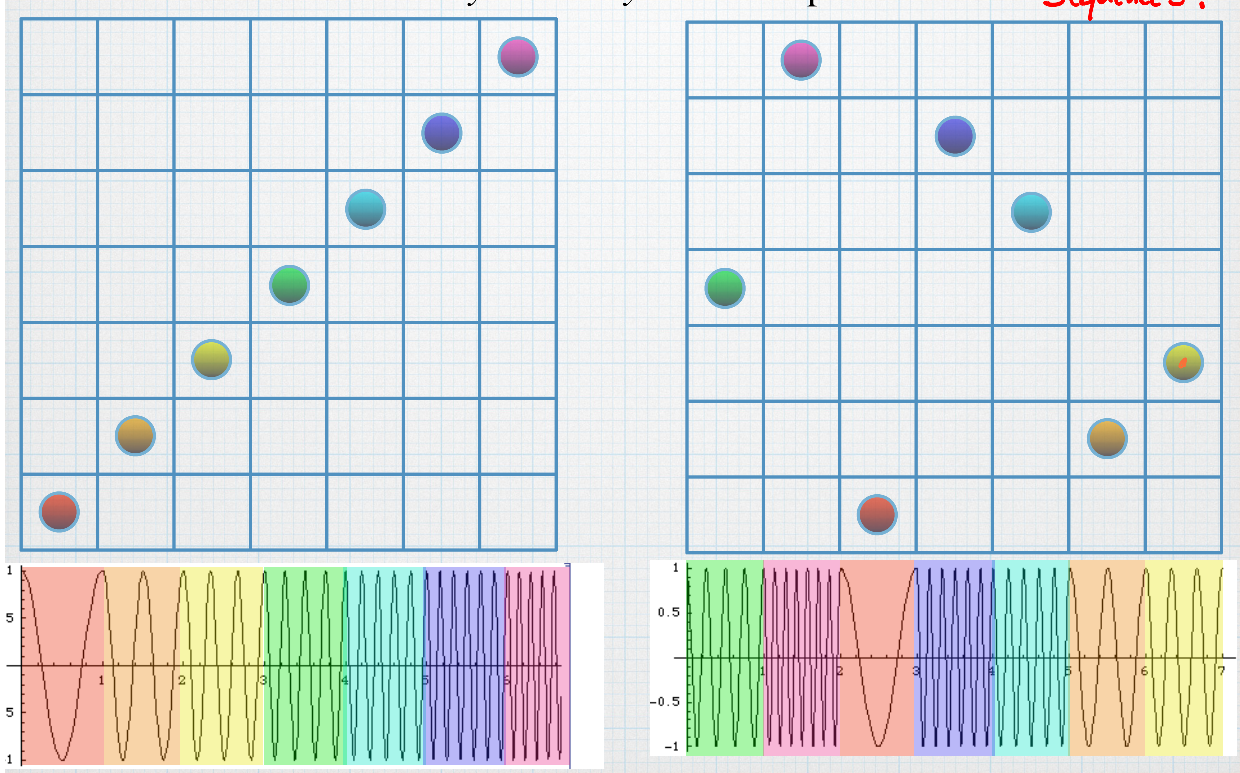
Session 35

Recall...

Frequency-Coded Waveforms

Geometric Array or *Binary Matrix* Representation

Costas sequences!



Recall...

Pushing Sequences:

A new class of Frequency-Coded Waveforms for Use in Adaptive Waveform Radar

Chieh-Fu Chang and Mark R. Bell, "Frequency-coded Waveforms for Enhanced Delay-Doppler Resolution," *IEEE Transactions on Information Theory,* vol. 49, no. 11, Nov. 2003, pp. 2960–2971.

Recall...

