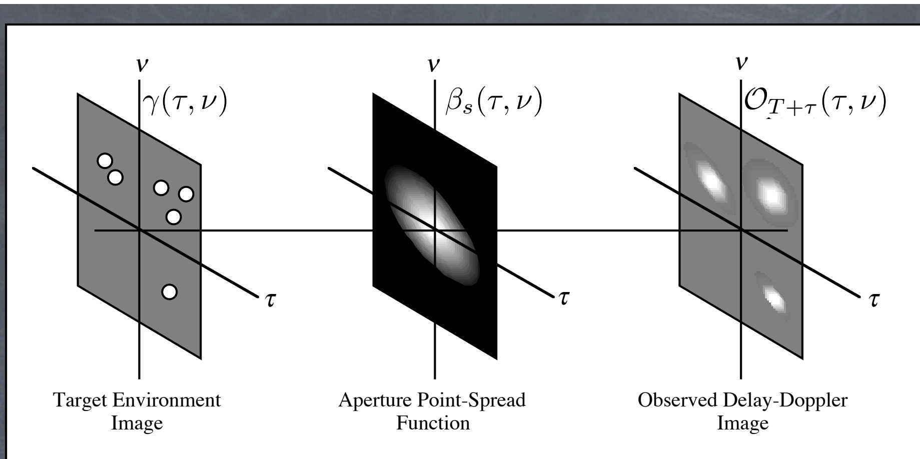
Session 28

Recall... 28.1



- If we think of a radar as an imaging system, then the ambiguity function behaves much like the point-spread function or impulse response of the system.

- High-resolution images require sharp ambiguity functions => waveform design prob.

The Imaging Interpretation of the Ambiguity Function

- Recall that the response of a matched filter matched to $s(t-\tau)e^{i2\pi\nu t}$ to a signal $s(t-\tau_0)e^{i2\pi\nu_0 t}$ is

$$\mathcal{O}_{T+\tau}(\tau,\nu) = e^{-i2\pi(\nu-\nu_0)\tau_0} \cdot \beta_s(\tau-\tau_0, \nu-\nu_0).$$

- Now consider two scatterers having returns with complex amplitudes c_1 and c_2, delays τ_1 and τ_2, and Doppler shifts ν_1 and ν_2, respectively. Then the output of a matched filter matched to the delay-Doppler pair (τ,ν) (ignoring multiple reflections between the two targets) is

$$\mathcal{O}_{T+\tau}(\tau,\nu) = c_1 e^{-i2\pi(\nu-\nu_1)\tau_1}\beta_s(\tau-\tau_1,\nu-\nu_1) + c_2 e^{-i2\pi(\nu-\nu_2)\tau_2}\beta_s(\tau-\tau_2,\nu-\nu_2).$$

- More generally, for M targets, it can be shown that

$$\mathcal{O}_{T+\tau}(\tau,\nu) = \sum_{m=1}^{M} c_m e^{-i2\pi(\nu-\nu_m)\tau_m}\beta_s(\tau-\tau_m,\nu-\nu_m).$$

- So the overall response is a linear combination of phase-shifted ambiguity functions centered about the target locations $(\tau_1,\nu_1),\ldots,(\tau_M,\nu_M)$.
- This looks a lot like convolution of the ambiguity function with impulses at the target locations, but the phase terms make this not quite right.

The Imaging Interpretation of the Ambiguity Function (Cont.)

Let's go back and look at the two scatterer case:

