Session 36

36.1

Phase-Coded Waveforms

If for a coded waveform

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t - nT) \exp\{i2\pi d_n t/T\} \exp\{j\phi_n\},$$

where

$$p(t) = 1_{[0,T]}(t),$$

we take

$$d_0 = d_1 = d_2 = \cdots = d_{N-1} = 0,$$

we get

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t - nT) \exp\{j\phi_n\}.$$

Such a signal is called a *phase-coded waveform.*

Such a waveform is characterized by the set of phases

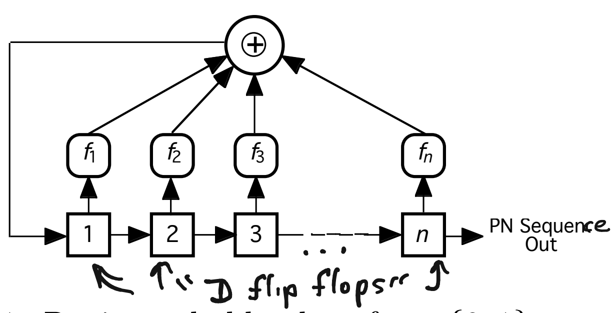
$$\{\phi_0, \phi_1, \phi_2, \ldots, \phi_{N-1}\}.$$

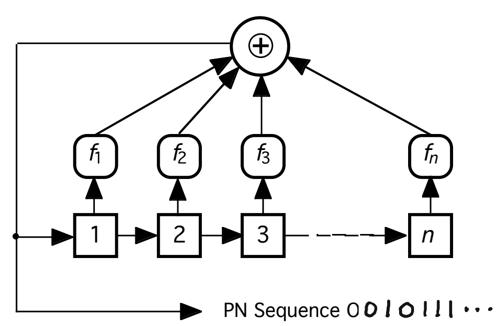
- There are a number of interesting Phase-Coded Waveforms. We will look at two:
  - Maximal Length *Linear Feedback Shift Register* (LFSR) sequences
  - Complementary Sequences

Maximum Length *Linear Feedback Shift Register* (LSRS) Sequences

Key references:

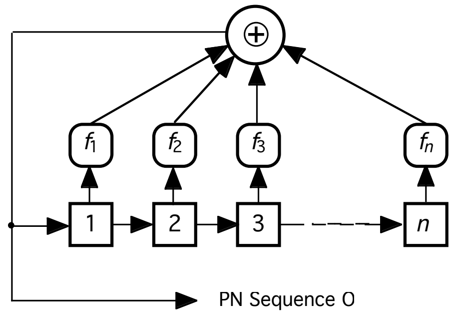
1. Solomon W. Golomb, *Shift Register Sequences,* Revised Edition, Aegian Park Press, 1982.
2. Robert J. McEliece, *Finite Fields for Computer Scientists and Engineers*, Kluwer, 1987.





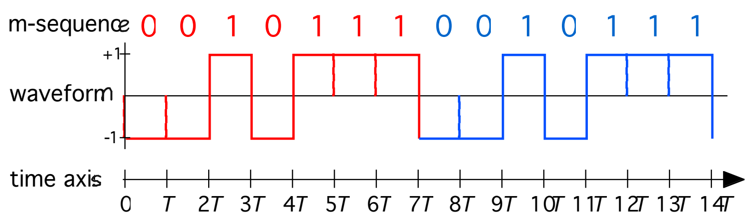
1. Registers hold values from $\{0, 1\}$.
2. Binary arithmetic (modulo-2 arithmetic).
3. Correct selection of binary coefficients $f_1, \ldots, f_n$ yields periodic sequences with period $N = 2^n - 1$.

Maximum Length *Linear Feedback Shift Register* (LSRS) Sequences



For $n = 3, f_1 = 0, f_2 = f_3 = 1$, we get the following length 7 periodic sequence when the initial state is not all zeros:

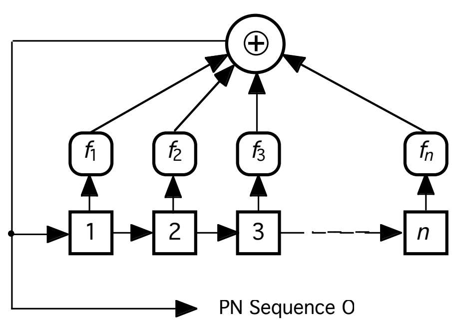
"0" → −1
"1" → +1



Here we map

- "0" → "−1", or phase $\phi_n = \pi$,
- "1" → "+1", or phase $\phi_n = 0$.

