Session 30

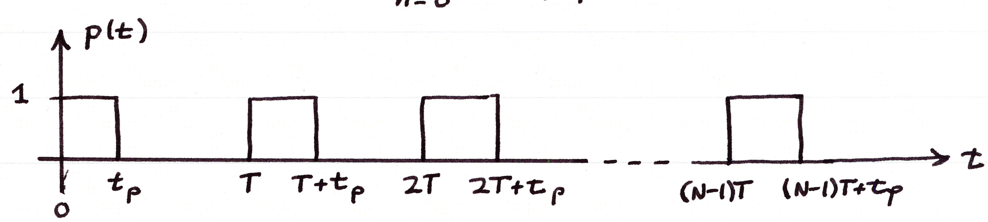
Processing a Coherent Pulse Train

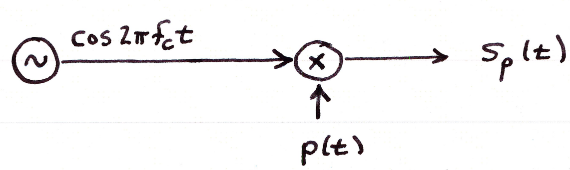
Assume a radar transmits a passband signal of the form

$$s_p(t) = p(t) \cos 2\pi f_c t$$

where

$$p(t) = \sum_{n=0}^{N-1} 1_{[0, t_p]}(t - nT).$$





30.2

The signal reflected back from a point-target with radial velocity $\dot{R}$ and received by the radar is of the form

$$r(t) = p(t) \cos\left[2\pi f_c t + \phi(t)\right].$$

(n.b. we have re-oriented the time origin to account for the round-trip delay)

Here, $\phi(t)$ is due to the Doppler shift:

$$\phi(t) = +2\pi f_D t = -2\pi \frac{2\dot{R}}{\lambda} t.$$

$f_D = \frac{2v_r}{\lambda}$ (monostatic)

n.b. We are assuming $p(t)$ is narrowband w.r.t. carrier f_c and that time dialation/compression of $p(t)$ is negligible.

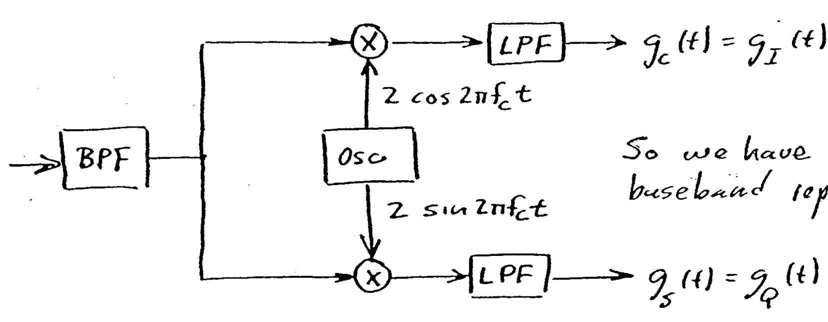
Narrowband Assumption

30.3

We would like to find the complex baseband representation of the received signal *r(t)* via processing:

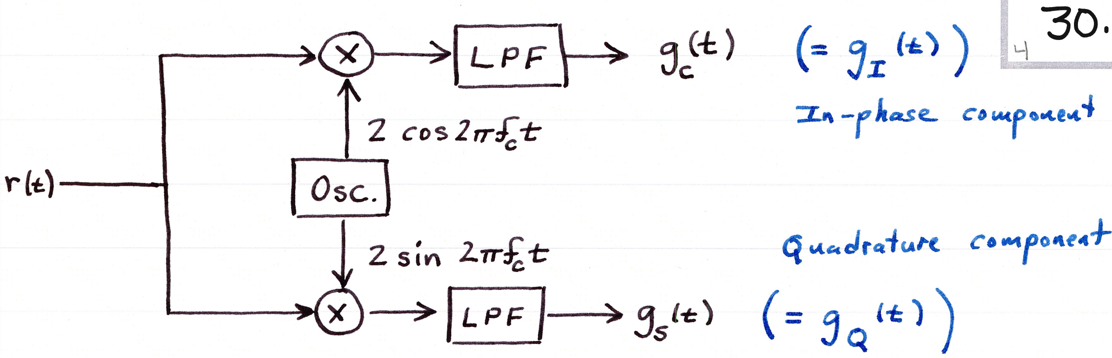
- Pass the signal through a bandpass filter that passes virtually all of the signal, but little outside of the signal's frequency band.

- Use an I/Q demodulator to get the in-phase and quadrature components.