The Ambiguity Function of Frequency-Coded Waveforms

The ambiguity function of $s(t) = \sum_{l=0}^{N-1} p(t - lT) e^{-j2\pi\Omega_l t}$ is

$$\chi_s(\tau,\nu) = \underbrace{\chi_s^{(1)}(\tau,\nu)}_{\text{main lobe}} + \underbrace{\chi_s^{(2)}(\tau,\nu)}_{\text{side lobes}},$$

where

$$\chi_s^{(1)}(\tau,\nu) = \sum_{m=0}^{N-1} e^{-j2\pi m\nu T} e^{-j2\pi\Omega_m\tau} \chi_p(\tau,\nu),$$

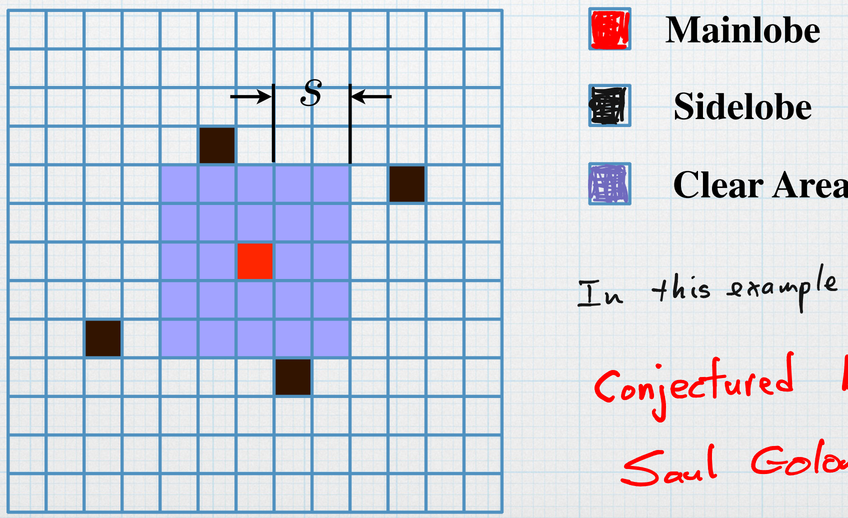
and

$$\chi_s^{(2)}(\tau,\nu) = \sum_{m=0}^{N-1} \sum_{n=0,n\neq m}^{N-1} e^{-j\pi(\Omega_m+\Omega_n)\tau} e^{-j\pi(m+n)T} \cdot \chi_p(\tau + (m-n)T, \nu + (\Omega_n - \Omega_m))$$

Recall...

Pushing Sequences

Definition: For the ambiguity function of a signal $s(t)$, a *clear area* of *size* s is a square area centered at the origin of the (τ, ν)-plane, where $|\tau| \leq sT_r$ and $|\nu| \leq s/T$, such that no sidelobe peaks are present in this area.



In this example, $s = 2$.

Conjectured by Saul Golomb.

Pushing Sequences

Definition: A sequence having the ambiguity function with a clear area of size s is called a *pushing sequence with power* s, where $s \geq 1$.

Any sequence $\{\underline{d}_N\}$ satisfying either $|i - j| > s$ or $|d_i - d_j| > s$ for all i, j, where $0 \leq i, j \leq N - 1$ and $i \neq j$, will have a clear area of size s and is thus a pushing sequence with power s. This property for a frequency coding sequence is called the *pushing property*.

We are interested in pushing sequences that efficiently fill the geometric array.

Recall...

Constructing Pushing Sequences

(*Lemma:* A Costas sequence derived from the Lempel T_4 construction is a pushing sequence of power 1.)

Boring!

Lee codes can be used to construct pushing sequences.

An *r*-error- correcting Lee code is a length 2 code having close-packed codewords in the geometric representation plane.

The *Lee metric* between codewords must be at least 2*r*+1.

Such codes exist for all positive *r*.

Recall...

Constructing Pushing Sequences

Theorem: For every positive integer r, the codewords $\{(k, (2r \oplus 1)k)\}$ form a close-packed r-error correcting dictionary in the Lee metric, where $k = 0, 1, 2 \ldots N-1$, $N = 2r^2 + 2r + 1$ and $\oplus$ represents addition modulo N. In that case, the Lee metric between each pair of codewords is at least $2r + 1$.

