Session 3

For a sinusoidal signal of cyclic frequency f_o, the Doppler shift is 3.1

$$f_D = \frac{2 v f_o}{c} = \frac{2v}{\lambda}$$

where

$$\lambda = \frac{c}{f_o} = \text{wavelength}.$$

- For signals of a single frequency, the Doppler effect corresponds to a shift in frequency.
- Doppler shift is proportional to carrier frequency and velocity.

Recall... In general 3.2

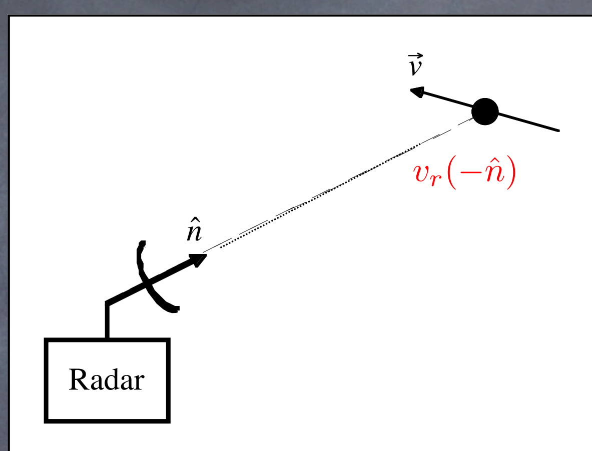
In general, if $s(t) \overset{\mathcal{F}}{\Leftrightarrow} S(\omega)$

$$\sqrt{\alpha}s(\alpha t) \overset{\mathcal{F}}{\Leftrightarrow} \frac{1}{\sqrt{\alpha}} S\left(\frac{\omega}{\alpha}\right)$$

- This is a scaling in frequency, not a frequency shift.
- But for narrowband signals and v<<c, the narrowband approximation is good.
- In sonar, the narrowband approximation is often bad.

3.3

Radial Velocity and Doppler

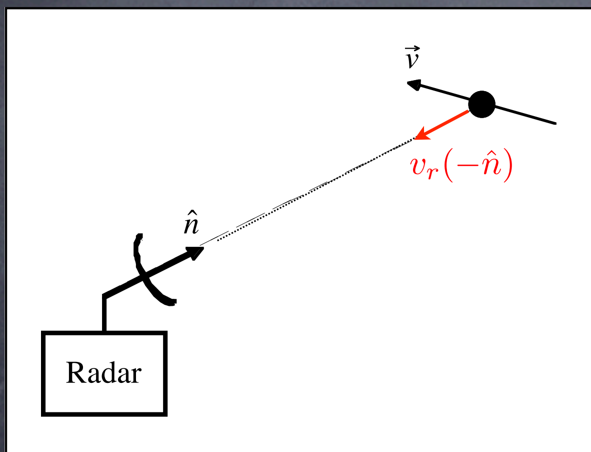


- It is the radial component of the velocity that determines the Doppler effect.
- The radial velocity is the velocity

3.4

Radial Velocity and Doppler

$R(t) = \text{Range of the target from antenna at time } t$



$$v_r = -\dot{R}(t) = -\frac{dR(t)}{dt}$$

Negative sign defines radial velocity directed at radar.

$$f_D = \frac{2 v_r f_c}{c} \quad \Longrightarrow \quad v_r = \frac{c f_D}{2 f_c}$$

