ECE642 Information Theory and Source Coding Fall 2023

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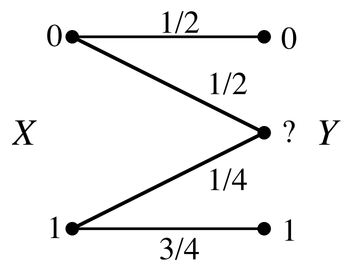
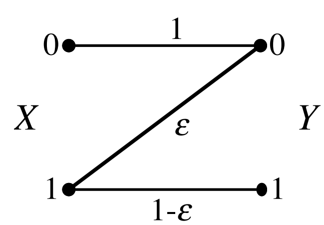
MSEE 336

Homework Assignment #3 Due:
Wednesday, October 18, 2021

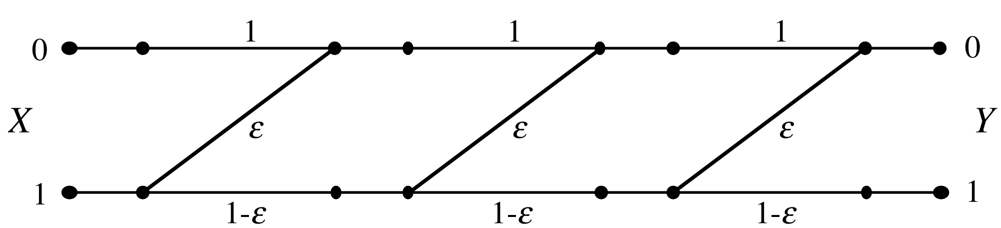
1. Let X be the outcome of the roll of a pair of fair dice. Let Y be a random variable taking on only two different values. How large can $I(X;Y)$ be? Explicitly describe at least one Y for which $I(X;Y)$ is maximum.
2. The channel we consider in this problem is called a *binary erasure channel.* Such a channel can be a useful model of a detection process in which one of two possible signals is sent and a decision is made at the receiver if it is clear which signal was sent. Otherwise an erasure "?" is declared. Assume the channel input X has probabilities $\Pr\{X=0\}=p$ and $\Pr\{X=1\}=1-p$. Compute $I(X;Y)$ as a function of p. For what value of p is $I(X;Y)$ maximum?



3. The channel below, called the *Z channel,* is useful for modeling binary optical communication channels in which the receiver threshold is sufficiently high such that the probability of receiving a noise photon is negligible. Find the capacity of the Z Channel.



4. Find the capacity of three independent Z channels with identical crossover probability ϵ cascaded in a row.



5. Find the capacity and optimizing input probability distribution for each of the following five DMC's: *(counts double)*

$$\begin{pmatrix} 1-\delta-\epsilon & \delta & \epsilon \\ \epsilon & \delta & 1-\delta-\epsilon \end{pmatrix}; \qquad (a)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1/2 & 1/2 \end{pmatrix}; \qquad (b)$$

$$\begin{pmatrix} 1-\epsilon & \epsilon & 0 \\ 0 & 1-\epsilon & \epsilon \\ \epsilon & 0 & 1-\epsilon \end{pmatrix}; \qquad (c)$$

$$\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/4 & 3/4 \end{pmatrix}; \qquad (d)$$

$$\begin{pmatrix} 1/3 & 1/3 & 0 & 1/3 \\ 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & 1/3 & 1/3 \end{pmatrix}. \qquad (e)$$

6. *Cover and Thomas* , Chapter 5, Problem 28 (First Edition Chapter 5, Problem 25 (p. 123).)