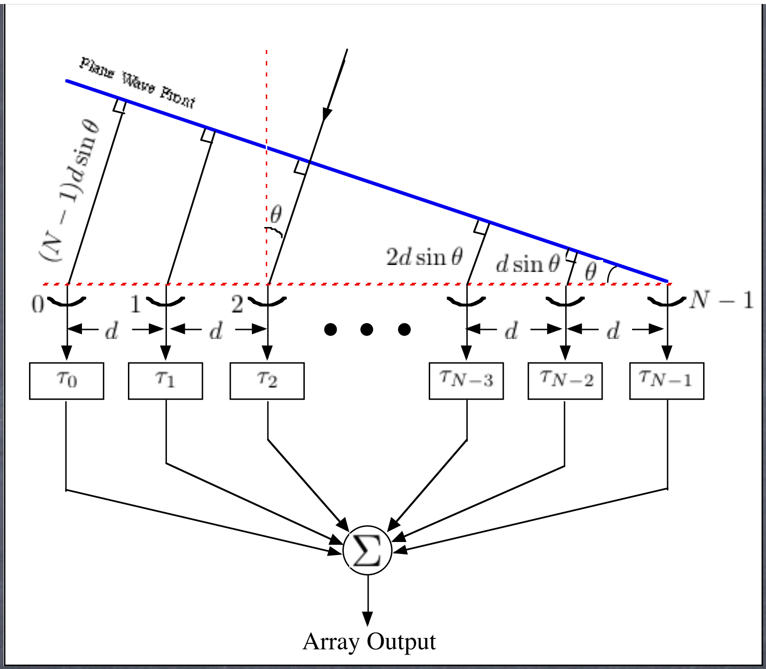
Session 38

Recall . . .

Linear Array Principle of Operation



Recall ...

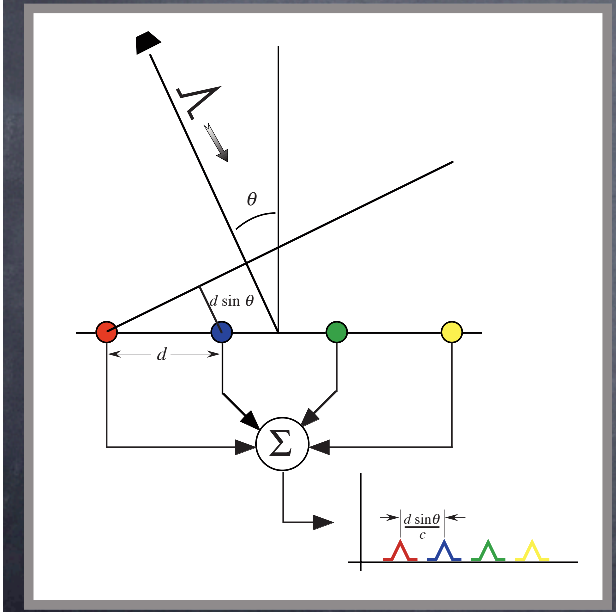


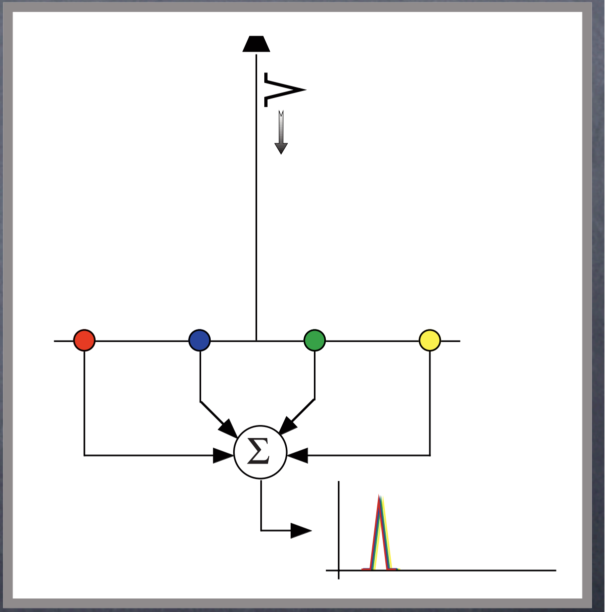
Setting the delays to

$$\tau_m = \frac{[(N-1)-m]d\sin\theta}{c}, \quad m = 0, \ldots, N-1$$

causes the received signals to add up "in-sync."

Signals adding in-sync result in a stronger version of the original signal.



