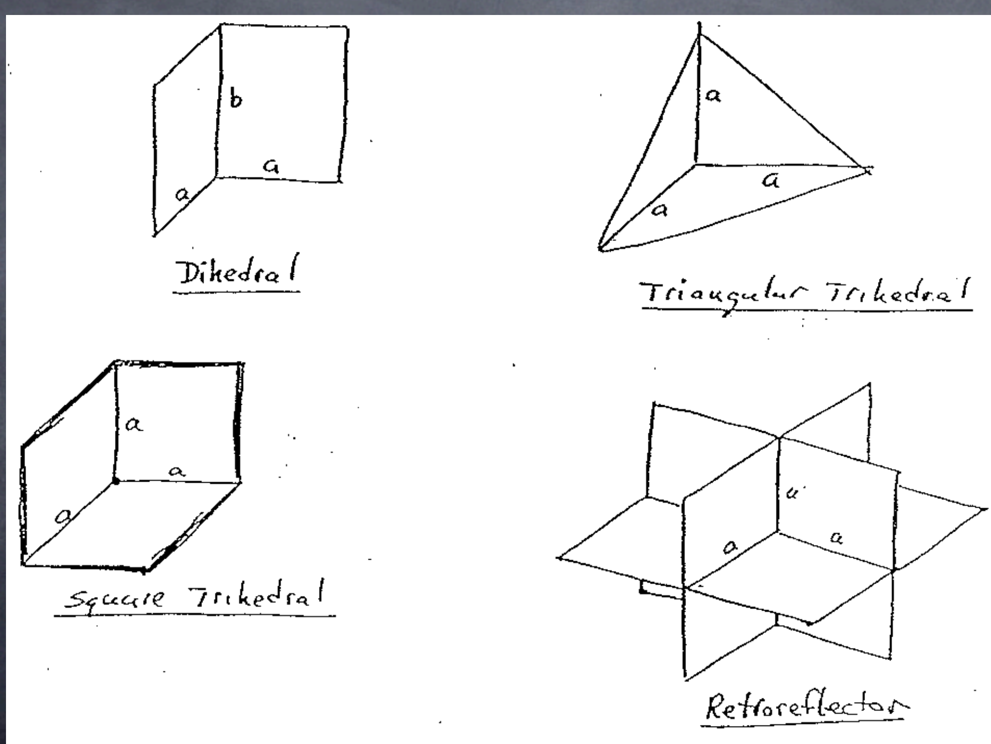
Session 6

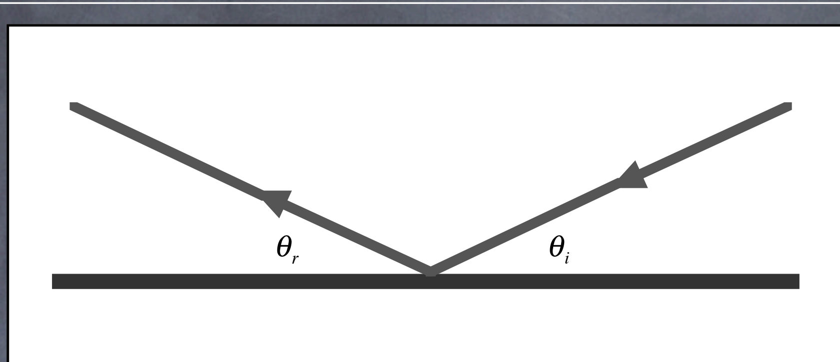
Recall...

Common Corner Reflector Geometries



Fresnel's Law of Reflection

6.2



"The angle of incidence equals the angle of reflection."