3.5

Radial Velocity and Doppler

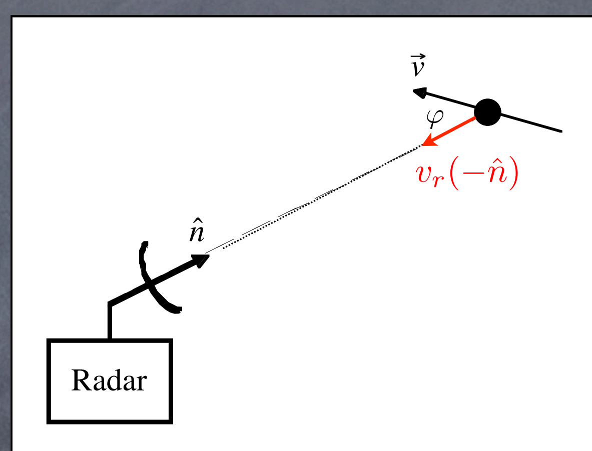
$$v_r = -\hat{n} \cdot \vec{v}$$

$$\Downarrow$$

$$f_D = \frac{-2(\hat{n} \cdot \vec{v}) f_c}{c}$$

In general

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\varphi$$



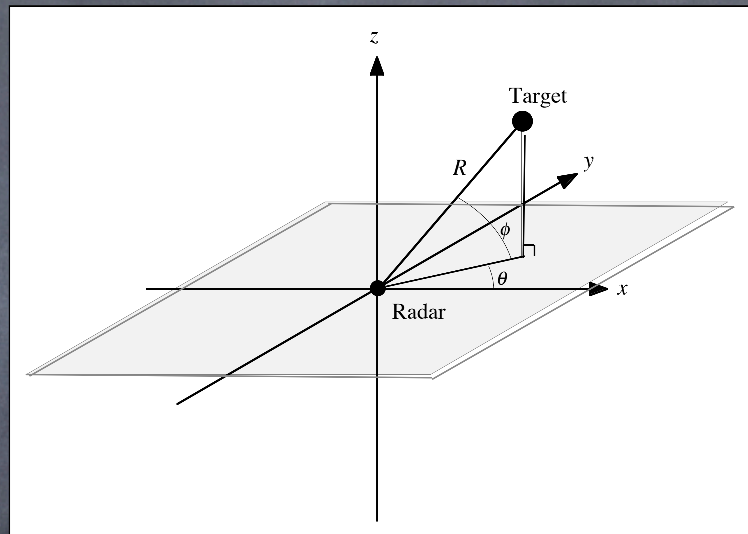
$$\Longrightarrow \quad f_D = \frac{-2|\vec{v}| f_c \cos\varphi}{c}$$

"cosine effect"

3.6

Azimuth and Elevation Angle Measurement

Radar measures azimuth and elevation angles using antenna



ϕ = elevation angle

θ = azimuth angle

R = range to target

$(R, \theta, \phi) \sim$ spherical coordinates

$(x, y, z) \sim$ cartesian coordinates $\Longrightarrow$

We can easily translate:

$$x = R\cos\theta\cos\phi$$
$$y = R\sin\theta\cos\phi$$
$$z = R\sin\phi$$

3.7

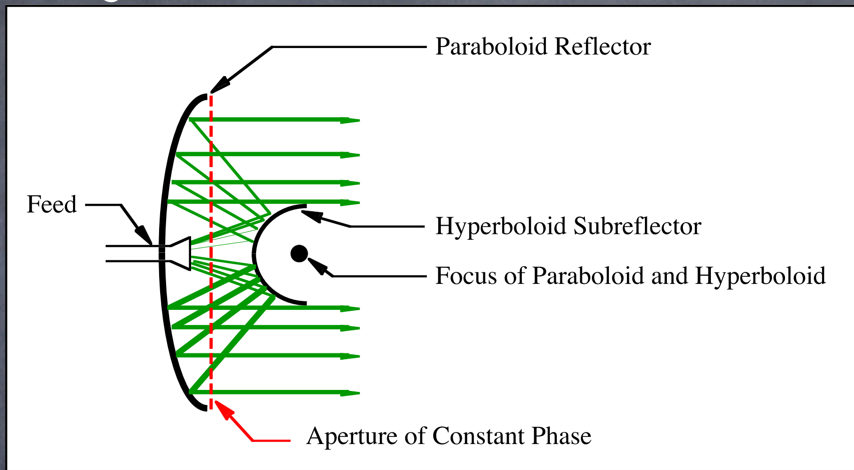
Antennas and the Transmission of EM Energy

- Ability of a radar to make useful measurements is ultimately limited by the amount of energy scattered and collected
- This is a function of
  - > Transmitted Energy
  - > Target Scattering Characteristics
  - > Transmit and Receive Antenna Characteristics

