Session 11

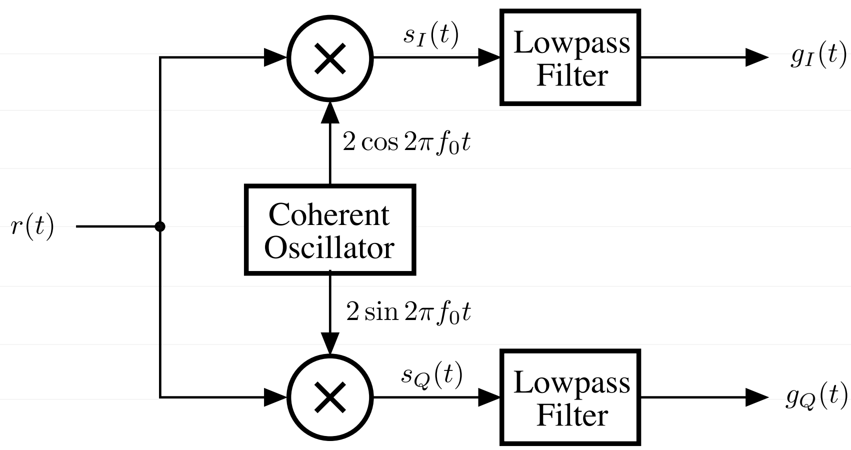
Radar Signal Models

11.1

- Radar, like most radio systems, modulates a signal of interest onto a high-frequency RF carrier.
- The information bearing signal or pulse typically carries the information of interest.
- In a radar receiver, the received waveform's amplitude, phase and delay must be extracted from the received signal.

11.2

- One very common way of doing this is by "beating the signal down to baseband" using a quadrature detector :



11.3

Suppose a pulse $p(t) = \mathbb{1}_{[0,T]}(t)$ amplitude modulates a carrier with frequency f_0 to produce a signal $s(t)$:

$$s(t) = \cos(2\pi f_0 t) \cdot \mathbb{1}_{[0,T]}(t)$$

Now if the received signal $r(t)$ is a τ-delayed version of $s(t)$, we have

$$r(t) = s(t-\tau) = \cos[2\pi f_0 (t-\tau)] \cdot \mathbb{1}_{[0,T]}(t-\tau)$$

$$= \cos(2\pi f_0 t - 2\pi f_0 \tau) \cdot \mathbb{1}_{[\tau, T+\tau]}(t)$$

Basic Trig. Identities

11.4

$$\cos A \cdot \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

$$\cos A \cdot \sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$$

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta} \leftarrow$$

Paul Nahin, Dr. Euler's Faboulous Formula

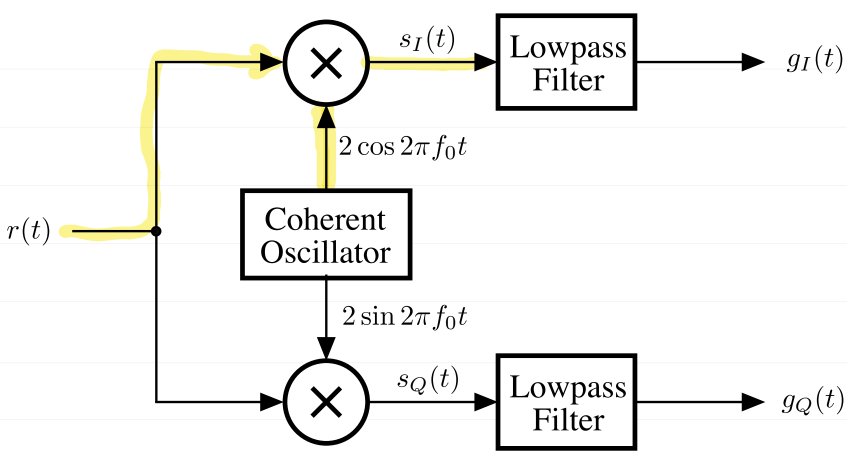
$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

e.g.

$$\cos A \cdot \cos B = \left(\frac{e^{iA}+e^{-iA}}{2}\right)\left(\frac{e^{iB}+e^{-iB}}{2}\right) = \cdots \frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$