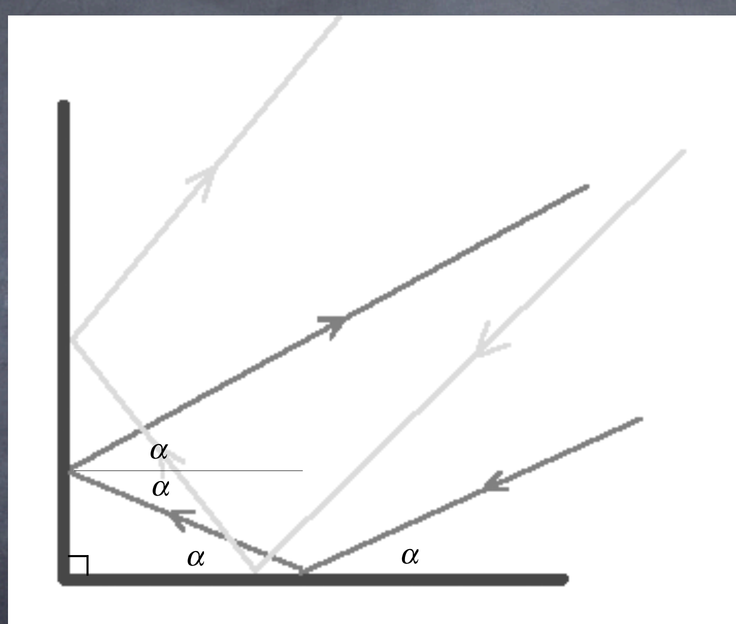
$$\theta_i = \text{Angle of Incidence}$$

$$\theta_r = \text{Angle of Reflection}$$

n.b., We assume surface is large compared to a wavelength (Ray Optics Approximation)

Consider a Dihedral Corner Reflector

6.3



From the Law of Reflection, simple geometry indicates that a ray is reflected back in the direction from which it came.

Consider a Trihedral Corner Reflector

Thought Experiment: Can you convince yourself that if the walls of this room were mirrors, that a laser beam shining into a corner would be reflected back in the direction from which it came?

Geometry Exercise: Can you show this mathematically in 3-D space using vectors to represent the rays?

See Ruck et al., Chapter 8, for more on corner reflectors

Thermal Noise in Microwave Receivers

- Thermal motion of charges in any conducting or lossy body produces fluctuating currents and voltages.
- Nyquist (1927) by considering the average energy in a resonator in thermal equlibrium with its environment.
- For a derivation, see any good book on Radio Astronomy (e.g., Krauss, Radio Astronomy, 1986) or for a brief outline, see Minkoff, Signals, Noise and Active Sensors, 1992.

Using Nyquist's approach, it can be shown that if an antenna is pointing at a black body at absolute temperature T, the power in a band of width Δf centered about frequency f, the power out of the antenna is

$$\Delta P_n = \frac{hf}{e^{hf/kT} - 1} \Delta f$$

Equivalently, the one-sided power spectral density (PSD) of the noise is

$$P_n(f) = \frac{hf}{e^{hf/kT} - 1}$$

$k =$ Boltzmann's constant $= 1.38 \times 10^{-23}$ (J/K)
$h =$ Planck's constant $= 6.62 \times 10^{-34}$ (J-sec)

$$\boxed{\Delta P_n = \frac{hf}{e^{hf/kT} - 1} \Delta f}$$

- This is independent of antenna gain, as long as the antenna sees a source of constant temperature.
- This would occur if the antenna was in a box with walls that were a black body at constant temperature T.
- This is well approximated if an object of temperature T fills the main beam.
- Not all sources encountered satisfy this (e.g. Radio Astronomy)

$$P_n(f) = \frac{hf}{e^{hf/kT} - 1}$$

When $hf/kT \ll 1$

$$P_n(f) = \frac{hf}{1 + \frac{hf}{kT} + o(\frac{hf}{kT}) - 1}$$

$$\approx \frac{hf}{hf/kT}$$

$$= kT$$

At microwave frequencies $hf \ll kT \Rightarrow hf/kT \ll 1$, so

$$P_n(f) = kT$$

Not a function of frequency

White Noise Approximation

- This is where the white noise assumption in microwave communications comes from.
- This is only a low frequency oximation

Clearly not true when $hf \approx kT$

(Average energy per mode approaches energy per photon)

At microwave frequencies...

