Session 33

Coded Radar Signals

Reference: N. Levanon and E. Mozeson, ***Radar Signals***, Wiley, 2004 (ISBN 0-471-47378-2)

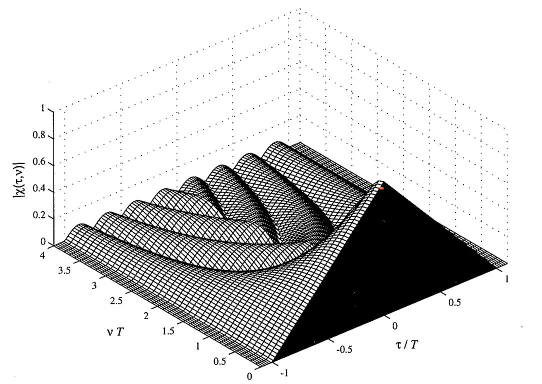
- We will consider coded radar "pulses" as opposed to pulse trains.
- We have already seen that modulation of a radar pulse (e.g. a "chirp") increases the range resolution of the signal through bandwidth expansion.
- Modulation of a radar pulse can significantly modify its ambiguity function (e.g. the "sheared" ambiguity function generated by a chirp)
- We want to consider intelligent approaches to modulating waveforms
- Coded waveforms provide a structured approach to designing waveforms.

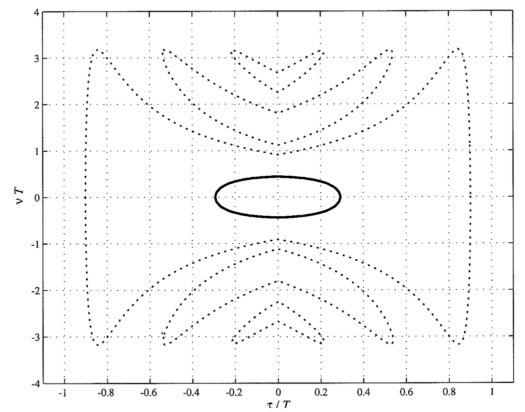
Rectangular Pulse

$$s(t) = 1_{[0,T]}(t).$$

$$\beta_s(\tau,\nu) = e^{-i\pi(T+\tau)}(T-|\tau|)\frac{\sin \pi\nu(T-|\tau|)}{\pi\nu(T-|\tau|)} \cdot 1_{[-T,T]}(\tau$$

\ plot of $|\beta_s(\tau,\nu)|$ for a pulse of duration $T=1$ appear as follows





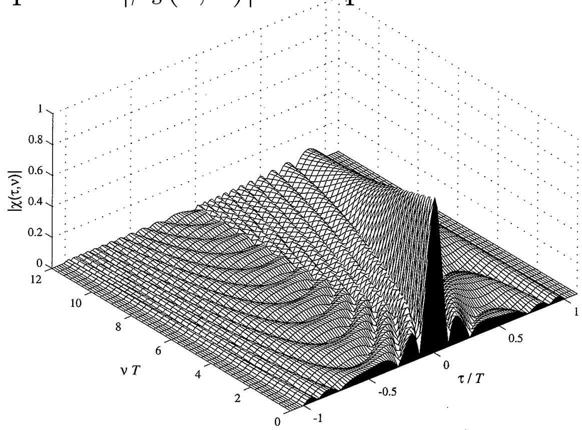
Linear FM Chirp Pulse

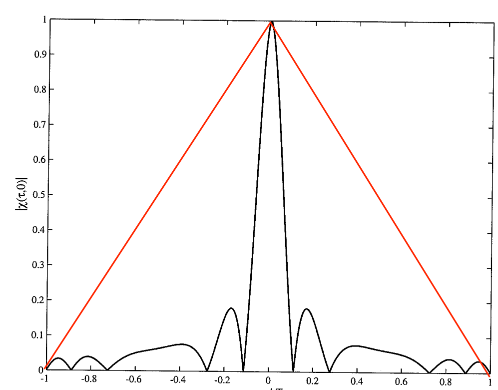
$$v(t) = e^{i\pi\alpha t^2} \cdot 1_{[0,T]}(t),$$

where $\alpha \in \mathbf{R}$.
Then

$$\beta_v(\tau,\nu) = e^{-i\pi(T+\tau)}(T-|\tau|)\frac{\sin \pi(\nu-\alpha\tau)(T-|\tau|)}{\pi(\nu-\alpha\tau)(T-|\tau|)} \cdot 1_{[-T,T]}(\tau).$$