So we have a complex baseband representation

30.4

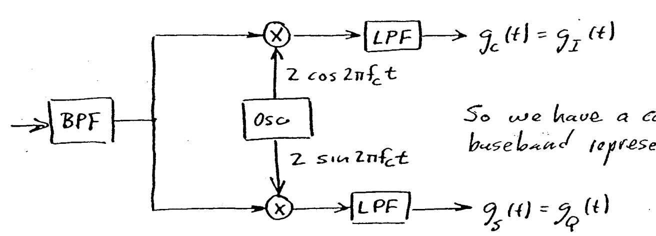


$$g_c(t) = p(t)\cos\phi(t)$$
$$g_s(t) = p(t)\sin\phi(t)$$

Now consider the complex envelope function

$$u(t) = g_c(t) + i\,g_s(t) = p(t)\cos\phi(t) + i\,p(t)\sin\phi(t)$$
$$= p(t)\left[\cos\phi(t) + i\sin\phi(t)\right]$$
$$= p(t)\,e^{i\phi(t)}.$$ (By Euler's identity)

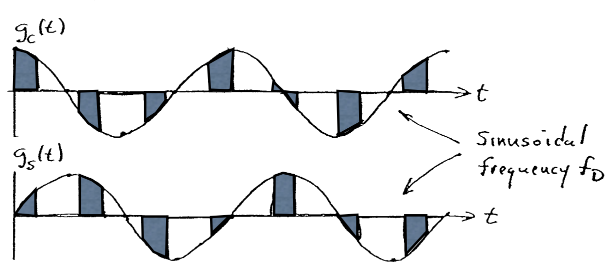
30.5

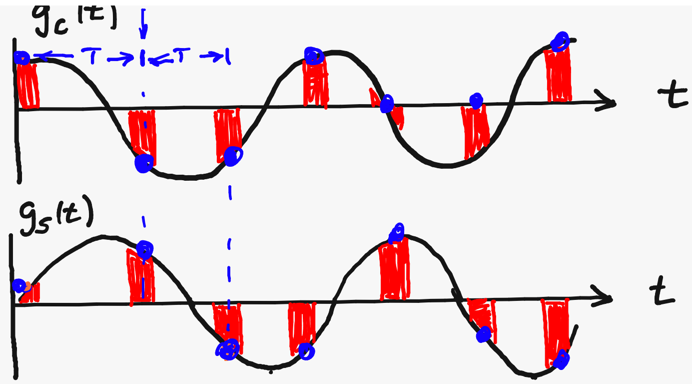


So we have a complex baseband representation

$$g_c(t) = p(t)\cos\phi(t)$$
$$g_s(t) = p(t)\sin\phi(t)$$

For $\phi(t) = 2\pi f_D t$ as in our case, the two signals appear as follows:





Now we can form the complex envelope function

$$u(t) = g_c(t) + i\, g_s(t)$$

If we sample $u(t)$ at sampling instants $t_0, t_1, \dots, t_{N-1}$ such that

$$t_n - t_{n-1} = T \quad , n = 1, \dots, N-1$$

with t_0 selected within the first pulse $(0 < t_0 < t_p)$ we get one complex sample from each pulse return

Note that if we define the complex sequence

$$u_n = u(t_n) = u(t_0 + nT), \; n = 0, 1, \dots, N-1$$

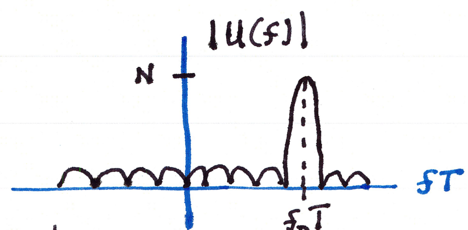
and compute the DTFT of $\{u_n\}$, we get

$$U(f) = \sum_{n=0}^{N-1} u_n e^{-i2\pi nTf} = \sum_{n=0}^{N-1} e^{i2\pi f_D (t_0 + nT)} e^{-i2\pi nTf}$$

$$= e^{i2\pi f_D t_0} \sum_{n=0}^{N-1} e^{-i2\pi (f - f_D) nT}$$

For $f = f_D \Rightarrow |U(f)| = N$,

For $f \not\approx f_D \Rightarrow |U(f)| \cong 0$.



n.b where we pick t_0 determines the delay to the target. (t_0 = delay to first pulse return.)