$hf/kT \ll 1$ for $T = 300°\text{K}$

$$\begin{aligned} hf &= \text{energy per photon at frequency } f \\ kT &= \text{energy per second per hertz} \\ &= \text{energy (averge energy per mode)} \end{aligned}$$

kT/hf = average number of photons per mode

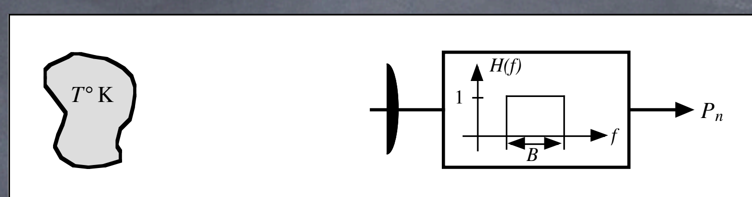
$kT/hf \gg 1 \;\Rightarrow$ Central Limit Theorem holds

$\Rightarrow$ Gaussian noise

So we can see where the white Gaussian Noise model comes from.

Noise Temperature and Noise Figure

Recall:

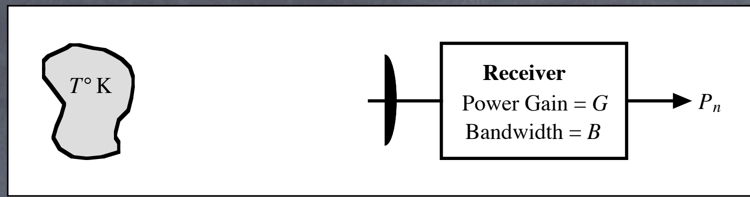


$$P_n(f) = \frac{hf}{e^{hf/kT} - 1}$$

$$\Delta P_n = \frac{hf}{e^{hf/kT} - 1}\Delta f$$

When $hf/kT \ll 1$ (true at microwave for $T = 300°$ K)

$$P_n(f) = \frac{hf}{1 + \frac{hf}{kT} + o(\frac{hf}{kT}) - 1} \approx \frac{hf}{hf/kT} = kT$$



Assume antenna pointing at object of temperature T.

Assume receiver has bandwidth B and power gain G.

The noise power at the output of the receiver is

$$P_n = kTGB$$

where

GB = *Gain-Bandwidth Product* of the receiver.

Gain-Bandwidth Product

Generally, the power gain of a receiver is a function of frequency.

We write the power gain as $G(f)$.

Then the *gain-bandwidth product* is

$$GB = \int_0^\infty G(f)\,df \qquad \text{(Gain-Bandwidth Product)}$$

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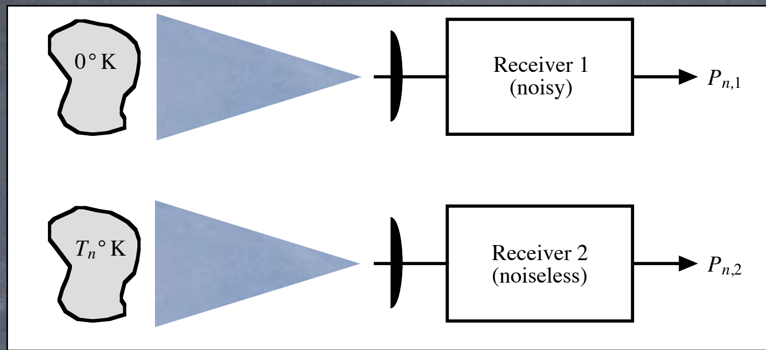
Noise Temperature

(Characterizing Microwave System Noise)



Receiver 1 is an *actual* noisy receiver. Its antenna pointed at a black body of temperature $T = 0°$ K.