3.8

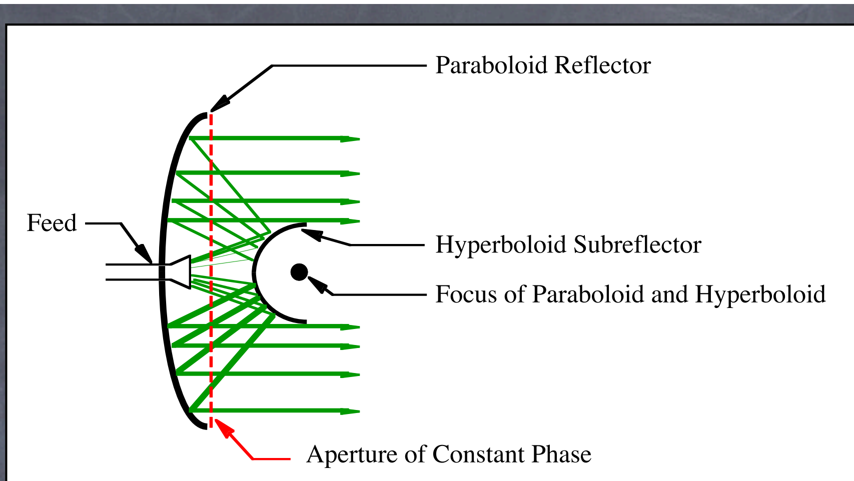
Typical Microwave Antenna

Cassagranian Reflector Antenna (1-30 GHz):



- Path length from Feed to aperture is constant regardless of particular path.
- Distance to distant point on axis to feed is constant regardless of particular path.

3.9

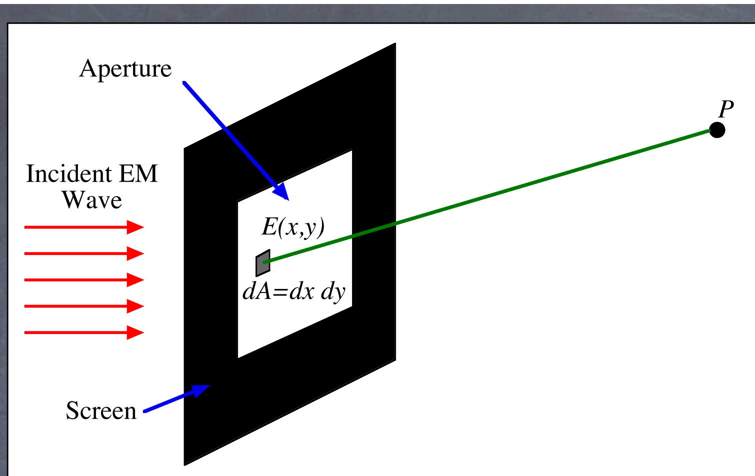


- Outgoing field crossing the aperture of the paraboloid is in phase at all points on the aperture.
- This is the condition required for focusing energy in a narrow beam (Fourier

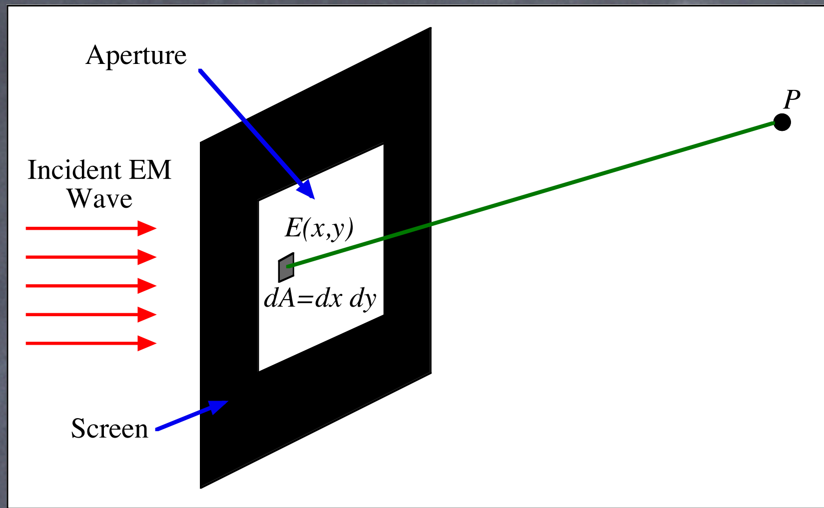
3.10

- <u>In General, for Apertures that are In-Phase</u>
- On Transmit: Greater the concentration of power, and the larger the power density.
- On Receive: Greater the collection area and thus the greater the power collected.
- Reciprocity Theorem states that these two facts are basically the same thing.