11.5



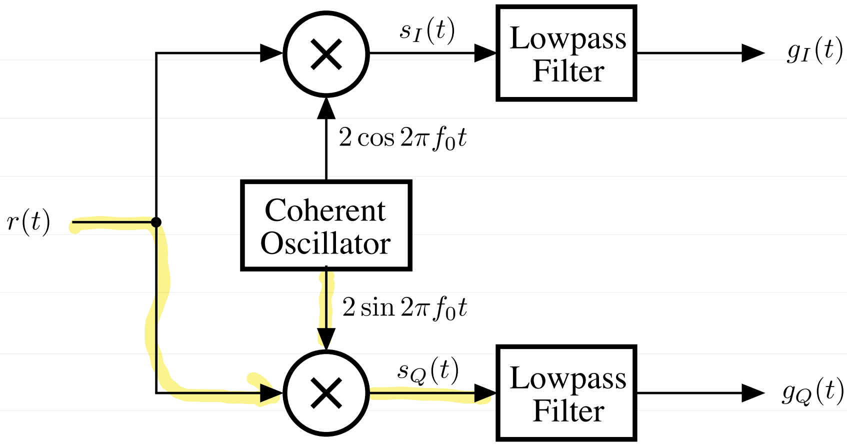
$$s_I(t) = r(t)\cdot 2\cos 2\pi f_0 t$$

$$= \cos(2\pi f_0 t - 2\pi f_0 \tau)\cdot 1_{[\tau, T+\tau]} \cdot 2\cos 2\pi f_0 t$$

$$= \left[\cos(2\pi(2f_0)t - 2\pi f_0\tau) + \cos(-2\pi f_0 \tau)\right]\cdot 1_{[\tau, T+\tau]}(t)$$

$$\left[\cos(2\pi(2f_0)t - 2\pi f_0\tau) + \cos(2\pi f_0 \tau)\right]\cdot 1_{[\tau, T+\tau]}(t)$$

11.6



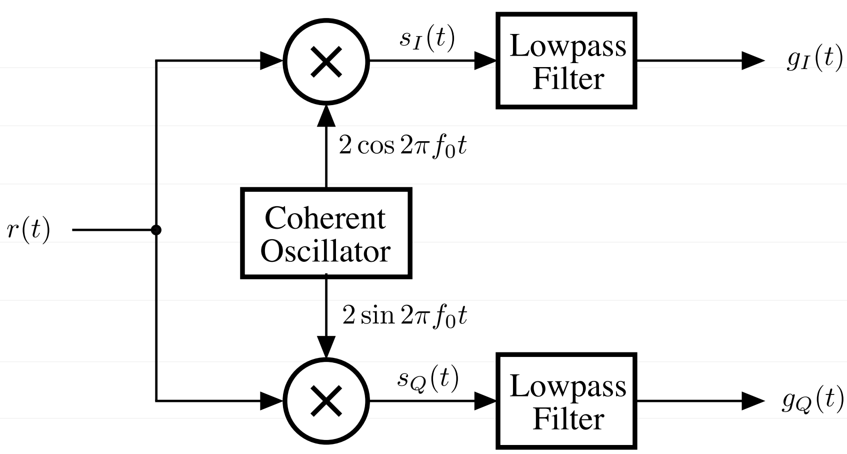
$$s_Q(t) = r(t)\cdot 2\sin(2\pi f_0 t)$$

$$= \cos(2\pi f_0 t - 2\pi f_0 \tau)\cdot 1_{[\tau,T+\tau]}(t) \cdot \sin(2\pi f_0 t)$$

$$= \left[\sin(2\pi(2f_0)t - 2\pi f_0\tau) - \sin(-2\pi f_0 \tau)\right]\cdot 1_{[\tau,T+\tau]}(t)$$

$$= \left[\sin(2\pi(2f_0)t - 2\pi f_0\tau) + \sin(2\pi f_0\tau)\right]\cdot 1_{[\tau,T+\tau]}(t)$$

