Session 27

Final Exam

FE.1

Two time slots:

- Wednesday, May 5, 3:00 – 5:00 pm (EDT)
- Thursday, May 6, 7:00 – 9:00 pm (EDT)

Please let me know which of these two time-slots you would like (mrb@ecn.purdue.edu)

Open Book, Open Notes, Calculator is allowed.

The Final Exam will consist of 4 questions from the following areas:

FE.2

- Antennas, Radar Equation / Friis Equation Application
- Detection Theory
- Ambiguity Functions
- Array Antenna Fundamentals
- SAR Basics
- Radar Target Detection

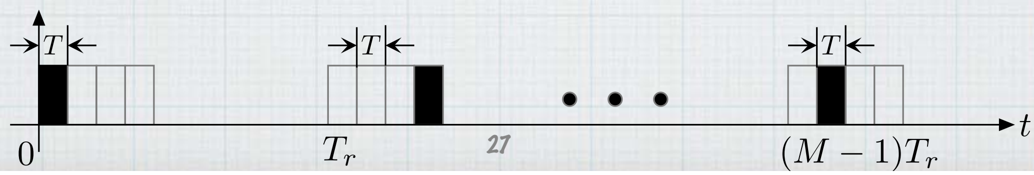
Computationally Efficient Pulse-Doppler Processing of Nonuniform Coherent Pulse Trains*

Shahzada Rasool and Mark R. Bell
School of Electrical and Computer Engineering
mrb@ecn.purdue.edu

* You are not responsible for the material in this lecture on the Final Exam.

Efficient Processing of Nonuniform Coherent Pulse Trains

- A number of investigators (e.g., Resnick, Rihaczek) have considered the use of nonuniform coherent pulse trains because of their low range sidelobes.
- Nonuniform pulse trains "break up" the "bed-of-nails" delay-Doppler response of uniform pulse trains processed with standard coherent pulse-Doppler processing.
- The most significant drawback to the use of coherently processed nonuniform pulse trains is the lack of a computationally efficient approach to formulating a bank of Doppler matched filters. (Pulse-Doppler Processing)
- In this work, we show a computationally efficient approach to generating a bank of Doppler matched filters for a class of nonuniform pulse trains.
- These nonuniform pulse trains are based on *Pulse Position Modulation* (PPM).



The Matched Filter

Matched Filter: The L.T.I. filter that maximizes SNR when detecting a signal $s(t)$ in the presence of wide-sense stationary additive noise with PSD $S_{nn}(f)$ is given by

$$H(f) = \frac{kS^*(f)e^{-i2\pi fT}}{S_{nn}(f)},$$

where k is an arbitrary constant, and

$$S(f) = \int_{\mathbf{R}} s(t)e^{-i2\pi ft}\,dt.$$

When the noise is white with PSD $S_{nn}(f) = N_0/2$, the filter that maximizes the SNR has impulse response