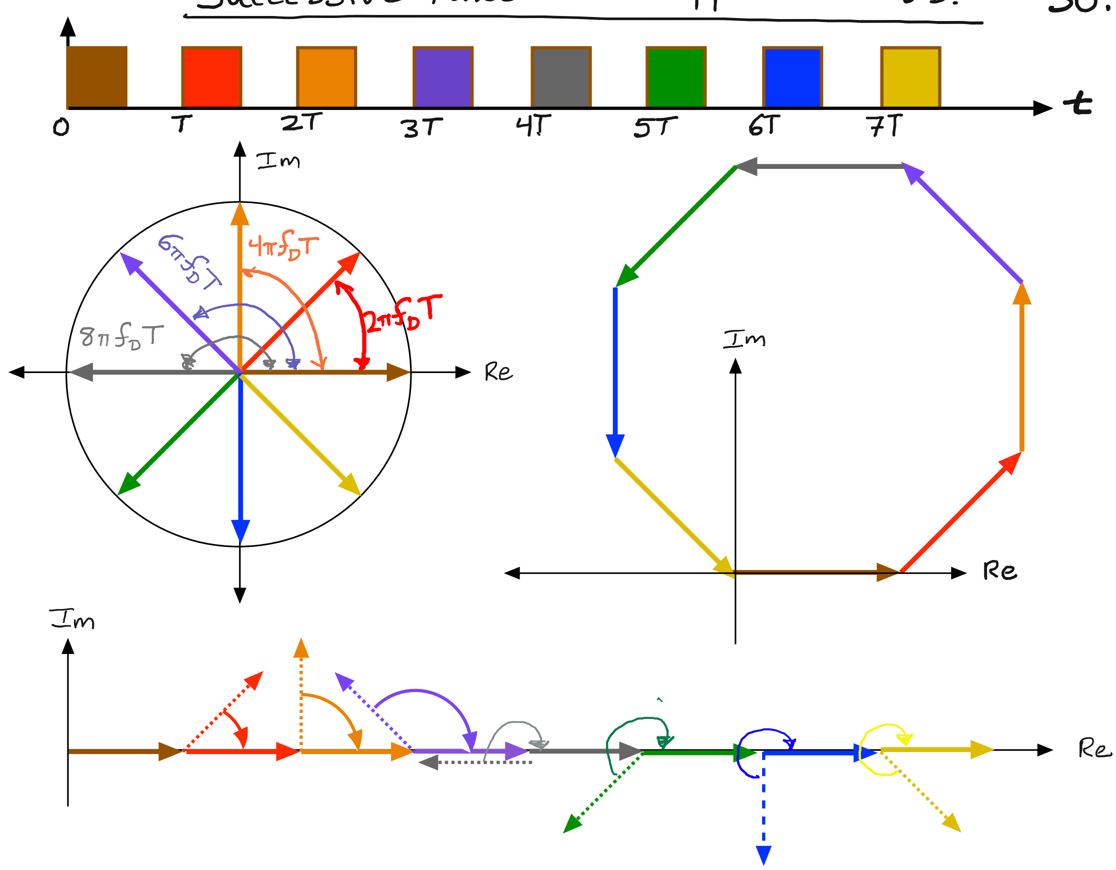
We could calculate the DFT of the sequence u_n:

$$U_k = \sum_{n=0}^{N-1} u_n \, e^{-i\frac{2\pi nk}{N}}, \quad k = 0, 1, \ldots, N-1.$$

Here again, we would get a large magnitude when $\frac{k}{N} \simeq T f_D$:

$$U_k = \begin{cases} N \cdot e^{i 2\pi f_D t_0}, & k \doteq N T f_D \\ 0, & \text{elsewhere.} \end{cases}$$

Successive Pulses with Doppler Shift f_D.



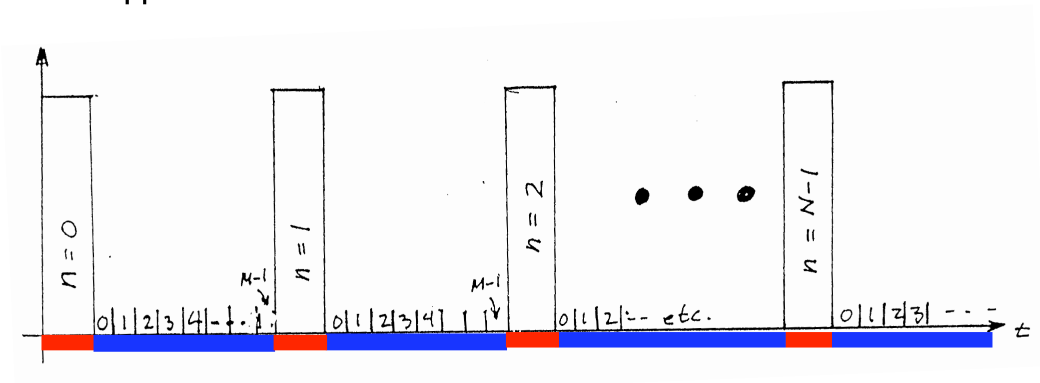
So for ease of implementation, the DFT (computed using an FFT algorithm) is commonly used to determine the Doppler of a target located at delay t_0 corresponding to the initial sampling time t_0.

