ECE678 Radar Engineering
Fall 2024

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MSEE 336

Homework Assignment #5
Due Midnight, Wednesday, November 13, 2024

Reading Assignment: Chapter 7: The Ambiguity Function in the draft text, and/or Levanon Chapters 6 and 7.

1. If the ambiguity function of $s(t)$ (an arbitrary finite-energy signal) is $\beta_s(\tau,\nu)$, show that the ambiguity function of $v(t) = s(\alpha t)$ is

$$\beta_v(\tau,\nu) = \frac{1}{|\alpha|}\beta_s\left(\alpha\tau, \frac{\nu}{\alpha}\right),$$

   for all real numbers $\alpha \neq 0$.

2. Show that for an energy normalized signal $u(t)$, (*i.e.*, having $E = 1$),

$$\frac{\partial^2 \beta_u(0,0)}{\partial \nu^2} = -4\pi^2 \int_{-\infty}^{\infty} t^2\, |u(t)|^2\, dt.$$

3. Prove that the two-dimensional Fourier transform of $|\beta_s(\tau,\nu)|^2$ is equal to the function itself. That is,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\beta_s(\tau,\nu)|^2 e^{i2\pi\nu t} e^{-i2\pi\tau f}\, d\tau\, d\nu = |\beta_s(t,f)|^2.$$

4. Calculate the asymmetric ambiguity function $\beta_s(\tau,\nu)$ of the signal

$$s(t) = Be^{-t^2/T^2}.$$

   What is the value of B that makes the signal unit energy?

5. Calculate the ambiguity function of the signal

$$s(t) = Be^{-t^2/T^2} e^{i\pi\alpha t^2}.$$

   (*n.b.*, This is the same signal as in the previous problem, except it is modulated by a linear FM chirp.)

6. *Levanon, 7.3*: Draw the following four cuts of the ambiguity function of three coherent single frequency pulses: $|\beta(\tau,0)|$, $|\beta(0,\nu)|$, $|\beta(T,\nu)|$, and $|\beta(2T,\nu)|$. The width of each

pulse is t_p, the pulse repetition interval is T, and $T \approx 5.5t_p$. In the last three cuts, find the value of the ambiguity function at $\nu = 1/2T$.

7. *Levanon, 7.4*: A 10 GHz radar utilizes a linear FM pulse of duration $t_p = 20\,\mu$s and frequency deviation $\Delta f = 1$ MHz. The pulse is relected from a point target approaching the radar with a radial velocity of 150 m/s. What will be the bias error in measuring the range? (Assume that the range is determined by the delay at which the matched filter ouput—the matched filter for a non-Doppler shifted pulse—reaches a peak.)

8. Prove that if a (baseband) $s(t)$ has an ambiguity function $\Gamma(\tau, \nu)$, then the (baseband) pulse $s(t)e^{i\pi\alpha t^2}$ has ambiguity function $\Gamma(\tau, \nu - \alpha\tau)$.

9. Using the result of problem 8, calculate $\Gamma(\tau, \nu)$ for the following waveforms
   (a) $s(t) = e^{i\pi\alpha t^2} \cdot 1_{[-T/2,T/2]}(t)$.
   (b) $s(t) = e^{i\pi\alpha t^2} \cdot e^{-\pi\beta t^2}$, where β is a positive real number.