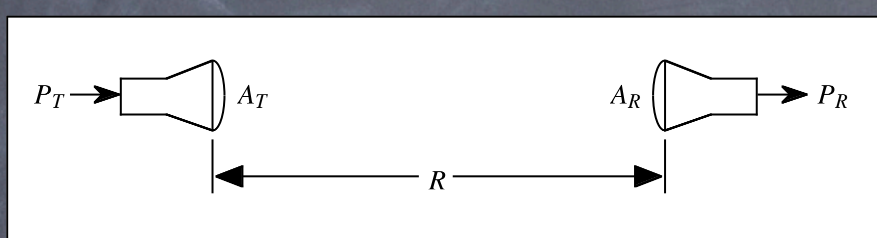
Session 4

Recall...

4.1

The Friis Equation

- Suppose we have two antennas "pointing at each other" a large distance R apart.



If

$$P_T = \text{transmitted power}$$
$$P_R = \text{received power}$$

then

$$\frac{P_R}{P_T} = \frac{A_T A_R}{\lambda^2 R^2}. \qquad \text{(Friis Equation)}$$

4.2

Antenna Gain over an Isotropic Radiator

- Isotropic Radiator: An antenna that radiates energy uniformly in all directions (transmit).

- On receive, it is equally sensitive to energy from all directions (by reciprocity).

If at a distance R from an isotropic radiator, we place a receive aperture A_R

$$\frac{P_R}{P_T} = \frac{A_R}{4\pi R^2}$$

surface area of sphere of radius R

$$\frac{P_R}{P_T} = \frac{A_R}{4\pi R^2} = \frac{A_i A_R}{\lambda^2 R^2} \quad \Rightarrow \quad \boxed{A_i = \frac{\lambda^2}{4\pi}}$$

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$$\boxed{A_i = \frac{\lambda^2}{4\pi}}$$

- Note that the usual requirements for aperture size are not met.

- However, it works in the Friis equation.

- For this reason we will find it useful.

The *gain* of A_T over A_i is

$$G = \frac{(P_R/P_T)_T}{(P_R/P_T)_i} = \frac{(P_R)_T}{(P_R)_i} = \frac{4\pi A_T}{\lambda^2}$$

So in general, the relation between the effective area A and gain G of antenna is

$$\boxed{G = \frac{4\pi A}{\lambda^2}} \qquad \boxed{A = \frac{\lambda^2 G}{4\pi}}$$

Gain is often expressed in dB:

$$G(\text{dB}) = 10 \log_{10} G \quad (\text{dB})$$

Antenna Gain...

- By reciprocity, the gain of an antenna on transmit is equal to the gain of an antenna on receive.
- The Friis Equation can be written in terms of antenna gains:

$$\frac{P_R}{P_T} = \frac{G_T G_R \lambda^2}{16\pi^2 R^2}$$

Antenna Directivity and Beam Pattern

For a uniformly illuminated aperture

$$G_T = \frac{4\pi A_{TG}}{\lambda^2}$$

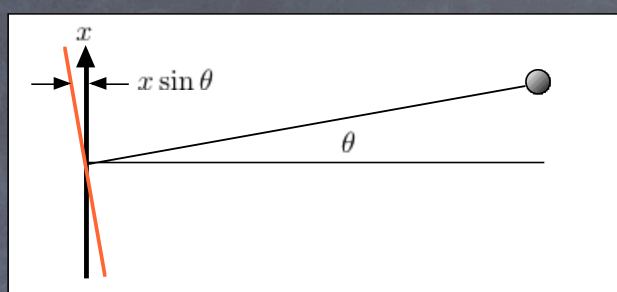
If A_{TG} is not uniformly illuminated

$$G_T = \frac{4\pi A_T}{\lambda^2} = \frac{4\pi A_{TG}}{\lambda^2}\eta$$

If we go off axis by θ (azimuth) and ϕ (elevation) the gain is not as large as in direction $(\theta, \phi) = (0, 0)$. It can be written as

$$G(\theta, \phi) = \frac{4\pi A_{\mathrm{TG}}}{\lambda^2} \cdot f(\theta, \phi)$$

Aperture Efficiency?