This is the beginning (and main idea) on which we build the delay-Doppler processor.

The Pulse Waveform Delay-Doppler Processor

In general, we are interested in looking at the returns from all possible ranges (delays).

The situation appears as follows:



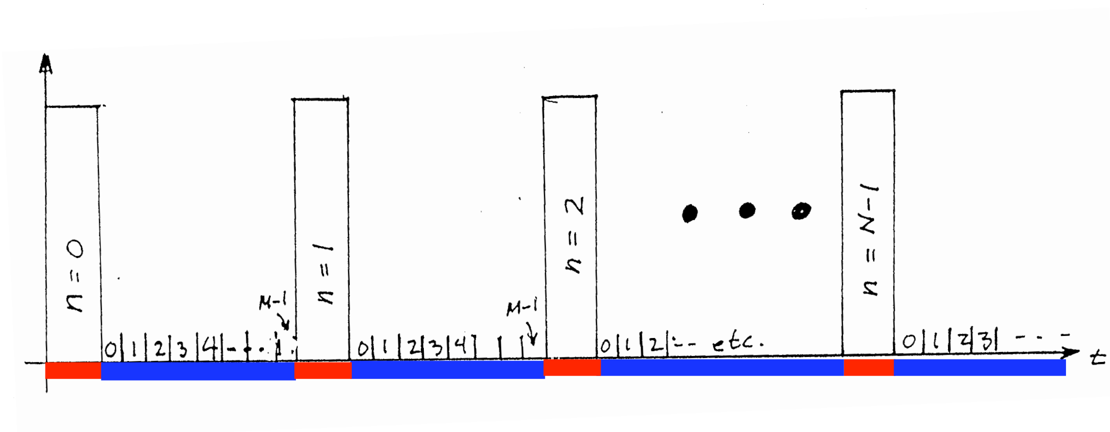
Because we transmit finite-bandwidth signals, we only have limited range or delay resolution.

Recall what $\beta_s(\tau, \nu)$ looks like.

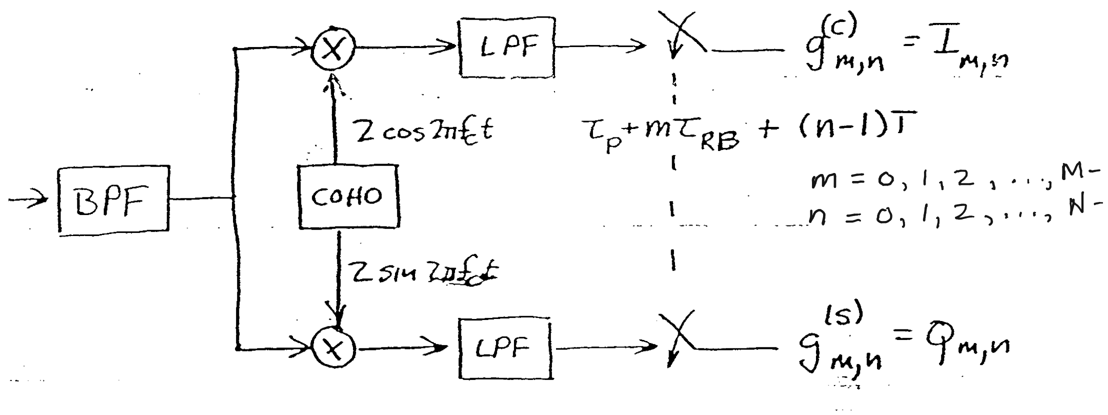
So we will divide the listening period into bins of duration τ_{RB}.

So we will divide the listening period into time bins of width τ_{RB} and sample the signal once per bin for each range bin of interest.

Label the range bins $0, 1, \ldots, M-1$.



We can get the sample returns from each of the range bins (both I and Q components to represent the complex baseband signal) by processing the received signal as follows:



n.b., τ_p is the transmitted pulse duration.