Maximum Length *Linear Feedback Shift Register* (LSRS) Sequences



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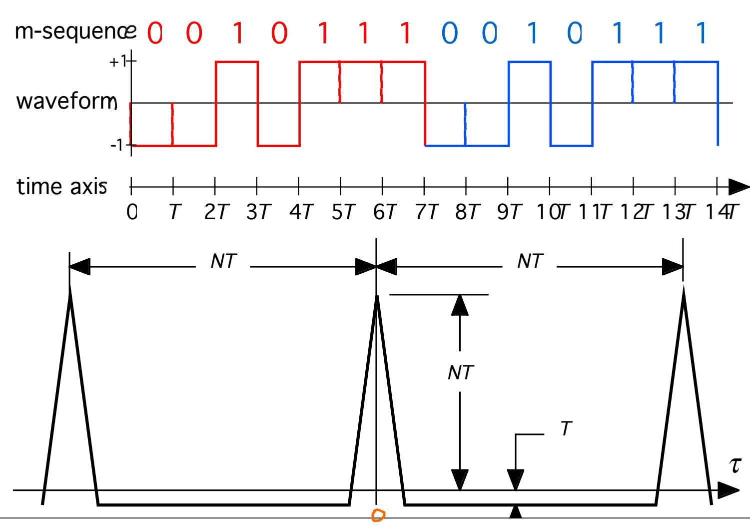
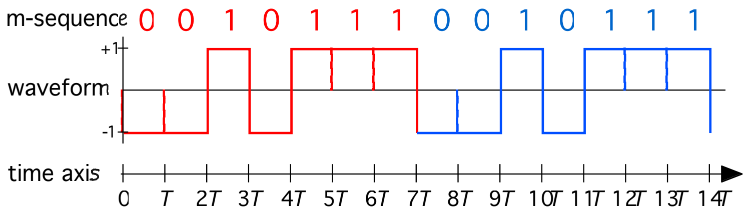
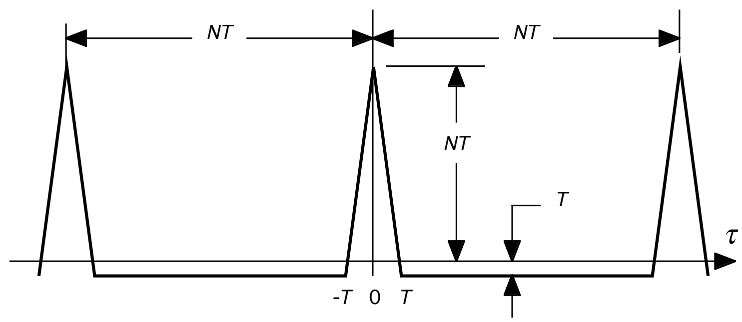




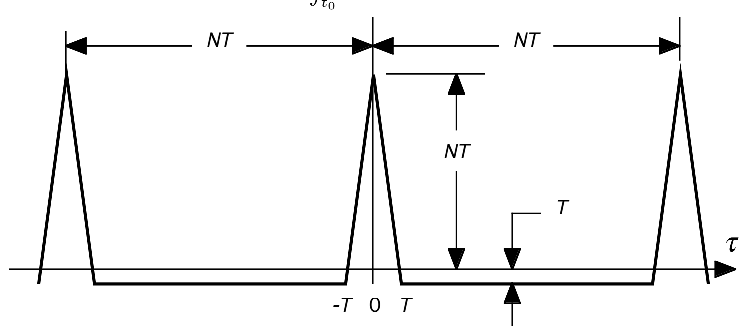
The time-autocorrelation function

$$R_s(\tau) = \int_{t_0}^{t_0+NT} s(t)s^*(t-\tau)\,dt$$

appears as follows:



$$R_s(\tau) = \int_{t_0}^{t_0+NT} s(t)s^*(t-\tau)\,dt$$



- With proper selection of N and T, periodic waveforms for high range resolution CW radar can be achieved.
- These waveforms are used in direct-sequence spread-spectrum systems as “pseudo-noise spreading sequences”.
- Can be used for “spread spectrum radar systems”.
- Multi-access characteristics of families of these waveforms useful for multistatic radar.
- Useful for Low Probability of Intercept (LPI) radar.
- Doppler ambiguity characteristics can be problematic.

Golay Complementary Sequences

- *Complementary Sequences* are two (or more) phase coded sequences that can be used together to yield good range resolution results.

- Initially introduced by Marcel Golay for the design of optical spectrometers (see Harwit and Sloane, *Hadamard Transform Optics.)*

- One of the first examples of diversity waveform techniques.