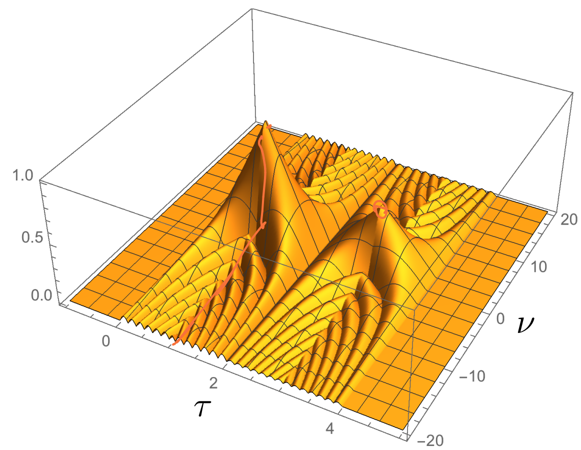
$$\mathcal{O}_{T+\tau}(\tau,\nu) = c_1 e^{-i2\pi(\nu-\nu_1)\tau_1}\beta_s(\tau-\tau_1,\nu-\nu_1) + c_2 e^{-i2\pi(\nu-\nu_2)\tau_2}\beta_s(\tau-\tau_2,\nu-\nu_2).$$

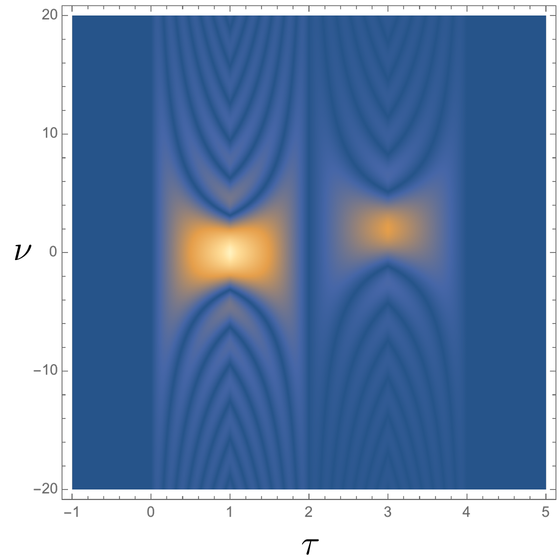
Let's take a look at one plots of $|\mathcal{O}(\tau,\nu)|$ for two targets at different locations in delay and Doppler, as well as both rectangular and chirp waveforms.

The Imaging Interpretation of the Ambiguity Function (Cont.)

This is what $|\mathcal{O}_{T+\tau}(\tau,\nu)|$ looks like when $(\tau_1,\nu_1,c_1)=(1,0,1)$ and $(\tau_2,\nu_2,c_2)=(3,2,0.5)$.

$$s(t)=1_{[0,1]}(t)$$





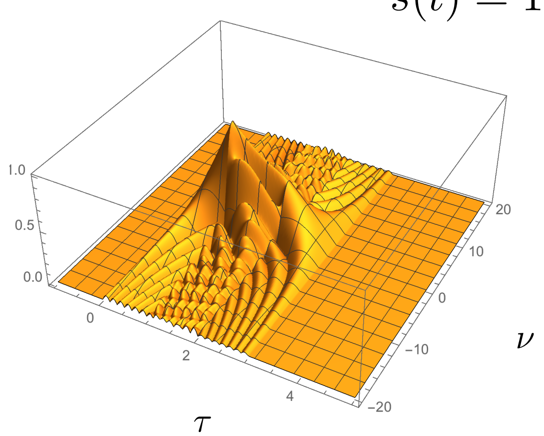
With no overlap in the responses of the two targets, we have

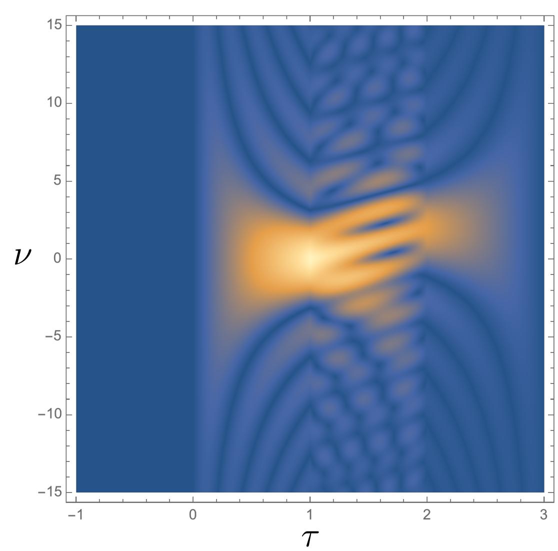
$$|\mathcal{O}(\tau,\nu)| = |c_1|\cdot|\beta_s(\tau-\tau_1,\nu-\nu_1)| + |c_2|\cdot|\beta_s(\tau-\tau_2,\nu-\nu_2)|.$$

The Imaging Interpretation of the Ambiguity Function (Cont.)