Theorem: If the hits exist at $(i, (2r \oplus 1)i)$ in the geometric array of $\{\underline{d}_N\}$, where $i = 0, 1, 2 \ldots N-1$, $N = 2r^2 + 2r + 1$, r is a positive integer and $\oplus$ represents addition modulo N, then $\{\underline{d}_N\}$ is a pushing sequence with power r.

So the geometric array of a pushing sequence of power *r* is given by the corresponding Lee Code and can be easily constructed.

Furthermore ...

Sidelobe Locations and Heights

Theorem: For a Lee pushing sequence with power r, the level of the sidelobe peak at

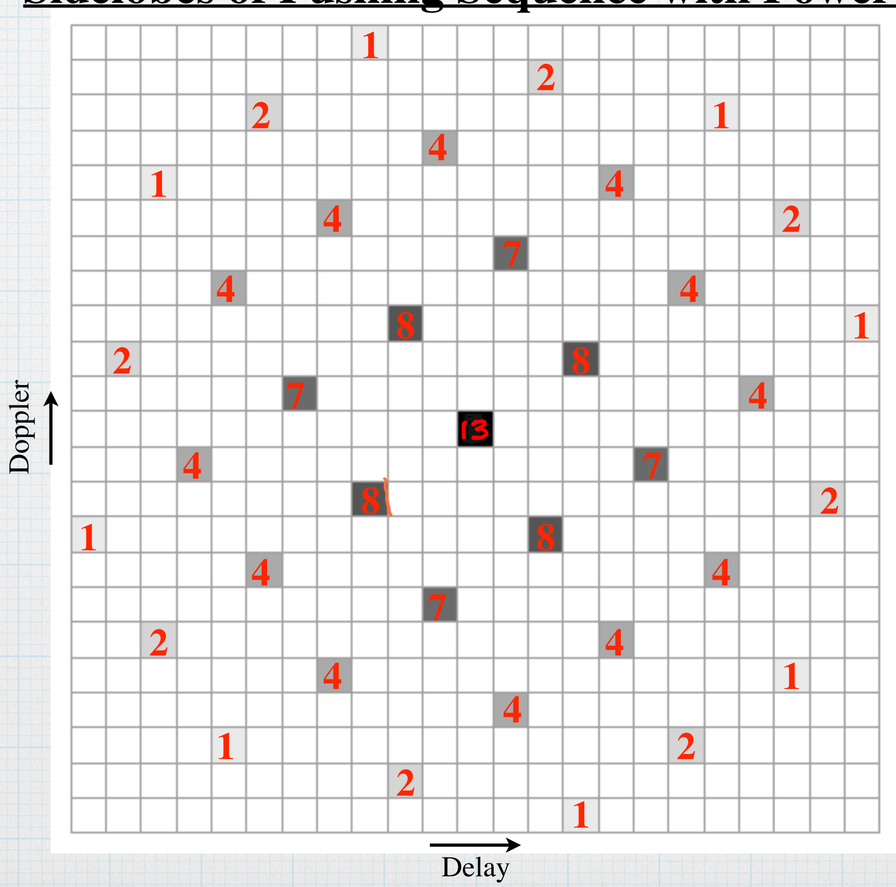
$$(\tau, \nu) = k_1 V_1 + k_2 V_2,$$

where k_1 and k_2 are integers, $V_1 = (r+1, r)$ and $V_2 = (r, -(r+1))$, is given by