Looking back at the aperture off-axis we see a "virtual aperture" with a phase shift across it.

In general, at angle (θ, ϕ) we have

$$G(\theta, \phi) = \frac{4\pi A_{\mathrm{TG}}}{\lambda^2} \cdot f(\theta, \phi)$$

$$f(\theta, \phi) = \left(\frac{1}{A_{\mathrm{TG}}}\right) \frac{\left| \int\int_{A_{\mathrm{TG}}} \exp\left\{ i\frac{2\pi}{\lambda}[x\sin\theta + y\sin\phi] \right\} E(x,y)\, dx\, dy \right|^2}{\int\int_{A_{\mathrm{TG}}} |E(x,y)|^2\, dx\, dy}.$$

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$$f(\theta,\phi) = \left(\frac{1}{A_{\mathrm{TG}}}\right) \frac{\left|\int\int_{A_{\mathrm{TG}}} \exp\left\{i\frac{2\pi}{\lambda}[x\sin\theta + y\sin\phi]\right\} E(x,y)\,dx\,dy\right|^2}{\int\int_{A_{\mathrm{TG}}} |E(x,y)|^2\,dx\,dy}$$

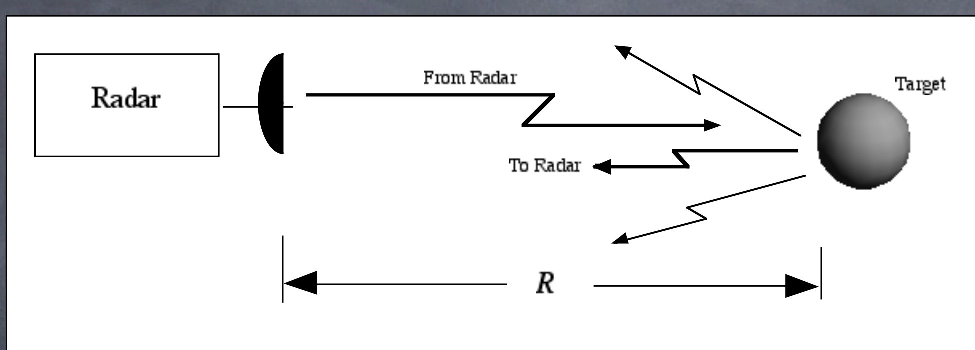
This expression acts like an efficiency, taking on values between 0 and 1.

It provides a measure of the directivity of the antenna for transmitting and receiving power.

We call this quantity the beam pattern of the antenna.

4.9

The Radar Equation



Given that power P_T is transmitted, what is the received power P_R?

To answer this, we must understand the target's behavior.

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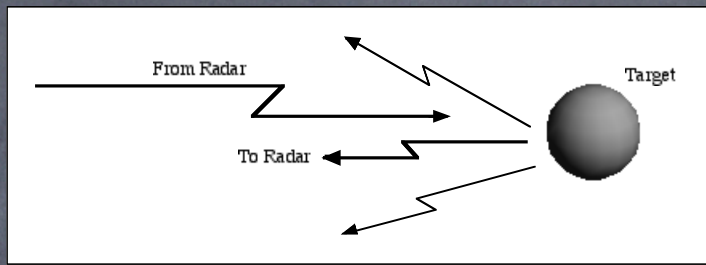
Target Scattering Characteristics



Assume target has following characteristics:

(i) As a receive aperture, it has $A_R = \sigma\,(\mathrm{m}^2)$;

(ii) It reradiates all of this received energy isotropically.