This is what $|\mathcal{O}_{T+\tau}(\tau,\nu)|$ looks like when $(\tau_1,\nu_1,c_1)=(1,0,1)$ and $(\tau_2,\nu_2,c_2)=(2,2,0.5)$.

$$s(t)=1_{[0,1]}(t)$$





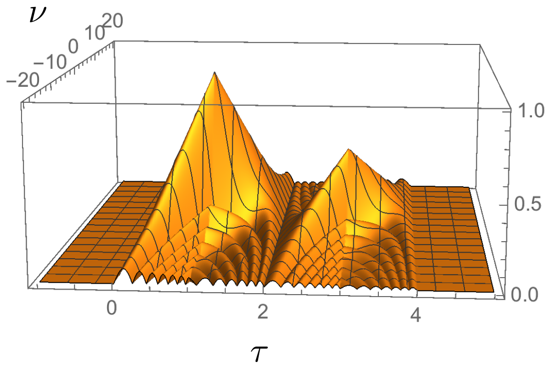
Because of the overlap in the responses of the two targets,

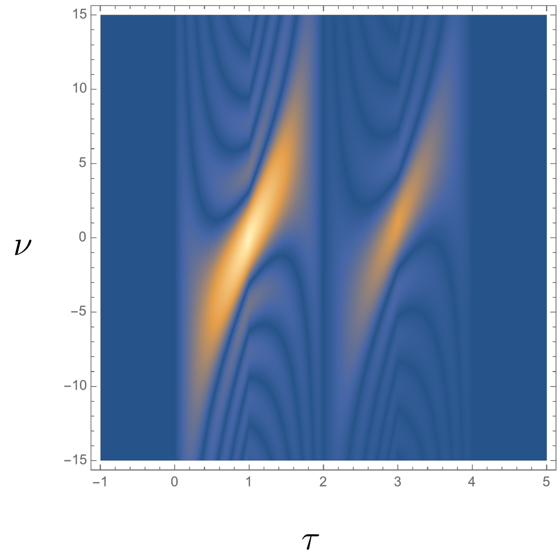
$$|\mathcal{O}(\tau,\nu)| \neq |c_1|\cdot|\beta_s(\tau-\tau_1,\nu-\nu_1)| + |c_2|\cdot|\beta_s(\tau-\tau_2,\nu-\nu_2)|.$$

The Imaging Interpretation of the Ambiguity Function (Cont.)

This is what $|\mathcal{O}_{T+\tau}(\tau,\nu)|$ looks like when $(\tau_1,\nu_1,c_1)=(1,0,1)$ and $(\tau_2,\nu_2,c_2)=(3,2,0.5)$.

$$s(t)=e^{i\pi 10t^2}\cdot 1_{[0,1]}(t)$$





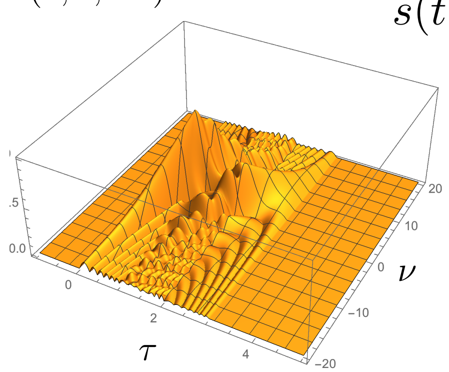
With no overlap in the responses of the two targets, we have