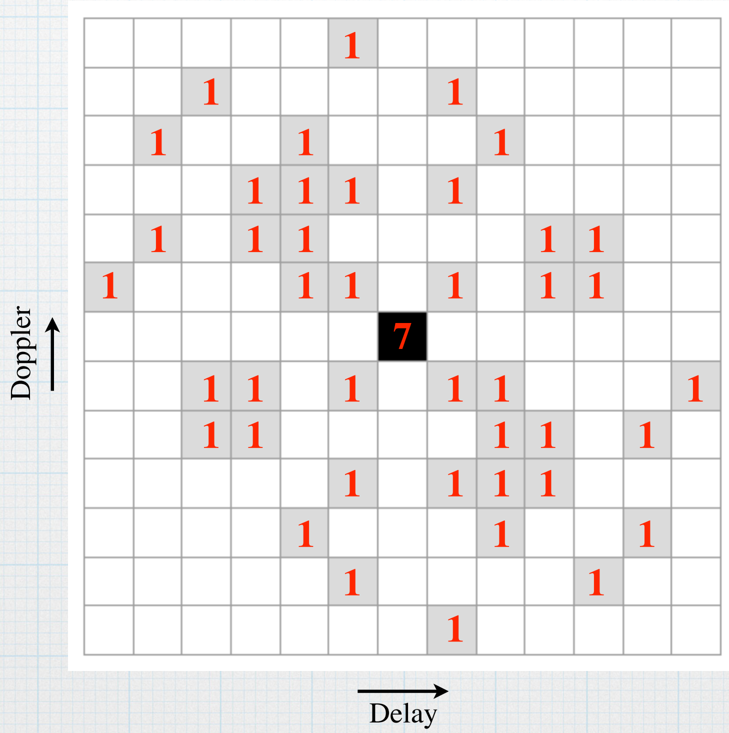
$$l(k_1, k_2) = \left\lfloor \frac{(2r+1-|k_1+k_2|)(2r+1-|k_1-k_2|)}{2} \right\rfloor$$

when $|k_1|, |k_2| \leq (2r-1)$ and $|k_1| + |k_2| \leq 2r$, and 0 otherwise. Furthermore, these are the only sidelobes.

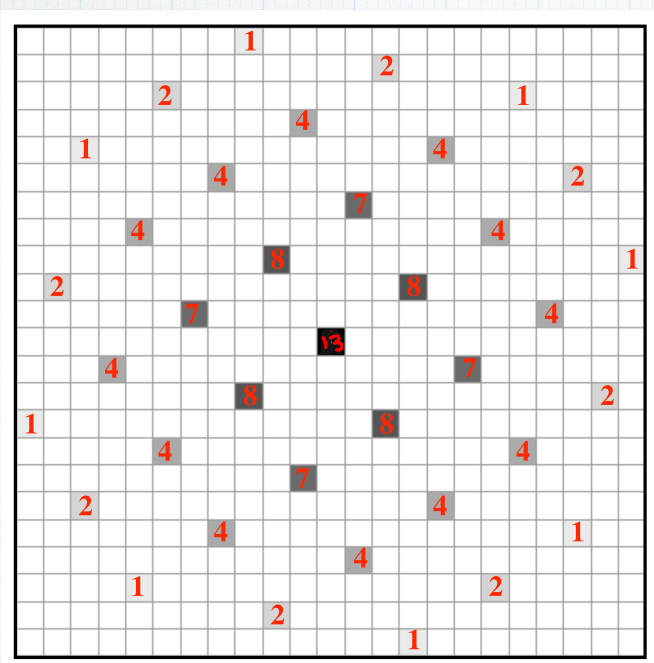
Sidelobes of Pushing Sequence with Power 2