A plot of $|\beta_s(\tau,\nu)|$ for a pulse of duration $T=1$ with $\alpha = 10$ appears as follows:





Coded Radar Signals

A coded waveform $s(t)$ is a signal of the form

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t - nT) \exp\{j2\pi d_n t/T\} \exp\{j\phi_n\},$$

Unit Energy

where

$$p(t) = 1_{[0,T]}(t),$$

$\{d_n\}_{n=0}^{N-1}$ = a sequence of integer frequency modulating indices,

$\{\phi_n\}_{n=0}^{N-1}$ = a sequence of real valued phases,

T = duration of a single waveform "chip,"

NT = total duration of the coded waveform.

We will initially take

$$\phi_0 = \phi_1 = \phi_2 = \cdots = \phi_{N-1} = 0,$$

which will give us a *frequency-coded signal*

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t - nT) \exp\left\{j2\pi \left(\frac{d_n}{T}\right) t\right\}.$$

Frequency-Coded Waveforms

A frequency coded waveform $s(t)$ is a signal of the form

$$s(t) = \sum_{l=0}^{N-1} p(t - lT) e^{-j2\pi\Omega_l t},$$

where

T = *chip duration*,

$p(t) = 1_{[0,T]}(t)$ (chip waveform),

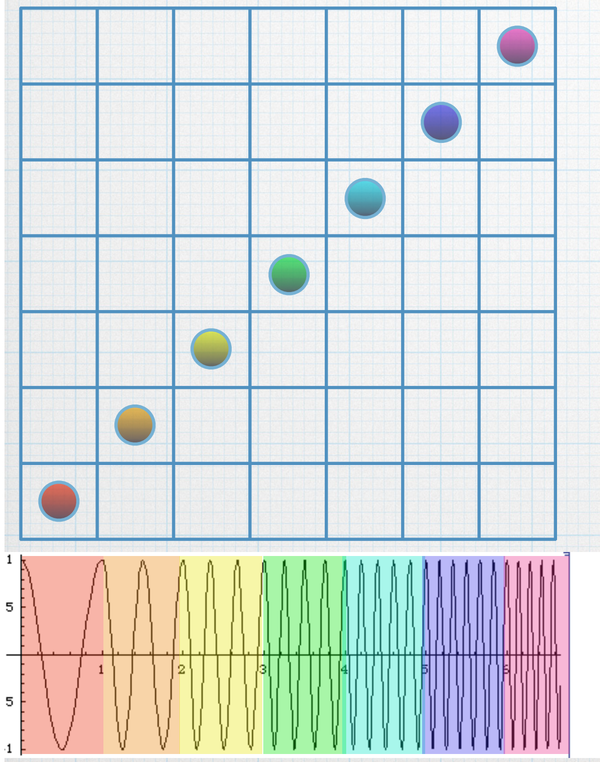
and

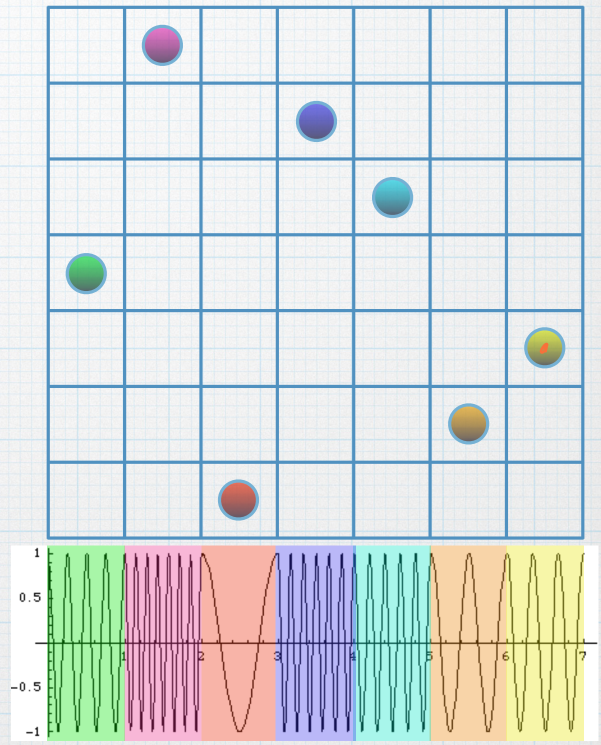
$$\Omega_l = d_l/T, \quad l = 1, 2, \dots N,$$

where $\{d_l\}$ is a permutation of the integers $1, 2, \dots N$.