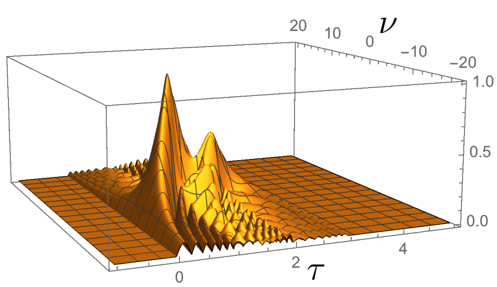
$$|\mathcal{O}(\tau,\nu)|=|c_1|\cdot|\beta_s(\tau-\tau_1,\nu-\nu_1)|+|c_2|\cdot|\beta_s(\tau-\tau_2,\nu-\nu_2)|.$$

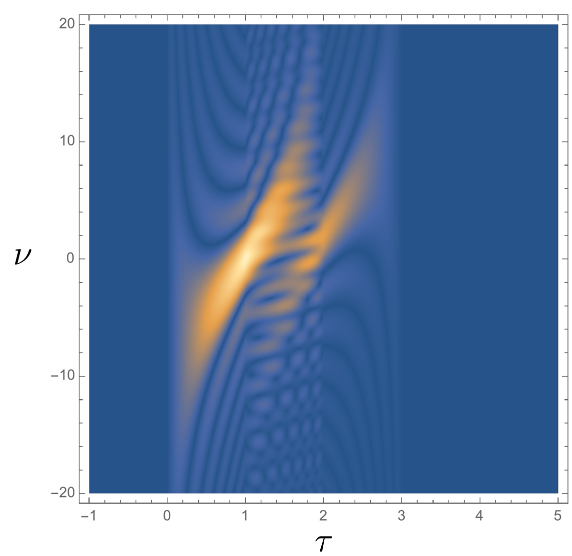
The Imaging Interpretation of the Ambiguity Function (Cont.)

This is what $|\mathcal{O}_{T+\tau}(\tau,\nu)|$ looks like when $(\tau_1,\nu_1,c_1)=(1,0,1)$ and $(\tau_2,\nu_2,c_2)=(2,2,0.5)$.

$$s(t)=e^{i\pi 10t^2}\cdot 1_{[0,1]}(t)$$







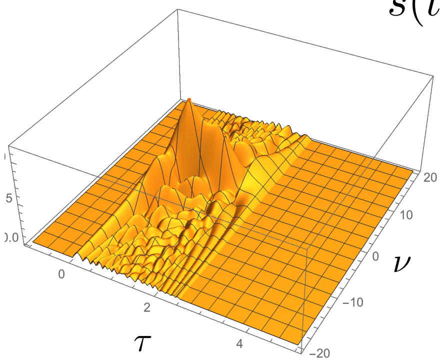
Because of the overlap in the responses of the two targets,

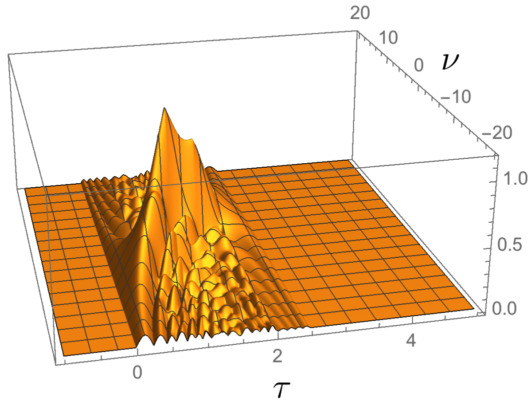
$$|\mathcal{O}(\tau,\nu)|\neq|c_1|\cdot|\beta_s(\tau-\tau_1,\nu-\nu_1)|+|c_2|\cdot|\beta_s(\tau-\tau_2,\nu-\nu_2)|.$$