$$h(t) = s^*(T-t),$$

(setting $k = N_0/2$.) The resulting SNR is

$$\mathrm{SNR}_T = \frac{2E_s}{N_0}.$$

27.4

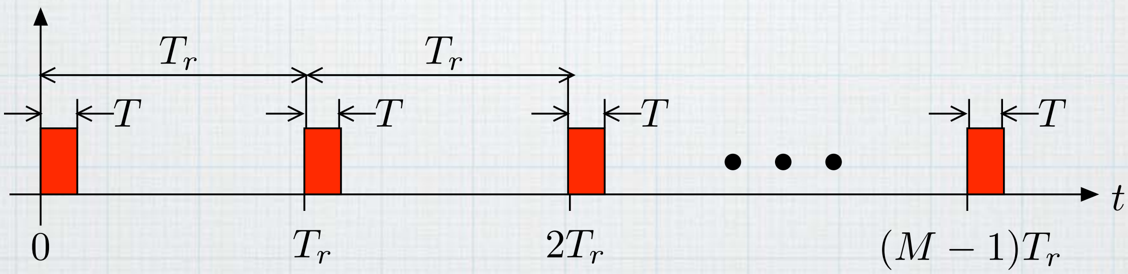
Processing a Uniform Pulse Train

Let $s(t)$ be a complex baseband uniform pulse train of the form

$$s(t) = \sum_{m=1}^{M} a_m p(t - (m-1)T_r),$$

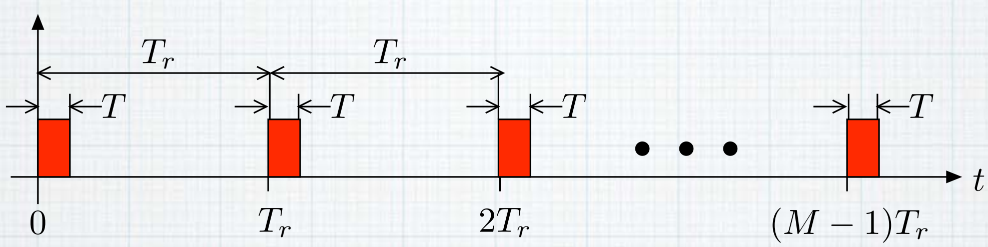
where the a_m are (generally complex) amplitude weighting coefficients and T_r is the interpulse period (IPP).

PRI = 1/PRF



27.5

Processing a Uniform Pulse Train



We find the matched filter that maximizes the output at time $t = MT_r$:

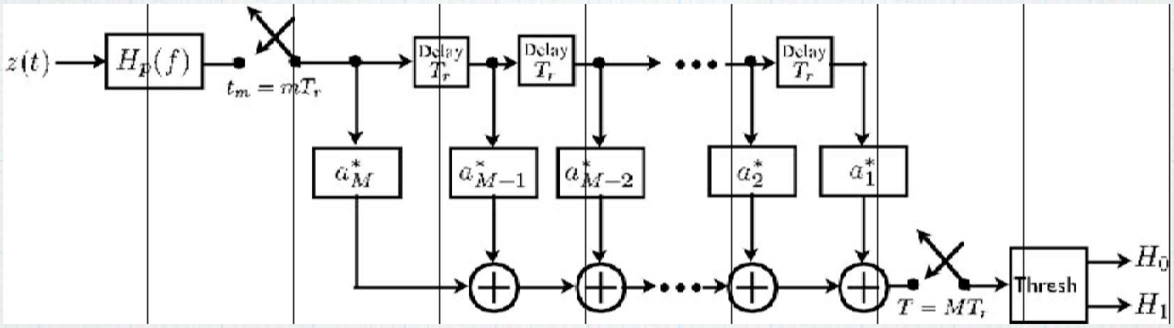
$$\begin{aligned}
\tilde{H}_M(f) &= \frac{S^*(f)e^{-i2\pi f MT_r}}{S_{nn}(f)} = \frac{\sum_{m=1}^{M} a_m^* P^*(f)e^{i2\pi f(m-1)T_r}}{S_{nn}(f)} e^{-i2\pi f MT_r} \\
&= \frac{P^*(f)}{S_{nn}(f)} e^{-i2\pi f T_r} \cdot \sum_{m=1}^{M} a_m^* e^{-i2\pi f(M-m)T_r} \\
&= H_p(f) \cdot \sum_{m=1}^{M} a_m^* e^{-i2\pi f(M-m)T_r},
\end{aligned}$$

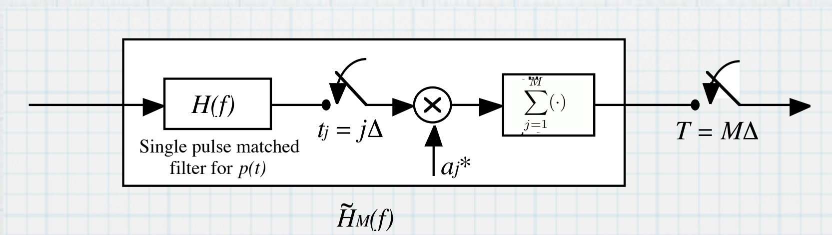
where $H_p(f)$ is the matched filter for the single pulse $p(t)$ sampled at time $t = T_r$.

27.6

Processing a Uniform Pulse Train

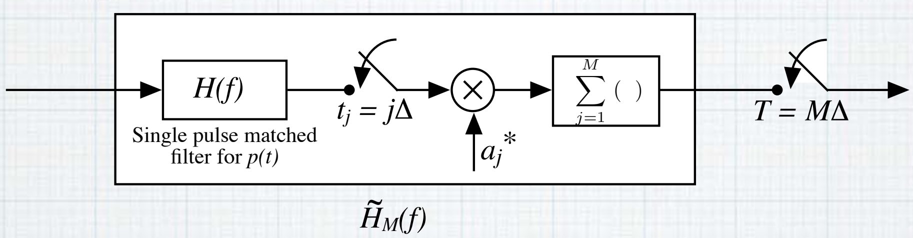
$$\tilde{H}_M(f) = H_p(f) \cdot \sum_{m=1}^{M} a_m^* e^{-i2\pi f(M-m)T_r}$$