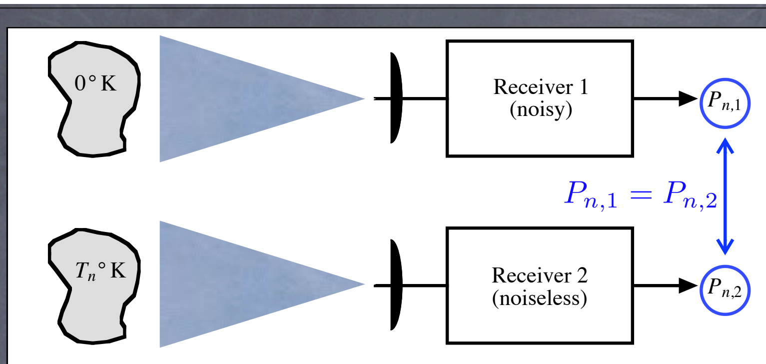
Receiver 2 is a *hypothetical* noiseless receiver. Its antenna is pointing at a black body of temperature T_n.

Assume we can adjust T_n in second scenario until

$$P_{n,1} = P_{n,2}$$

We call the T_N achieving this the *noise temperature* of Receiver 1.

6.15



- When we quote a noise temperature for a real receiver, we are referring internal receiver noise to a hypothetical external noise source.
- This is a convenient accounting trick, as it allows us to look at all noise contributions at the same location—the input to the receiver.

6.16

Noise Figure or Noise Factor

The noise figure or noise factor NF of a receiver is

$$\mathrm{NF} = \frac{T_n}{290^\circ\ \mathrm{K}} + 1$$

It is often expressed in dB:

$$\mathrm{NF}_{\mathrm{dB}} = 10\log_{10}\frac{T_n}{290\ \mathrm{K}} + 1 \qquad \text{(dB)}$$

6.17

Typical Noise Figures

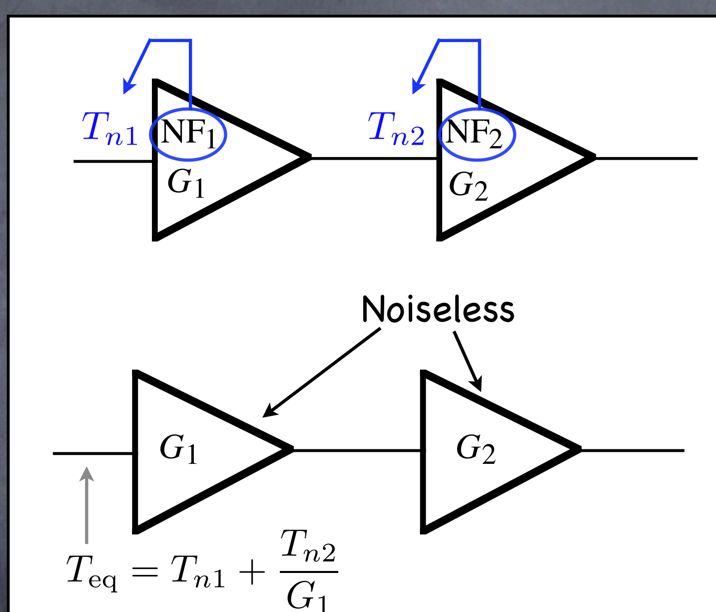
- Noise figures for typical microwave receivers used in satellite communications are in the range of 2–5 (3-7 dB).
- Radar systems are typically higher, because they are clutter—not thermal noise—limited.
- NASA DSN Stations have an overall system noise temperature of about 20°K. NF=1.069

Big Antennas, "Cool" Receivers

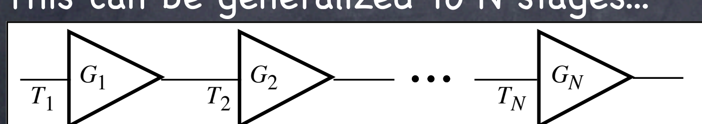
NASA DSN Antenna
70m diameter

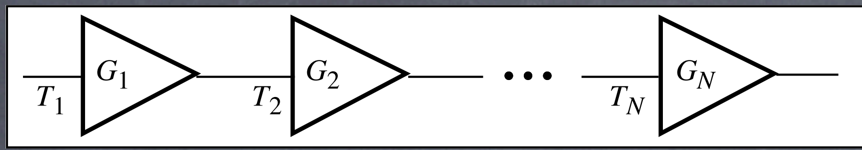
Arecibo (PR) Radio/Radar
Telescope Antenna
305m diameter

Noise Temperature/Figure of a Cascade



This can be generalized to N stages...