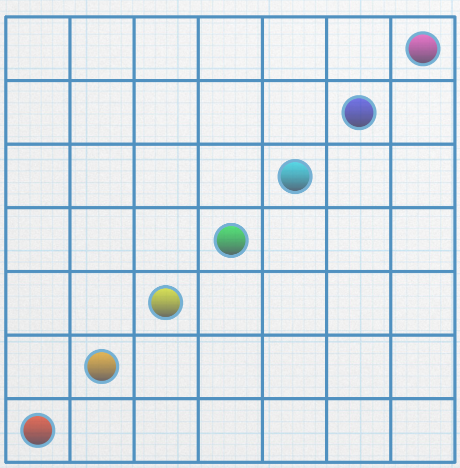
Frequency-Coded Waveforms

Geometric Array or *Binary Matrix* Representation

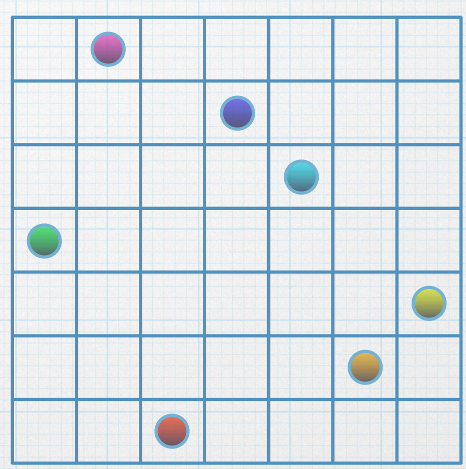




Frequency Coding Matrices for Frequency-Coded Signals



Stepped Frequency Approximation to a Chirp (LFM)



Costas Sequence

The Ambiguity Function of Frequency-Coded Waveforms

The ambiguity function of $s(t) = \sum_{l=0}^{N-1} p(t - lT) e^{-j2\pi\Omega_l t}$ is

$$\chi_s(\tau, \nu) = \chi_s^{(1)}(\tau, \nu) + \chi_s^{(2)}(\tau, \nu),$$

where $\chi_s(\tau,\nu) \triangleq \int_{-\infty} s(t)\, s^*(t-\tau)\, e^{+i2\pi\nu t}\, dt$

$$\chi_s^{(1)}(\tau, \nu) = \sum_{m=0}^{N-1} e^{-j2\pi m\nu T} e^{-j2\pi\Omega_m \tau} \chi_p(\tau, \nu),$$

and

$$\chi_s^{(2)}(\tau, \nu) = \sum_{m=0}^{N-1} \sum_{n=0, n\neq m}^{N-1} e^{-j\pi(\Omega_m+\Omega_n)\tau} e^{-j\pi(m+n)T} \cdot \chi_p(\tau + (m - n)T, \nu + (\Omega_n - \Omega_m))$$

n.b. $\beta_s(\tau,\nu) = \chi_s(\tau,-\nu)$.

The Ambiguity Function of Frequency-Coded Waveforms

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n.b. $\beta_s(\tau,\nu) = \chi_s(\tau,-\nu)$.

The sidelobes are given by

$$\chi_s^{(2)}(\tau,\nu) = \sum_{m=0}^{N-1} \sum_{n=0,n\neq m}^{N-1} e^{-j\pi(\Omega_m+\Omega_n)\tau} e^{-j\pi(m+n)T} \cdot \chi_p(\tau + (m-n)T, \nu + (\Omega_n - \Omega_m))$$

$$\chi_p(\tau + (m-n)T, \nu + (d_n - d_m)/T)$$

Large contribution when *these* equal zero!