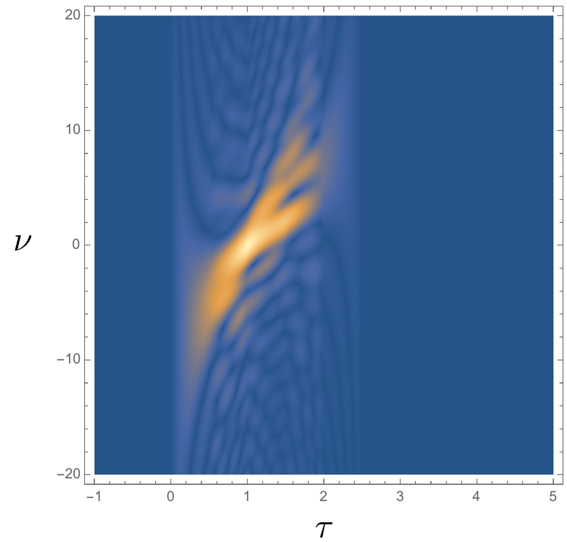
The Imaging Interpretation of the Ambiguity Function (Cont.)

This is what $|\mathcal{O}_{T+\tau}(\tau,\nu)|$ looks like when $(\tau_1,\nu_1,c_1)=(1,0,1)$ and $(\tau_2,\nu_2,c_2)=(1.5,2,0.5)$.

$$s(t)=e^{i\pi 10t^2}\cdot 1_{[0,1]}(t)$$







Clearly the shape of the ambiguity function has a profound effect on the ability to resolve closely spaced targets in delay and Doppler.

Numerical Evaluation of Ambiguity Functions

28.9

- The asymmetric ambiguity function is given by

$$\beta_s(\tau,\nu)=\int_{-\infty}^{\infty} s(t)s^*(t-\tau)\,e^{-i2\pi\nu t}\,dt.$$

- If $s(t)$ has support $[0,T]$, we can write this as

$$\beta_s(\tau,\nu)=\int_{0}^{T} s(t)s^*(t-\tau)\,e^{-i2\pi\nu t}\,dt.$$

- We can interpret this as the limit of a Riemann sum

$$\beta_s(\tau,\nu)\approx\sum_{n=0}^{N-1} s(n\Delta t)s^*((n-m)\Delta t)\,e^{-i2\pi\nu n\Delta t}\cdot\Delta t,$$

where

$$\tau=\tau_m=m\Delta t \qquad \text{and} \qquad N=[T/\Delta t].$$

Numerical Evaluation of Ambiguity Functions (Cont.)

- Now define the discrete-time signal

$$s[n] = s(n\Delta t), \quad n = 0, 1, 2, \ldots, N-1.$$

- Then we can write $\beta_s(\tau, \nu)$ in terms of $s[n]$ as

$$\beta_s(\tau, \nu) \approx \Delta t \cdot \sum_{n=0}^{N-1} s[n]s^*[n-m]\, e^{-i2\pi\nu n\Delta t}.$$

  Now consider the closely related N-point DFT

$$\mathrm{DFT}_n\left\{s[n]s^*[n-m]\right\} = \sum_{n=0}^{N-1} s[n]s^*[n-m]\, e^{-i\frac{2\pi nk}{N}}.$$

- If we take

$$\tau = \tau_m = m\Delta t$$

  and

$$\nu = \nu_k = \frac{k}{N\Delta t},$$

  then we have

$$\beta_s(\tau_m, \nu_k) = \left(\frac{T}{N-1}\right) \sum_{n=0}^{N-1} s[n]s^*[n-m]\, e^{-i\frac{2\pi nk}{N}} = \mathrm{DFT}_n\left\{s[n]s^*[n-m]\right\}.$$

Cross-Ambiguity Functions

Definition:(Asymmetric cross-ambiguity function) The *asymmetric cross-ambiguity function* of two finite-energy signals $r(t)$ and $s(t)$ is defined as

$$\beta_{rs}(\tau, \nu) = \int_{-\infty}^{\infty} r(t)s^*(t-\tau)e^{-i2\pi\nu t}\, dt.$$

Definition:(Symmetric cross-ambiguity function) The *symmetric cross-ambiguity function* of two finite-energy signals $r(t)$ and $s(t)$ is defined as

$$\Gamma_{rs}(\tau, \nu) = \int_{-\infty}^{\infty} r(t+\tau/2)s^*(t-\tau/2)e^{-i2\pi\nu t}\, dt.$$

We will focus primarily on the asymmetric form, but the symmetric form is of interest in signal theory.