You can show that

$$T_e = T_{n,1} + \frac{T_{n,2}}{G_1} + \frac{T_{n,3}}{G_1 G_2} + \cdots \frac{T_{n,N}}{G_1 G_2 \ldots G_{N-1}},$$

and

$$NF_e = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} + \cdots + \frac{NF_N - 1}{G_1 G_2 \ldots G_{N-1}}$$

Hence the emphasis on Low-Noise, High-Gain front ends is obvious.

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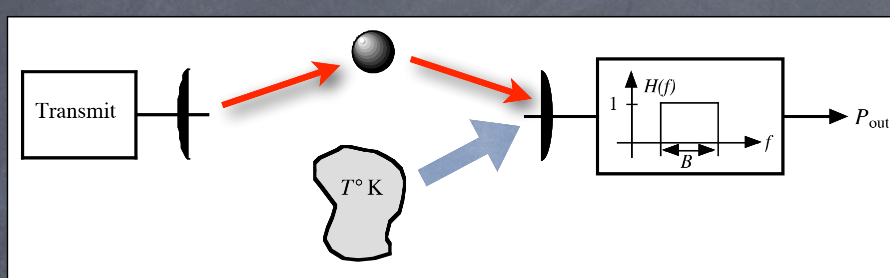
Signal-to-Noise Ratio in Radar

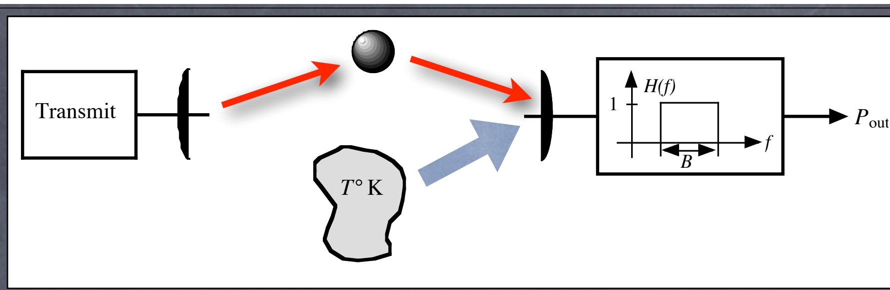
Assume we transmit a pulse of bandwidth B at frequency f_0



When we look at the output power, some is due to signal, and some is due to noise.

If the noise power is too great, we will not "see" the signal.

Signal-to-Noise Ratio is a measure of how "visible" the signal is.



We transmit a pulse of bandwidth B at frequency f_0.

The received signal power is

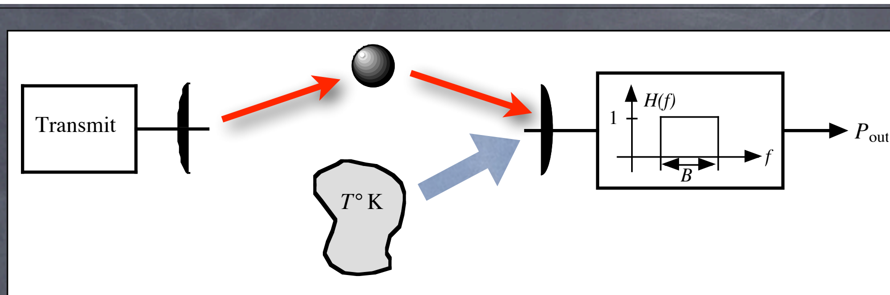
$$P_r = \frac{P_t A^2 \sigma}{4\pi \lambda^2 R^4} \qquad \text{(Radar Equation)}$$

Assume ideal bandpass filter H(f) does not effect the signal

(*n.b.* We must carefully interpret P_t and P_r:
"Peak Power", "Average Power"?—more on this later.)