Once we have carried out this procedure, we can think of the I and Q samples $\{I_{m,n}, Q_{m,n}\}$ as being arranged in an array as follows:

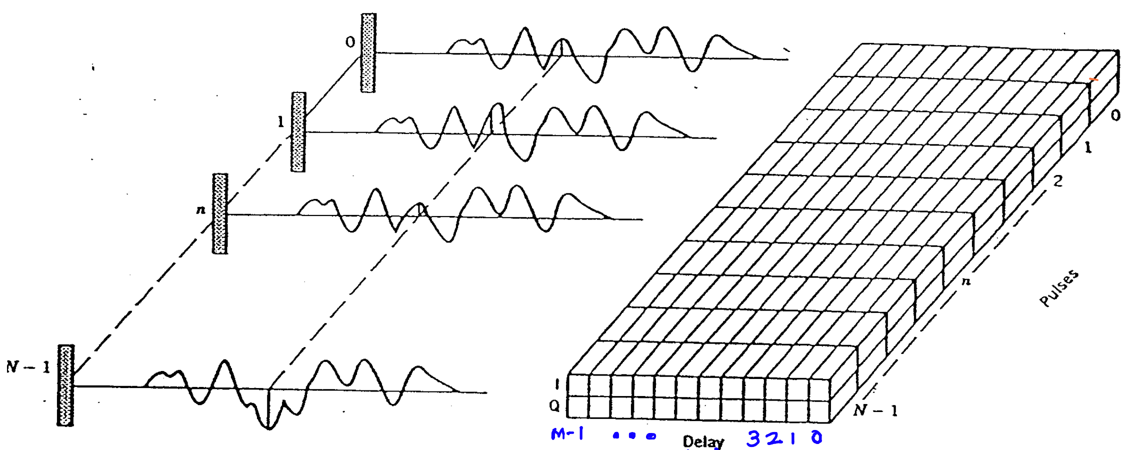
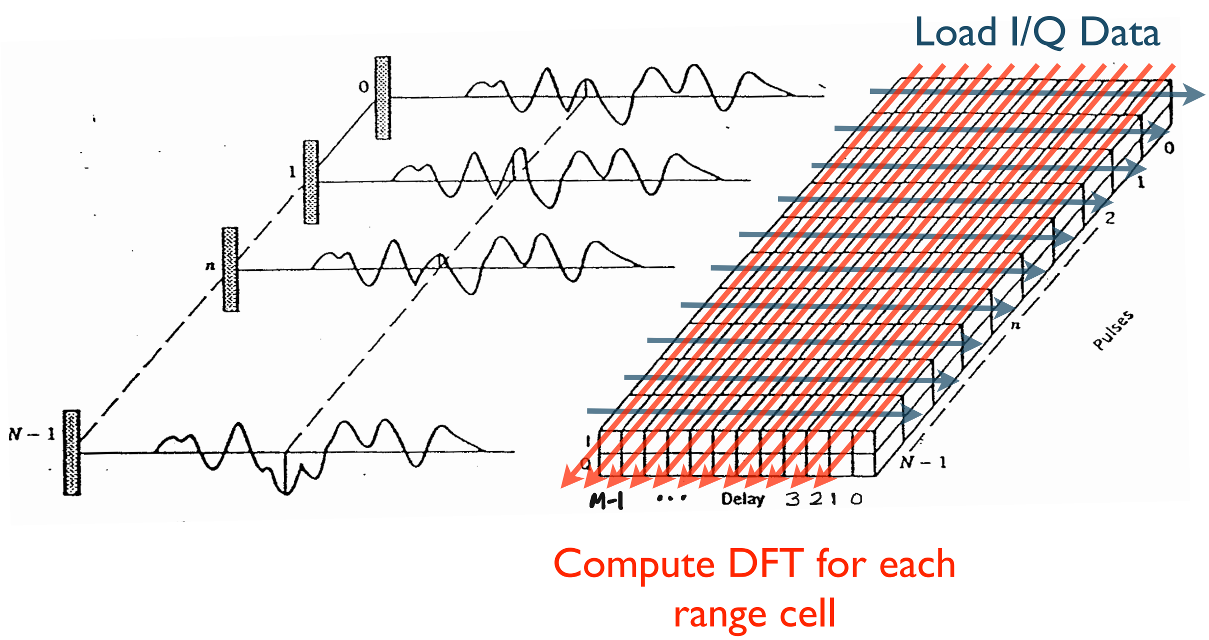


Figure 10.3 A conceptual diagram of the accumulation of I & Q samples at various delays, from N consecutive pulses.

(From: N. Levanon, *Radar Principles*, Wiley, 1987)



Load I/Q Data
0
1
n
N − 1
0
1
2
n
N − 1
Pulses
1
0
M-1
...
Delay
3 2 1 0
Compute DFT for each range cell