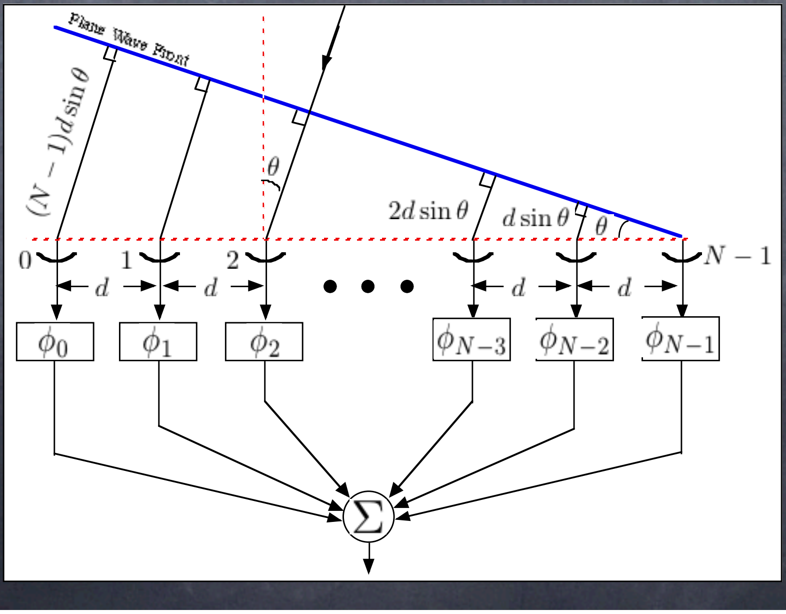


By the time shift theorem of Fourier transforms, we have

$$s(t) \overset{\mathcal{F}}{\leftrightarrow} S(f) \quad \Rightarrow \quad s(t-\tau) \overset{\mathcal{F}}{\leftrightarrow} S(f)\, e^{-i2\pi f \tau}.$$

So for a sinusoidal or narrowband signal at frequency f_0, we can replace the delay τ_m by phase shift

$$\phi_m = 2\pi f_0 \tau_m.$$



Assuming narrowband waves and phase shifters with

$$\phi_0 = \phi_1 = \phi_2 = \cdots = \phi_{N-1} = 0$$

and N identical elements with effective area $A_e(\theta)$ (gain $G_e(\theta)$) for a wave from direction θ, it can be shown the effective area of the array is

$$A(\theta) = A_e(\theta) \cdot \frac{1}{N} \left| \sum_{n=0}^{N-1} e^{i2\pi n (d/\lambda)\sin\theta} \right|^2$$

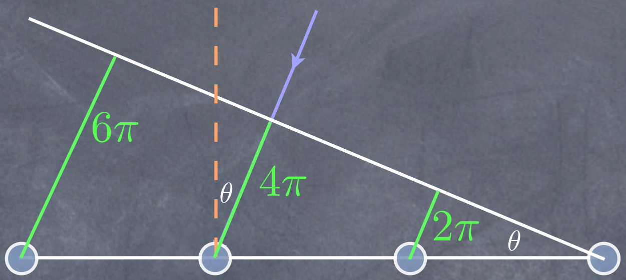
or equivalently

$$G(\theta) = G_e(\theta) \cdot \frac{1}{N} \left| \frac{\sin\left[N\pi(d/\lambda)\sin\theta\right]}{\sin\left[\pi(d/\lambda)\sin\theta\right]} \right|^2$$

$$\text{Array Length} = (N-1)d$$

Larger d implies higer resolution, but there is a price to pay.

If $d > \lambda/2$, we get grating lobes due to constructive interference at Bragg angles:



In order to reduce grating lobes, you must have $d \lesssim \lambda/2$.

You can also

1. Use nonuniform spacing of elements;
2. Use an $A_e(\theta)$ that reduces the most problematic grating lobes. (elements may be large)

Radio astronomy arrays often have severe grating lobes.

In radar they can be more problematic. Usually take $d \approx$

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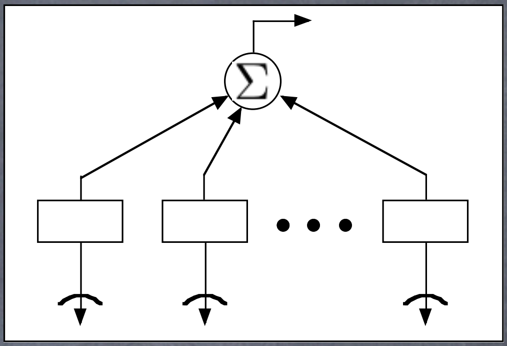
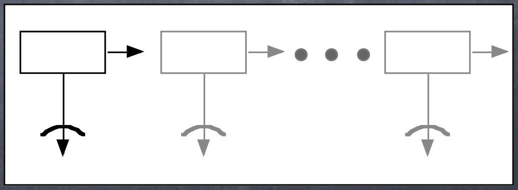
Usually take $d \approx \lambda/2$.

Synthetic Arrays

A real array:



A synthetic array:



Another approach is to use a single element and move it between observations

Signal processing is used to synthesize an “equivalent” array.