3.11

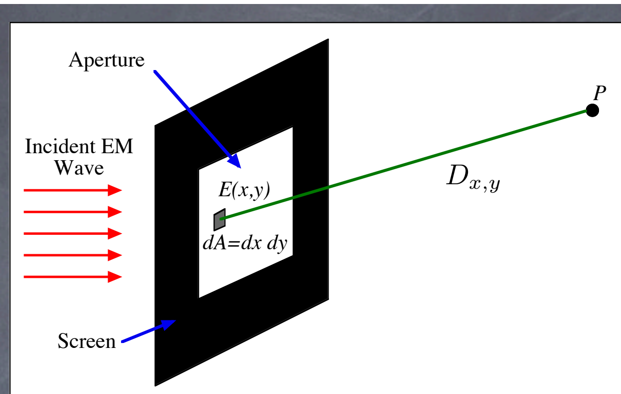


- Consider a plane wave normally incident on an aperture A
- Assume Electric field (for some fixed polarization) is E(x,y)
- We want field F(P) at point P.



> Huygens's Principle states that we sum the contributions from each differential element dA of the aperture to get the total field at P.

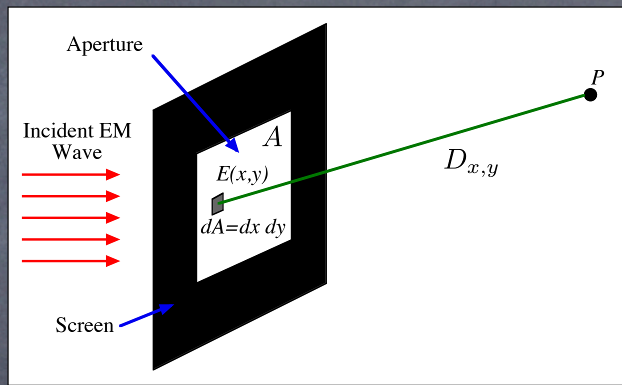
Translating into Mathematics:



$$dF_{x,y}(P) = \frac{C}{D_{x,y}(P)} \exp\left\{-i\frac{2\pi}{\lambda} D_{x,y}(P)\right\} dA$$

$C =$ a constant

The total field F(p) at P is obtained by integrating over the aperture A



$$F(P) = \int\!\!\int_A \frac{C}{D_{x,y}(P)} \exp\left\{-i\frac{2\pi}{\lambda}D_{x,y}(P)\right\} dx\, dy$$

Scalar Diffraction Theory

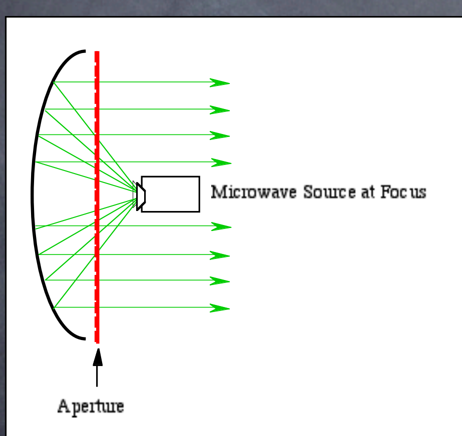
- This approach to calculating a scalar component of a electric field is called scalar diffraction theory.
- Useful because a linearly polarized wave traveling through free-space retains its polarization.
- This allows us to treat EM fields as scalar quantities instead of vector fields.
- Quite accurate for propagation of waves from large apertures at small diffraction

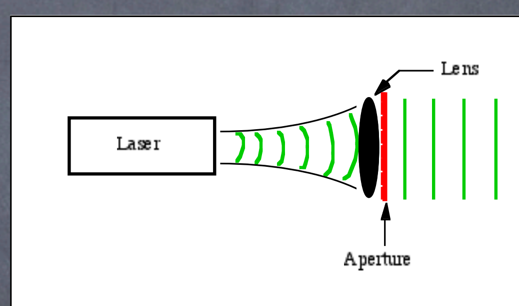
Scalar Diffraction

- Using Scalar diffraction, we can characterize the field arising at a point in space from the the field across the aperture that gave rise to it.
- References on Scalar Diffraction:
- Joseph Goodman, Fourier Optics
- M. Born and E. Wolf, Principles of Optics

Antenna Apertures

An antenna aperture is a surface of constant phase near the “face” of the antenna.





The aperture of an antenna has an area A.

This area characterizes the antenna’s behavior.