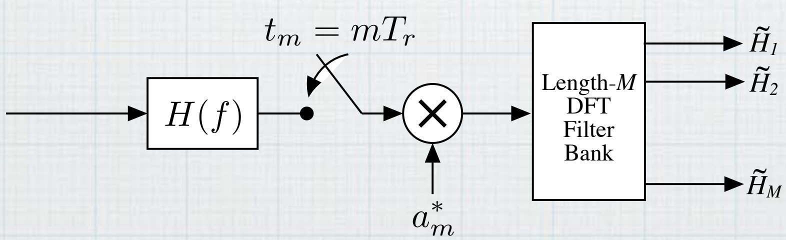


27.7

Processing a Uniform Pulse Train



- The pulse-Doppler processor implementing a bank of matched filters for this pulse train has the following form:



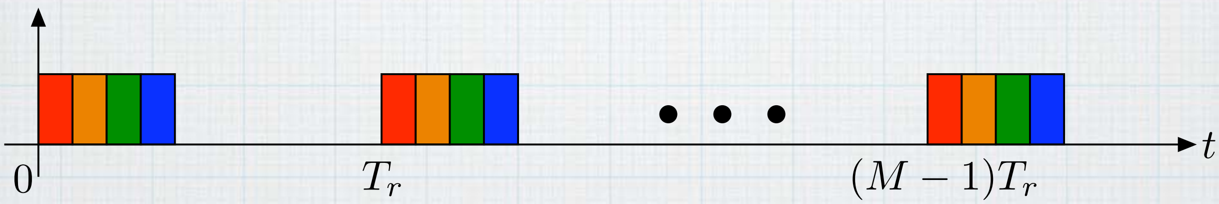
27.8

Processing Nonuniform Pulse Trains

Let $s(t)$ be a complex baseband uniform pulse train of the form

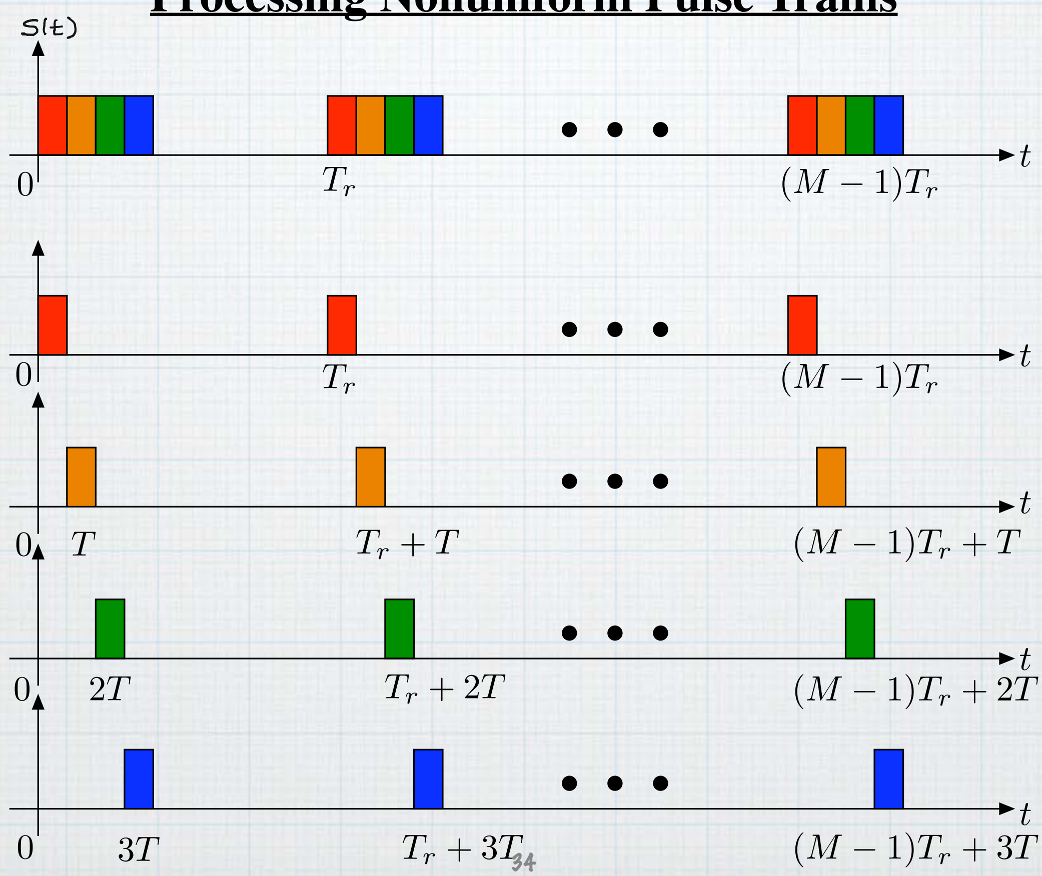
$$s(t) = \sum_{m=1}^{M} \sum_{j=1}^{J} a_{mj} p(t - (m-1)T_r - (j-1)T),$$

