ECE600 Random Variables and Waveforms — Mark R. Bell
Spring 2024 — MSEE 336

Homework Assignment #3
Should be completed by Session 8

Reading Assignment: Sections 3-1, 3-2, 4-1, 4-2, and 4-3 of Papoulis.

1. (*Papoulis,* Problem 2-10) Show that for n events $A_1, \ldots, A_n$,

$$P(A_n \cap A_{n-1} \cap \cdots \cap A_2 \cap A_1) = P(A_n | A_{n-1} \cap \cdots \cap A_2 \cap A_1) \cdots P(A_2|A_1)P(A_1).$$

2. (*Papoulis,* Problem 2-11) We select at random m objects from a set B of n objects and we denote the set of selected objects by A_m. Show that the probability p that a particular element ξ_0 is in A_m is equal to m/n. (*Hint:* p is equal to the probability that a randomly selected element of B is in A_m.)

3. (*Papoulis,* Problem 2-12) A call occurs at time t, where t is a randomly selected point in the interval $(0, 10)$ (all points in the interval being equally likely). (a) Find $P(\{6 \le t \le 8\})$. (b) Find $P(\{6 \le t \le 8\}|\{t > 5\})$.

4. (*Papoulis,* Problem 2-13) Let the sample space $\mathcal{S}$ consist of all positive real numbers, and let t be the outcome of the random experiment. Show that if

$$P(\{t_0 \le t \le t_0 + t_1\}|\{t \ge t_0\}) = P(\{t \le t_1\})$$

for all positive t_0 and t_1, then

$$P(\{t \le t_1\}) = 1 - e^{-ct_1},$$

where c is a constant.

5. (*Papoulis,* Problem 2-16) A box contains n identical balls labeled 1 through n. Suppose k balls are drawn in succession (without replacement.) (a) What is the probability that m is the largest number drawn? (b) What is the probability that the largest number drawn is less than or equal to m?

6. (*Papoulis,* Problem 2-19) A box contains m white and n black balls. Suppose k balls are drawn. Find the probability of drawing at least one white ball.

7. (*Papoulis,* Problem 2-20) A player tosses a penny from a distance onto the surface of a square table ruled in 1 inch squares. If the penny is 3/4 inches in diameter, what is the probability that it will fall entirely inside a square (assuming that the penny lands on the table)?

8. (*Papoulis,* Problem 3-1) Let p be the probability of an event A. (a) What is the probability that A occurs at least twice in n independent trials? (b) What is the probability that A occurs at least three times in n independent trials?

9. (*Papoulis*, Problem 3-2) A pair of dice is rolled 50 times. Find the probability of obtaining a double six at least three times. *Hint:* Consider $(p+q)^n$ and $(p-q)^n$.

10. (*Papoulis*, Problem 3-3) A pair of fair dice are rolled 10 times. Find the problem that "seven" will show at least once (*i.e.* by "seven", we mean that the sum of the two die on a toss equals 7.)

11. (*Papoulis*, Problem 3-8) Suppose there are r successes in n independent Bernouli trials. Fnd the conditional probability that there is a success on the i-th trial.