11.7



$$g_I(t) = LPF\{s_I(t)\} = \cos(2\pi f_0\tau)\cdot 1_{[\tau,T+\tau]}(t)$$

"In phase" ~ real

$$g_Q(t) = LPF\{s_Q(t)\} = \sin(2\pi f_0\tau)\cdot 1_{[\tau,T+\tau]}(t)$$

"Quadrature" ~ imaginary

We can construct a complex waveform

11.8

$$z(t) = g_I(t) + i\, g_Q(t)$$

$$= \cos(2\pi f_0 \tau)\cdot \mathbb{1}_{[\tau, T+\tau]}(t) + i \sin(2\pi f_0 \tau)\cdot \mathbb{1}_{[\tau, T+\tau]}(t)$$

$$= [\cos(2\pi f_0 \tau) + i \sin(2\pi f_0 \tau)] \cdot \mathbb{1}_{[\tau, T+\tau]}(t)$$

$$= e^{i 2\pi f_0 \tau} \cdot \mathbb{1}_{[\tau, T+\tau]}(t)$$

This is a rectangular pulse of duration T starting at time τ.

It has phase $2\pi f_0 \tau$ radians.

The form

11.9

$$z(t) = e^{i2\pi f_0 \tau} \cdot \mathbb{1}_{[\tau, T+\tau]}(t)$$

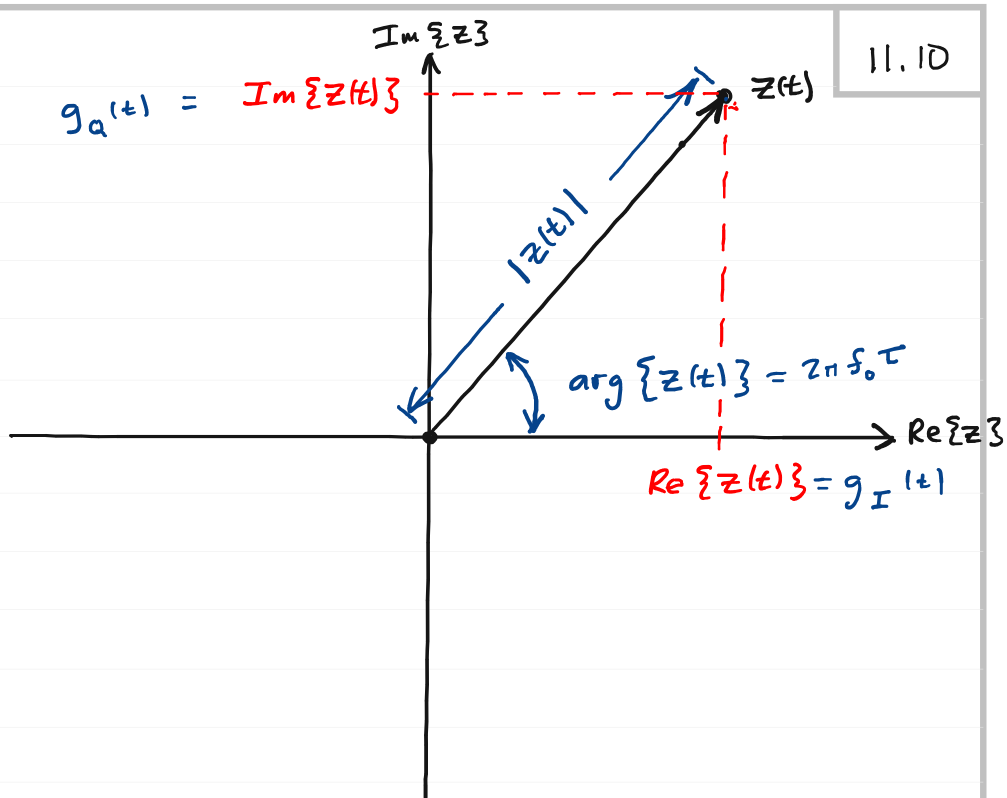
is called the complex baseband form of real received passband signal

$$r(t) = \cos(2\pi f_0 (t-\tau)) \cdot \mathbb{1}_{[0,T]}(t-\tau)$$

$$= \cos(2\pi f_0 t - 2\pi f_0 \tau) \cdot \mathbb{1}_{[\tau, T+\tau]}$$

We can represent $z(t)$ at time t using a phasor diagram:

11.10



11.11

The Narrowband Complex Baseband Signal

We now generalize the form of the received signal to be

$$r(t) = a(t) \cos(2\pi f_0 t + \theta(t))$$

where

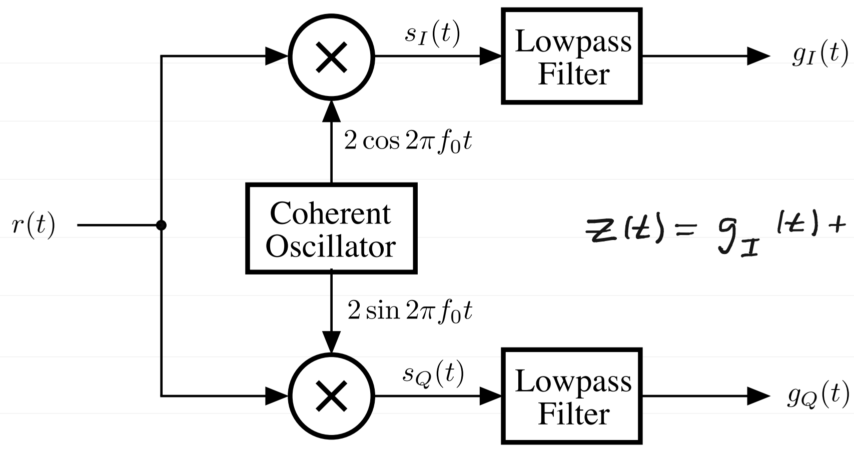
$$a(t) = \text{real-valued amplitude function}$$

and

$$\theta(t) = \text{real-valued phase function.}$$

Assume $a(t)$ and $\theta(t)$ vary slowly compared to $\cos 2\pi f_0 t$.

11.12



$$z(t) = g_I(t) + i\, g_Q(t)$$

$$g_I(t) = LPF\{ s_I(t) \} = a(t) \cos \theta(t)$$

$$g_Q(t) = LPF\{ s_Q(t) \} = -a(t) \sin \theta(t)$$

and we have the complex baseband waveform

$$z(t) = g_I(t) + i\, g_Q(t) = a(t)\, e^{-i\theta(t)} .$$

11.13

$$r(t) = a(t) \cos(2\pi f_0 t + \theta(t))$$

(real passband form)

$$\Updownarrow$$

$$z(t) = a(t)\, e^{-i\theta(t)}$$

(complex baseband form)

n.b. $r(t) = \mathrm{Re}\{ z(t)\, e^{i 2\pi f_0 t} \}$.

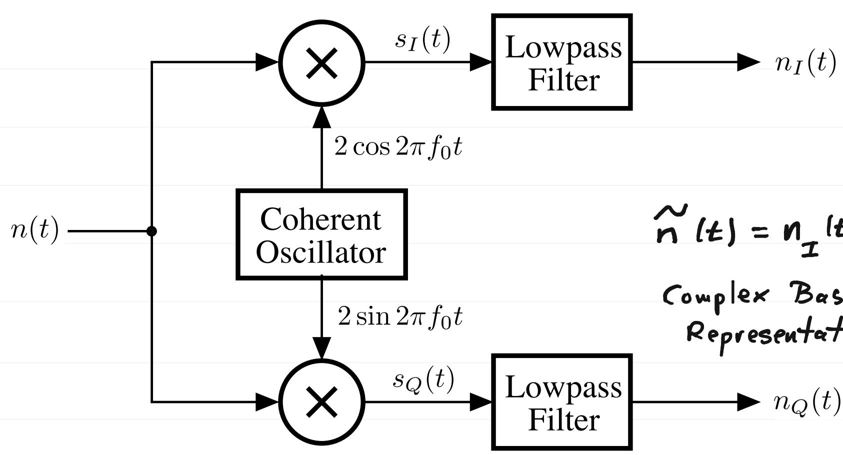
Narrowband Complex Baseband Noise

Consider a real, wide-sense stationary random process $n(t)$.

Assume that $n(t)$ is wide-sense stationary white Gaussian noise.