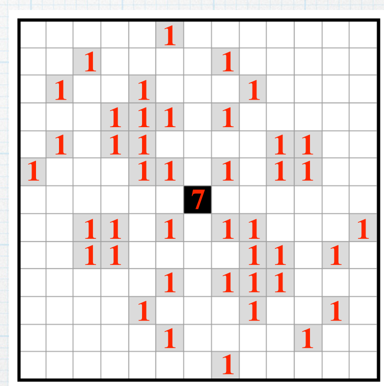


Sidelobes of a Length 7 Costas sequence

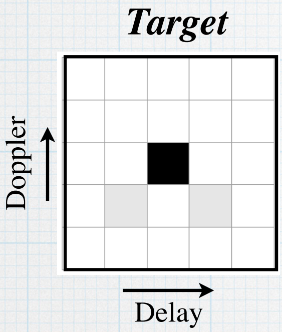


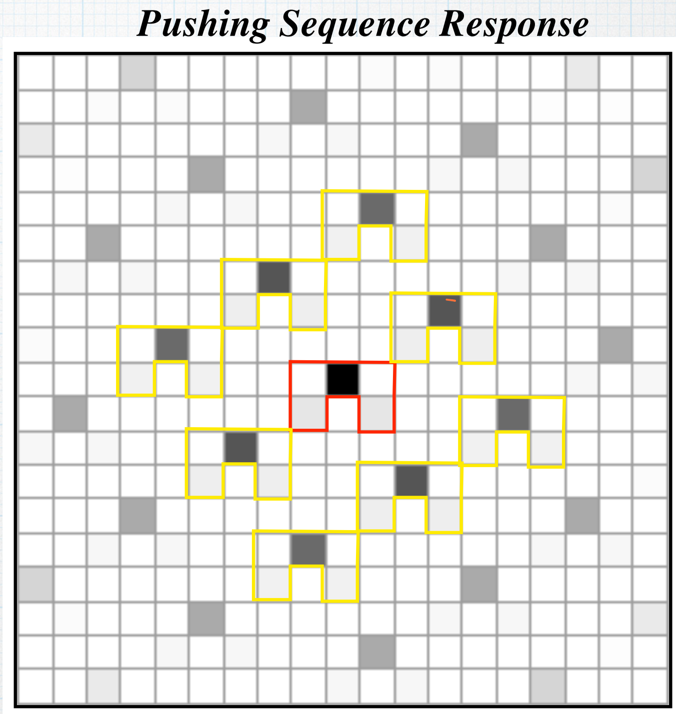
Comparison of *Pushing* and *Costas* Sidelobe Matrices