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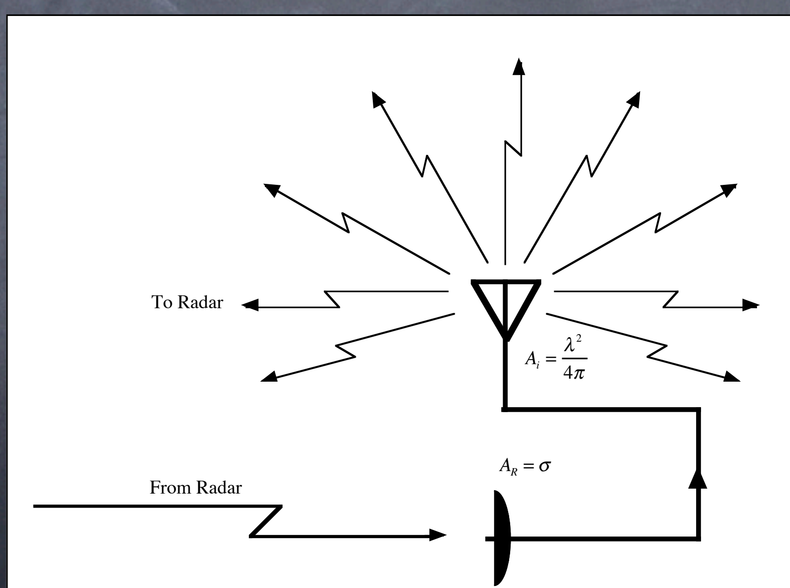
Assumed Target Characteristics

(i) As a receive aperture, it has $A_R = \sigma\,(\mathrm{m}^2)$;

(ii) It reradiates all of this received energy isotropically.



The power received by the target is given by

$$\frac{P_\sigma}{P_T} = \frac{A\sigma}{\lambda^2 R^2}$$

The fraction of the reradiated power received is

$$\frac{P_R}{P_\sigma} = \frac{A_i A}{\lambda^2 R^2} = \frac{(\lambda^2/4\pi)A}{\lambda^2 R^2} = \frac{A}{4\pi R^2}$$

It follows that

$$\frac{P_R}{P_T} = \frac{P_R}{P_\sigma} \cdot \frac{P_\sigma}{P_T} = \frac{A^2\sigma}{4\pi\lambda^2 R^4}$$

The Radar Equation

$$\boxed{\frac{P_R}{P_T} = \frac{A^2\sigma}{4\pi\lambda^2 R^4}}$$

(i) $\dfrac{P_R}{P_T}$ proportional to $\dfrac{1}{R^4}$

(ii) $\dfrac{P_R}{P_T}$ proportional to σ

Notes on Radar Equation

- Alternative form using antenna gain(s) instead of effective area can be derived.
- Bistatic version with different transmit and receive ranges and effective areas can be derived (requires generalization of radar cross section.)
- As we will see, radar cross section and geometric cross section can be quite different—don't let this throw you for now.

Radar Targets

$$\frac{P_R}{P_T} = \frac{A^2\sigma}{4\pi\lambda^2 R^4}$$

Radar Cross Section

- RCS is used to characterize the scattering characteristics of target.
- Defined in terms of hypothetical target—defines an equivalence class of targets.
- Is used to describe physical targets that behave nothing like the hypothetical target that defines it. This is OK!

RCS Contributing Factors

- Size of Object
- Shape of Object
- Wavelength of Radiation
- Material(s) Object is Made of
- Orientation w.r.t. Radar

Table 4.1: Typical Values fo the Radar Cross Section of some Common Objects.

Object	RCS (m^2)
Small Insect (fly)	10^{-5}
Large Insect (locust)	10^{-4}
Medium-Sized Bird	0.001
Large Bird	0.01
Small Open Boat	0.02
Small Missile	0.1
Man	1
Small Single-engine Airplane	1
Small Fighter or Four-Passenger Jet	2
Helicopter	2
Bicycle	2
Small Pleasure Boat (20–30 ft.)	2
Large Tactical Fighter Airplane	6
Cabin Cruiser (40–50 ft.)	10
Large Bomber or Commercial Airliner	40
Jumbo Jet	100
Automobile	100
Pickup Truck	200
Ship	3000–1000000

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For real targets, we almost never know the value of the RCS a priori.