If we beat $n(t)$ down to baseband, we get a corresponding complex baseband waveform

$$\tilde{n}(t) = n_I(t) + i\, n_Q(t).$$

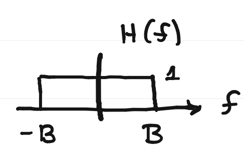


$$\tilde{n}(t) = n_I(t) + i\, n_Q(t)$$

Complex Baseband Representation

If we have ideal low-pass filters

$$H(f) = \mathbb{1}_{[-B,B]}(f),$$



then

$$S_{n_I n_I}(f) = S_{n_Q n_Q}(f) = \frac{N_0}{2}\, \mathbb{1}_{[-B,B]}(f).$$

Recall...

Narrowband Complex Baseband Noise

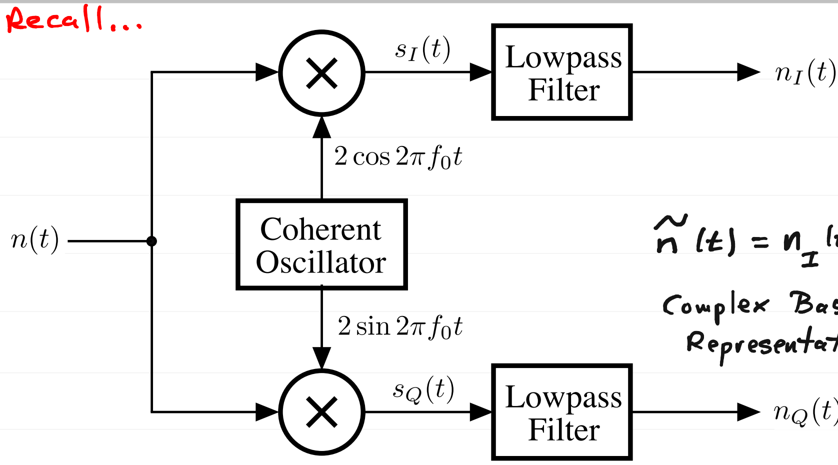
Consider a real, wide-sense stationary random process $n(t)$.

Assume that $n(t)$ is wide-sense stationary white Gaussian noise.

If we beat $n(t)$ down to baseband, we get a corresponding complex baseband waveform

$$\tilde{n}(t) = n_I(t) + i\, n_Q(t).$$

Recall...

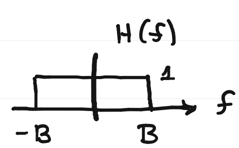


$\tilde{n}(t) = n_I(t) + i\, n_Q(t)$

Complex Baseband Representation

If we have ideal low-pass filters

$$H(f) = \mathbb{1}_{[-B,B]}(f),$$



then

$$S_{n_I n_I}(f) = S_{n_Q n_Q}(f) = \frac{N_0}{2}\, \mathbb{1}_{[-B,B]}(f).$$

Furthermore, $n_I(t)$ and $n_Q(t)$ have identical autocorrelation functions:

$$R_{n_I}(\tau) = R_{n_Q}(\tau) = \int_{-B}^{B} \frac{N_0}{2} e^{i2\pi f\tau}\, df$$

$$= N_0 B \left(\frac{\sin 2\pi B\tau}{2\pi B\tau} \right)$$

$$= N_0 B \operatorname{sinc}(2B\tau)$$

where

$$\operatorname{sinc}(x) \triangleq \frac{\sin \pi x}{\pi x}.$$

Furthermore, it can be shown that

$$E[n_I(t+\tau)\, n_Q(t)] = -E[n_Q(t+\tau)\, n_I(t)]$$

$$\Rightarrow \quad R_{n_I n_Q}(\tau) = -R_{n_Q n_I}(\tau)$$

It follows that

$$R_{\tilde{n}}(\tau) = E[\tilde{n}(t+\tau)\, \tilde{n}^*(t)]$$

$$= E[(n_I(t+\tau) + i n_Q(t+\tau))(n_I(t) - i n_Q(t))]$$

$$= R_{n_I}(\tau) + R_{n_Q}(\tau) + i\left(E[n_Q(t+\tau) n_I(t)] - E[n_I(t+\tau) n_Q(t)]\right)$$

$$= 2\left(R_{n_I}(\tau) + R_{n_Q n_I}(\tau)\right).$$

From this, we make the following observations...