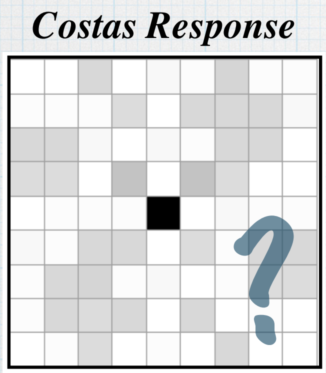




Waveform Response to a Target

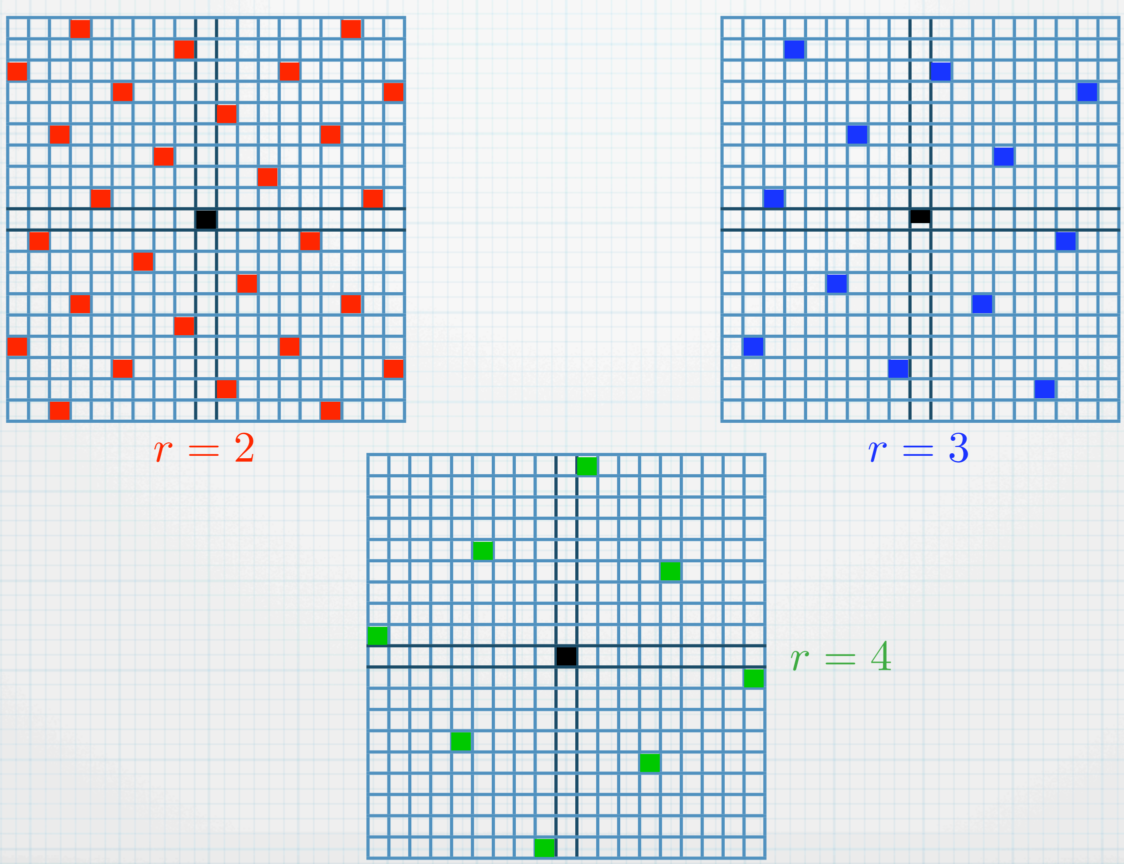




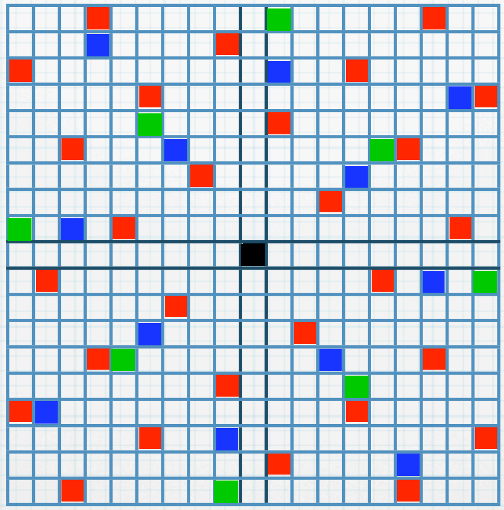


Of course you must know that there are no targets present at another sidelobe location

Sidelobe Distributions Available for Power r = 2, 3 and 4



Sidelobe Distributions Available for Power *r* = 2, 3 and 4



$r = 2$
$r = 3$
$r = 4$

Sequence Length *N* Increases with Power *r*

$$N = 2r^2 + 2r + 1$$

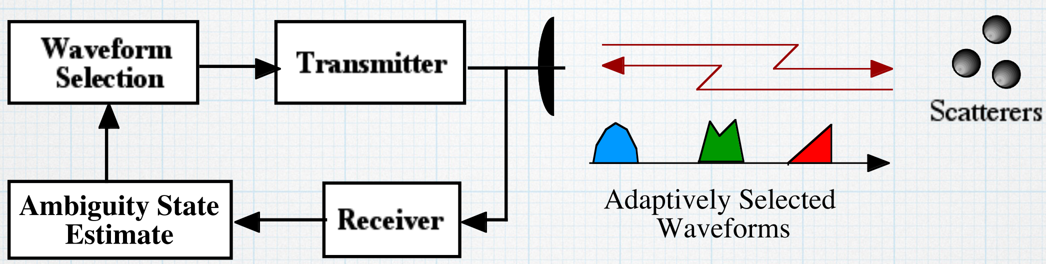
r	N
2	13
3	25
4	41
5	61
6	85

Increasing *r* significantly increases *N*, and hence the total waveform duration, total bandwidth, and time bandwidth-product.

Summary of Pushing Sequence Characteristics

- Frequency coded waveforms that are easy to generate and process.
- Mainlobe of ambiguity function surounded by clear area of arbitrarily large size (power)
- Large sidelobes located on a regular lattice outside of clear region.
- Sidelobe locations and sizes completely determinable.
- Need to know targets are not present at sidelobes.

