ECE600: Random Variables and Waveforms

Prof. Mark R. Bell
mrb@ecn.purdue.edu
(765)494-6412

ECE600 Contact Info.

Instructor: Prof. Mark R. Bell
Phone: (765) 494-6412
email: mrb@ecn.purdue.edu
TA: To Be Determined
Webpage: http://www.ece.purdue.edu/~mrb
(and follow the ECE600 link.)
Prerequisites

Graduate Standing

Solid Understanding of Calculus and Fourier Transforms.

Some mathematical maturity.

Textbook


The third edition of Papoulis is acceptable if you already have it.
(Alternate Textbook)


This is a modern, rigorous, well written treatment of the material in Papoulis. More detail than Papoulis provides.

Course Grading

- 3 Midterms Exams: 20% Each
- 1 Final Exam: 40%

Homework will not be collected—but you must do it!!!
# Course Schedule

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<td>Lecture</td>
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# Course Grading (Cont.)

- **No Make-up Exams.**
  
  If you miss a midterm exam, your final exam score will be used in its place.

- **Homework:**
  
  There will be weekly homework assignments.

- **Homework solutions will be posted to the course website.**

- **Do the homework!!!**
Prof. Bell’s Office Hours

- **Off Campus Students:**
  - W: 2:00 – 3:00pm
  - Th: 2:00 – 3:00pm

- **On Campus Students:**
  - T: 2:00 – 3:00pm
  - W: 3:00 – 4:00pm

Random Models in ECE

- Communications and Information Theory
- Computer Networks
- Solid State (Quantum Mechanics)
- Optics
- Control Theory
- Electromagnetics and Antennas
- Statistics, pattern recognition and data science.
Probability is Used to Model Uncertainty

- Systems that are too complex to model deterministically: (Ignorance)
  - Maxwell: Theory of Gases
  - Boltzmann: Statistical Mechanics
- Systems that are inherently random:
  - Games of Chance
  - Quantum Mechanics
  - Other “fundamentally random” systems.

Set Theory

- Why Set Theory?
- A Random Experiment: *roll a fair die.*

\[ S = \{1, 2, 3, 4, 5, 6\} \]

- We can define events
  - \( A_1 = \text{outcome is odd} = \{1, 3, 5\} \)
  - \( A_2 = \text{outcome is divisible by 3} = \{3, 6\} \)
  - \( A_3 = \text{outcome is prime} = \{2, 3, 5\} \)
• Each event of interest can be described by a subset of
  \[ S = \{1, 2, 3, 4, 5, 6\}. \]

• There are \(2^6 = 64\) distinct subsets of \(S\).

• In order to fully characterize a random experiment, we must know the
  probability of each of these sets.

Events:

• Events are subsets of \(S\).

• The collection of all events is called the event space
  \[ \mathcal{F}(S) = \{A_1, A_2, \ldots, A_{64}\} \]

Our random experiment is completely characterized by

\[ \{S, \mathcal{F}(S), P(\cdot)\} \]

where

\[ P(\cdot) : \mathcal{F}(S) \to [0, 1] \]

and assigns probability to each of the events.

This framework—with minor modifications—will be used to describe all of the random experiments we will consider.

A solid understanding of set theory is essential to this task.
Basic Set Theory Definitions

- A set is simply a collection of objects—we intentionally leave this undefined.

**Defn:** In any given problem, the set containing all possible elements of interest is called the **universe**, **universal set**, or **space**. We typically denote the space by $S$.

- There are a number of basic set operations we must be familiar with:

**Defn:** The **union** of two sets $A$ and $B$, denoted $A \cup B$, is defined as

$$A \cup B = \{ \omega \in S : \omega \in A \text{ or } \omega \in B \}.$$ 

**Defn:** The **intersection** of two sets $A$ and $B$, denoted $A \cap B$, is defined as

$$A \cap B = \{ \omega \in S : \omega \in A \text{ and } \omega \in B \}.$$ 

**Defn:** The **complement** of a set $A$ (with respect to $S$), denoted $\overline{A}$, $A'$, or $A^c$ is defined as

$$\overline{A} = \{ \omega \in S : \omega \notin A \}.$$ 

**Defn.** The set containing no elements is called the **empty set** or **null set**, and is denoted by $\emptyset$ or $\{\}$. (n.b. $\{\emptyset\}$ is not correct notation.)

**Defn:** If two sets $A$ and $B$ have no elements in common, then $A \cap B = \emptyset$, and $A$ and $B$ are said to be **disjoint**.

- The **set difference** of two sets $A$ and $B$ is defined as

$$A - B = \{ \omega \in S : \omega \in A \text{ and } \omega \notin B \} = A \cap \overline{B}.$$ 

**Defn:** The **symmetric difference** of two sets $A$ and $B$, is defined as

$$A \Delta B = \{ \omega \in A \text{ or } \omega \in B, \text{ but not both} \} = (A \cup B) - (A \cap B) = \ldots = (A \cap B) \cup (A \cap \overline{B})$$ 

**Defn:** Two sets $A$ and $B$ are **equal** if they contain exactly the same elements.

**Fact:** Two sets $A$ and $B$ are equal if and only if (iff) $A \subseteq B$ and $B \subseteq A$.

**Proof:** exercise.
Algebra of Set Theory

1. \( A \cup B = B \cup A \).  \text{ Union is commutative. }
2. \( A \cap B = B \cap A \).  \text{ Intersection is commutative. }
3. \( A \cup (B \cup C) = (A \cup B) \cup C \).  \text{ Union is associative. }
4. \( A \cap (B \cap C) = (A \cap B) \cap C \).  \text{ Intersection is associative. }
5. \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \).  \text{ Intersection is distributive over union. }
6. \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \).  \text{ Union is distributive over intersection. }
7. \( \overline{A} = A \).
8. \( A \cap B = A \cup B \).
9. \( A \cup B = A \cap B \).
10. \( S = \emptyset \).
11. \( A \cap S = A \).
12. \( A \cap \emptyset = \emptyset \).
13. \( A \cup S = S \).
14. \( A \cup \emptyset = A \).
15. \( A \cup \overline{A} = S \).
16. \( A \cap \overline{A} = \emptyset \).

Demorgan’s Laws.

• An indexed collection of sets is a set of sets \( \{ A_i ; i \in I \} \), where \( I \) is an index set.

So \( \{ A_i ; i \in I \} \) is a “set of sets” or a family of sets or a collection of sets.
Example: \( J = \{ 1, 2, 3 \} \)

\[ A_1 = [0, 1] = \{ x \in \mathbb{R} : 0 \leq x \leq 1 \} \]

\[ A_2 = [1, 2] \]

\[ A_3 = [2, 3] \]

So

\[ \bigcup_{i \in J} A_i = \bigcup_{i \in J} \{ 0, 1, 2, 3 \} \]