Homework Assignment #2(RevB)
Reading Assignment: Chapters 3 and 5.
Due: Midnight, Tuesday, Sept. 26, 2023.

1. Consider a binary memoryless source X with $\Pr\{X = 0\} = 0.995$ and $\Pr\{X = 1\} = 0.005$. A binary codeword is provided for every 100-symbol sequence containing 3 or fewer 1's.
   (a) If the codewords are all the same length, find the minimum length required to provide distinct codewords for the specified set of source words.
   (b) Find the probability of getting a source sequence for which no codeword has been provided.
2. Consider a source with alphabet $A = \{a_1, a_2, a_3, a_4\}$ and associated probabilities $\mathbf{p} = \{1/2, 1/4, 1/8, 1/8\}$. Now consider the 4^8 8-letter words that can be formed from the 4 letters.
   (a) Estimate as closely as you can the number of ϵ-typical sequences of length 8 words, for $\epsilon = 0.0$ and $\epsilon = 0.25$.
   (b) For each of these two values of ϵ, find the smallest rate R such that the ϵ-typical sequences can all be represented by distinct binary sequences of length Rn (here, $n = 8$).
3. In this problem, you are to try to construct UD codes over the code alphabet $B = \{0, 1, 2, 3\}$ with the prescribed codeword lengths. In the following table, K_i denotes the number of codewords of length i in the putative code. Construct a UD code in each case if possible.

	K_i			
i	Case 1	Case 2	Case 3	Case 4
1	3	2	1	0
2	3	7	7	7
3	3	3	3	3
4	3	3	7	11
5	4	5	4	3
6	0	0	0	4

4. A source has alphabet $A = \{a_1, \ldots a_8\}$ with probabilities 0.50, 0.15, 0.12, 0.10, 0.04, 0.04, 0.03, 0.02. Construct a binary Huffman code for this source. What is the resulting average codeword length ~~n~~? How does this compare to the entropy H of the source?
5. A set of eight messages with probabilities of 0.2, 0.15, 0.15, 0.1, 0.1, 0.1, 0.1, 0.1 is to be encoded into a ternary prefix-free code. Construct two sets of codewords whose lengths have the same minimum average value but different variances. Evaluate the common average length and the two variances. State a reason (or reasons) why one code might be preferable to the other for implementation purposes.
6. Consider a game of "twenty questions" in which you are required to determine, after asking a certain number of questions that can be answered "yes" or "no," the sum of the outcomes on the roll of a pair of fair dice. What is the minimum number of questions you need to ask, on the average. [*Hint:* If you asked "Is it 2?" "Is it 3?" etc., you would average a little under six questions. It is possible to do better, however.]
7. *Cover and Thomas*, Ch. 2, Problem 4 (Problem 5 in first edition.)
8. *Cover and Thomas*, Ch. 2, Problem 7 (a) (Problem 13 (a) in first edition.)
9. *Cover and Thomas*, Ch. 2, Problem 8 (Problem 14 in first edition.)
10. *Cover and Thomas*, Ch. 3, Problem 8 (Problem 4 in first edition.)
11. *Cover and Thomas*, Ch. 3, Problem 9 (Problem 5 in first edition.)