Adaptive Waveform Radar



- We need waveforms that are easy to generate and transmit.
- We need waveforms for which we can estimate ambiguity functions and the inverse problem of ambiguity state.
- Frequency-coded waveforms seem like a good choice.

Adaptive Waveform Radar

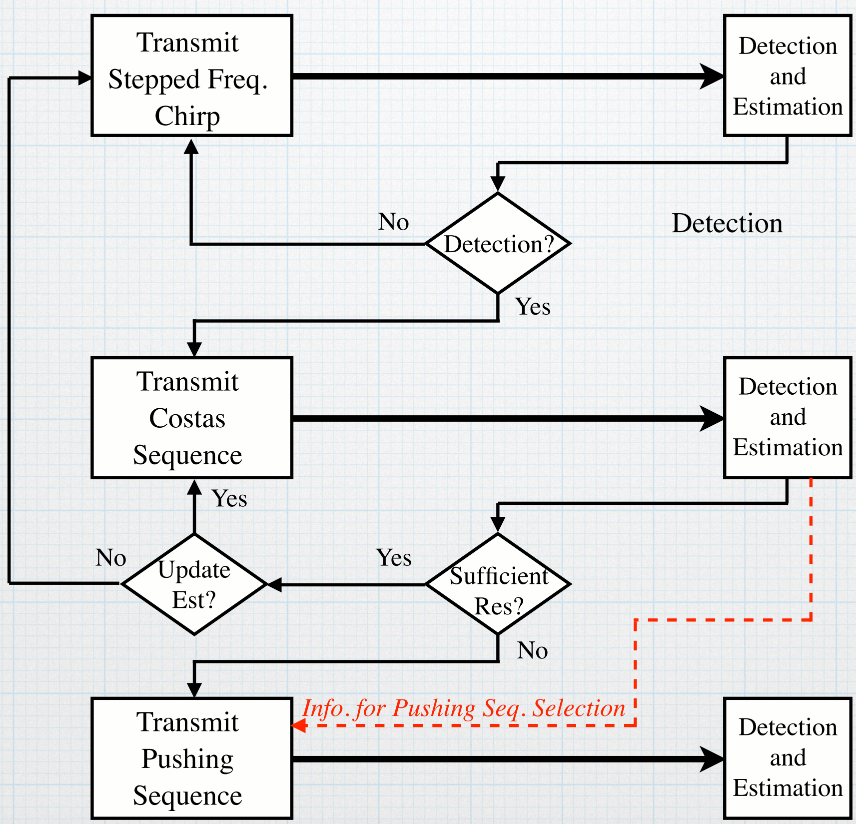
1. Transmit *frequency coded stepped chirp* for Doppler tolerant detection.

2. If a target is detected, transmit a *Costas sequence* for a high-resolution delay-Doppler measurement.

3. If higher resolution is required for small targets that may be masked is Costas sidelobes, transmit *pushing sequence*

   - Target locations from Costas sequence measurement are needed for appropriate *pushing sequence* selection.

Adaptive Waveform Radar



Phase Coded Waveforms

Phase-Coded Waveforms

If for a coded waveform

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t - nT) \exp\{i2\pi d_n t/T\} \exp\{j\phi_n\},$$

where

$$p(t) = 1_{[0,T]}(t),$$

we take

$$d_0 = d_1 = d_2 = \cdots = d_{N-1} = 0,$$

we get

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t - nT) \exp\{j\phi_n\}.$$

Such a signal is called a *phase-coded waveform.*

Such a waveform is characterized by the set of phases

$$\{\phi_0, \phi_1, \phi_2, \ldots, \phi_{N-1}\}.$$

- There are a number of interesting Phase-Coded Waveforms. We will look at two:
  - Maximal Length *Linear Feedback Shift Register* (LFSR) sequences McEliece, Finite Fields for Computer Scientists and Engineers
  - Complementary Sequences