$\tau = (n-m)T$ and $\nu = (d_n - d_m)/T$

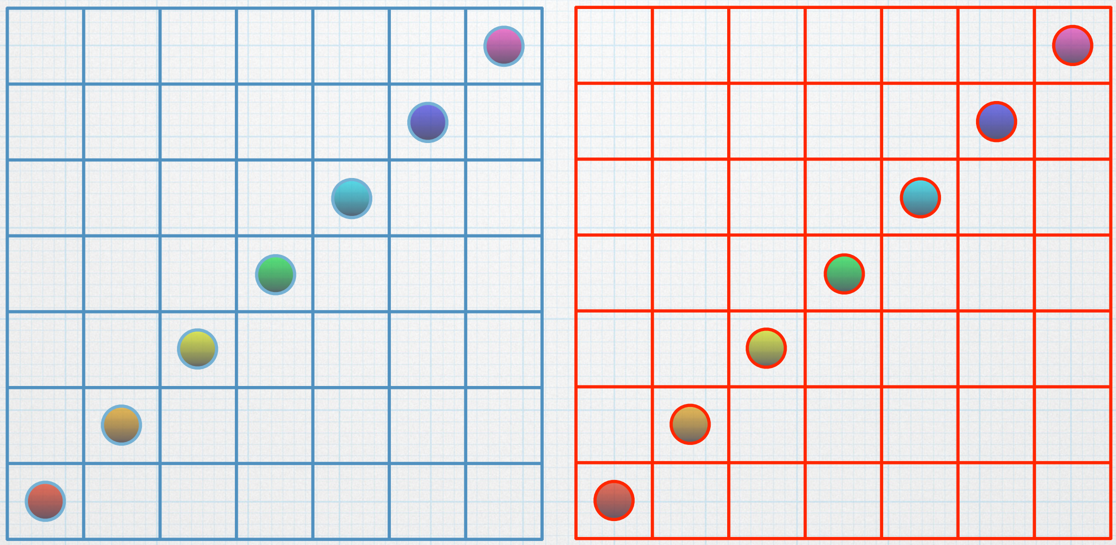
or taking $T = 1$ for simplicity...

$\tau = n - m$ and $\nu = d_n - d_m$

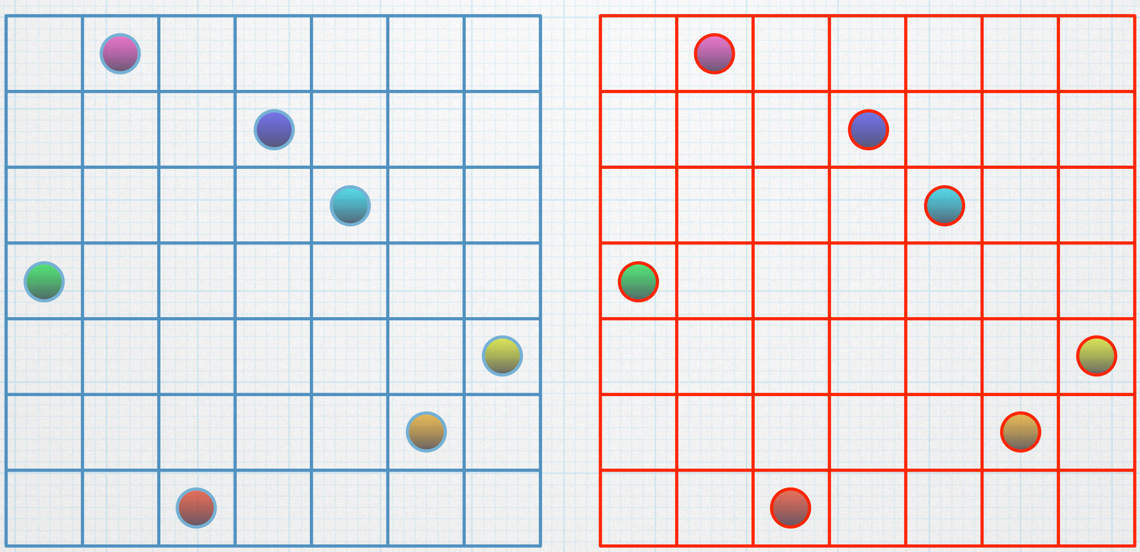
Coincident Sidelobe Approximation

- If we consider only the sidelobe contributions due to the situations where both arguments of the ambiguity function is zero, we want to minimize the number of situations where this occurs.
- We especially want to minimize multiple “hits” for any given delay and Doppler shift.
- While this approach only minimizes an approximation of the ambiguity function sidelobes, it is surprisingly effective.
- It is, in fact, the approach John Costas used in designing Costas sequences.

LFM Chirp Sidelobe Overlay Demo



Costas Sidelobe Overlay Demo



Characteristics of Stepped-Frequency Waveforms

- A wide variety of waveforms with different ambiguity functions can be generated.
- These waveforms can be easily generated and amplified for transmission.
- The ambiguity characteristics of these waveforms can be easily visualized because of their localization in time and frequency.
- Provides a straightforward approach to characterizing “ambiguity state” of a target environment.
- These characteristics make them ideal for adaptive waveform radar.