The received noise power is

$$P_n = kT_eB$$



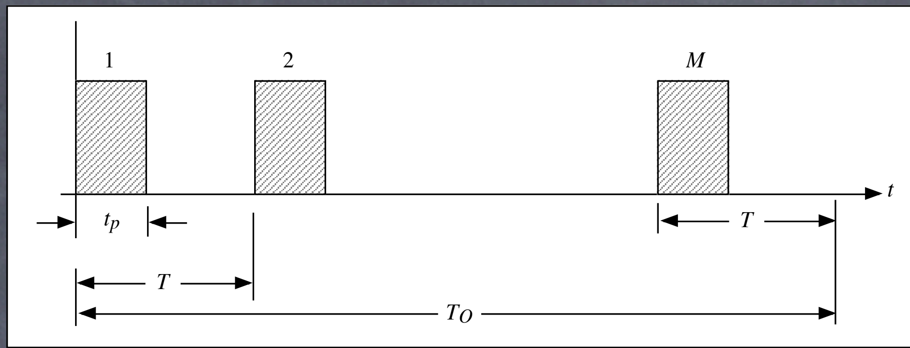
The signal-to-noise ratio is

$$\text{SNR} = \frac{P_r}{P_n} = \frac{P_t A^2 \sigma}{4\pi \lambda^2 R^4 kT_eB} \quad \longleftarrow \text{Effective Aperture}$$

$$= \frac{P_t \lambda^2 G^2 \sigma}{(4\pi)^3 R^4 kT_eB} \quad \longleftarrow \text{Antenna Gain}$$

We can think of this as the SNR for a single pulse when the pulse is "on", but many radars transmit a sequence of pulses.

6.24



t_p = Pulse Width

T = Pulse Period (PRI~Pulse Repetition Interval)

T_O = Observation Time = MT

Similarly, we can define bandwidths/frequencies

$B = 1/t_p$ = Pulse Bandwidth (unmodulated)

$f_R = 1/T$ = Pulse Repetition Frequency (PRF)

$\Delta B_{\text{Dop}} = 1/T_O$ = Doppler Resolution of Radar

6.25

If the source is not transmitting all of the time, we must be careful to note whether we are talking about Peak Power or Average Power.

Peak Power → P_T Average Power → P_{AVE}

If P_T is the peak power (power when transmitter is on)

$$P_{\text{ave}} = \frac{P_T t_p}{T} = P_T \cdot (\text{duty cycle})$$

If a coherent detector and matched filter are used to process returns, the voltage received from each pulse adds coherently:

Signal power increases as M^2

Noise power increases as M

Thus we have

$$\text{SNR}_M = M \cdot \text{SNR}_1 = \frac{M P_T A^2 \sigma}{4\pi\lambda^2 R^4 k T_e B}$$

Because P_T is the peak power when the transmitter is "on" and the signal bandwidth $B \approx 1/t_p$, we have

$$\text{SNR}_M = \frac{M P_{\text{ave}} \frac{T}{t_p} A^2 \sigma}{4\pi\lambda^2 R^4 k T_e B} = \frac{P_{\text{ave}} \frac{B}{\triangle B_{Dop}} A^2 \sigma}{4\pi\lambda^2 R^4 k T_e B}$$

$$= \frac{P_{\text{ave}} A^2 \sigma}{4\pi\lambda^2 R^4 k T_e \triangle B_{Dop}}.$$

Typically, $\Delta B_{\text{Dop}} \ll B$ and it can be that $P_{\text{ave}} \ll P_T$.

So, "who wins?"

$$\text{SNR}_M = \frac{P_{\text{ave}} A^2 \sigma}{4\pi\lambda^2 R^4 k T_e \triangle B_{Dop}}$$

Typically P_{ave} is constant. Bounded from above by hardware limitations

Noise bandwidth varies as $1/M$

Thus, SNR_M is proportional to M.

$\triangle B_{Dop}$ becomes the effective noise bandwidth.

(We will see this later.)