We may know a range of values that σ may lie in:

$$\sigma_L \le \sigma \le \sigma_U$$

Sometimes it makes sense to treat σ as a random variable:

$$\sigma(\omega) \text{ defined on } (\mathcal{S}, \mathcal{F}, P)$$

Sometimes it makes sense to treat σ as a random process:

$$\sigma(t, \omega) \text{ defined on } (\mathcal{S}, \mathcal{F}, P)$$

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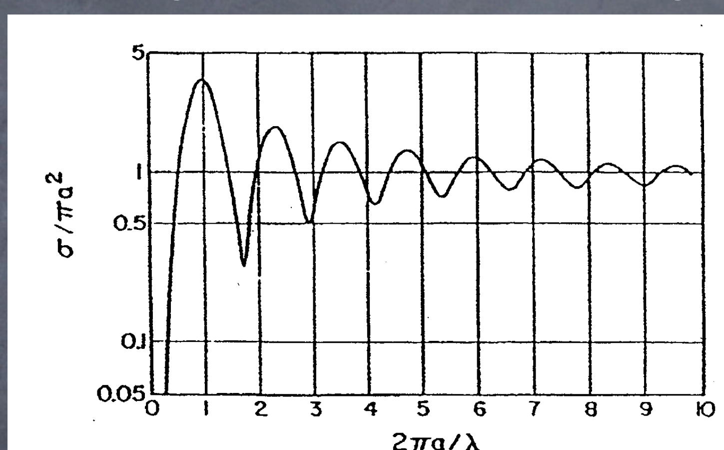
RCS of a Sphere

A strong function of wavelength



The *Optical Region*, where $2\pi a/\lambda > 20$

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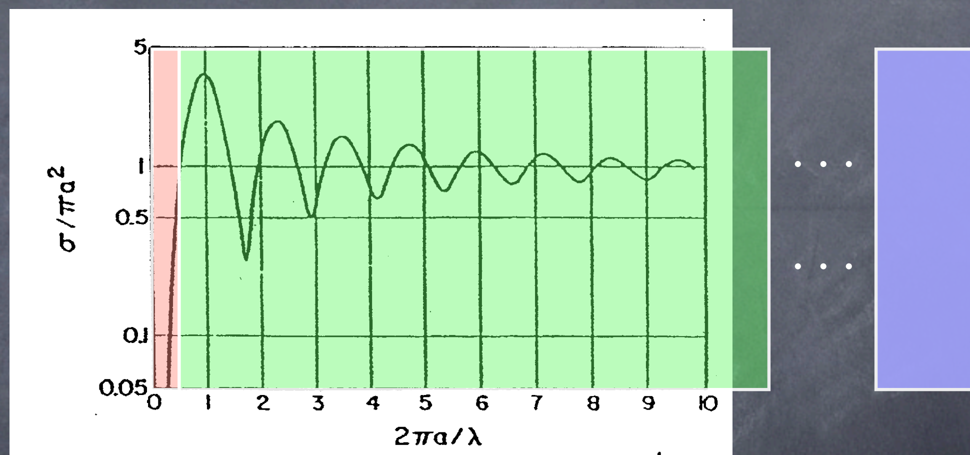
RCS of a (Perfectly Conducting) Sphere

A strong function of wavelength



The *Rayleigh Region*, where $2\pi a/\lambda < 0.4$

The *Mie Region*, where $0.4 < 2\pi a/\lambda < 20$

The *Optical Region*, where $2\pi a/\lambda > 20$

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RCS of a (Perfectly Conducting) Sphere

In the *Optical Region*, $\sigma \approx \pi a^2$. This is the geometric cross section of a sphere.

Because a sphere is invariant to changes in its orientation, it makes a cconvenient calibration target.

In the *Rayleigh Region*, where $\lambda >> a$,

$$\sigma \approx 9\pi a^2 \left(\frac{2\pi a}{\lambda}\right)^4 = \pi a^2 \left[9(ka)^4\right]$$

RCS of a (Perfectly Conducting) Sphere

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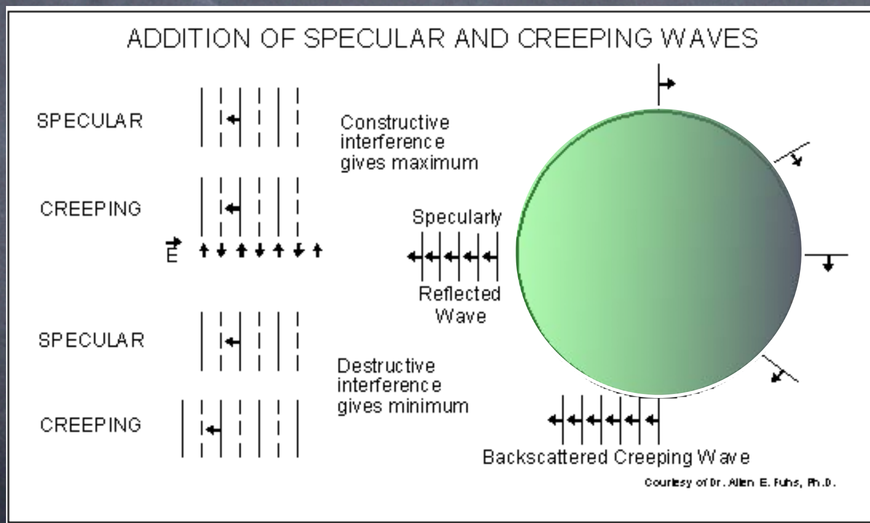
In the *Mie Region*, where $\lambda \approx a$, "creeping waves" travel around the sphere and interfere with the specular reflection:



This gives rise to the "resonance" seen in this region.

RCS of a Perfectly Conducting Plate

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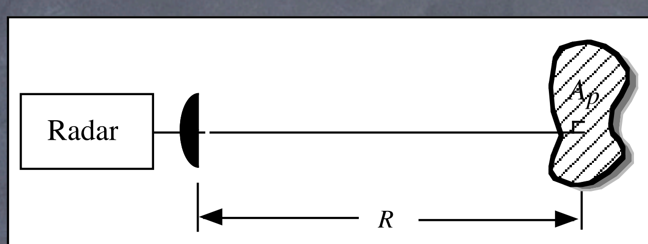
Consider a perfectly conducting plate with dimensions. much greater than λ. Assume area A_p.



Assume perpendicular orientation to incident wave.

In the far field, the plate is uniformly illuminated.

The plate reflects or radiates the wave as if it were a uniformly illuminated aperture of area A_p. (It is!)

Applying the Friis equation twice—once for each trip—we get

$$\frac{P_R}{P_T} = \frac{P_\sigma}{P_T} \cdot \frac{P_R}{P_\sigma} = \frac{AA_p}{\lambda^2 R^2} \cdot \frac{A_p A}{\lambda^2 R^2} = \frac{A^2 A_p^2}{\lambda^4 R^4}$$

But

$$\frac{P_R}{P_T} = \frac{A^2 \sigma}{4\pi \lambda^2 R^4}$$

Equating these expressions, we have

$$\frac{A^2 A_p^2}{\lambda^4 R^4} = \frac{A^2 \sigma}{4\pi \lambda^2 R^4} \quad \Longrightarrow \quad \boxed{\sigma = \frac{4\pi A_p^2}{\lambda^2}.}$$