where the a_{mj} are (generally complex) amplitude weighting coefficients,T_r is the interpulse period (IPP), and T is the pulse width.



27.9

Processing Nonuniform Pulse Trains



27.10

Processing Nonuniform Pulse Trains

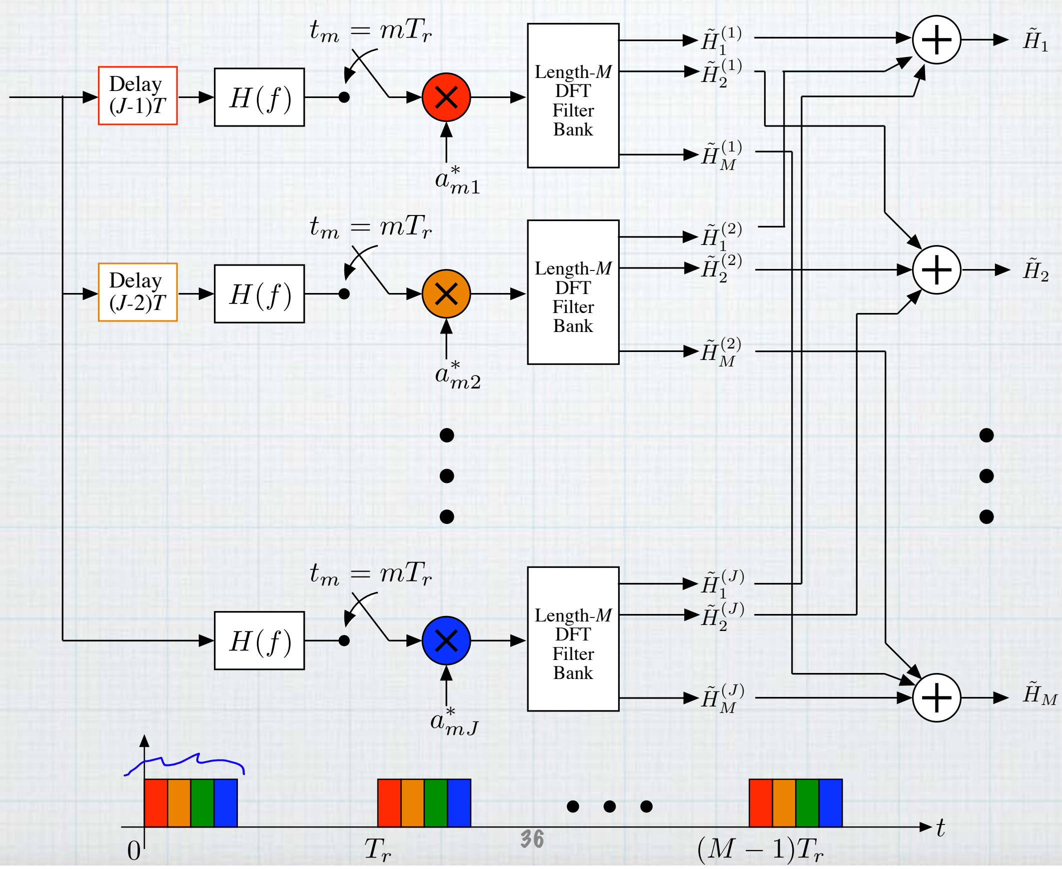
We find the matched filter that maximizes the output at time $t = MT_r$:

$$
\begin{aligned}
\tilde{H}_M(f) &= \frac{S^*(f)e^{-i2\pi f M T_r}}{S_{nn}(f)} \\
&= \frac{\sum_{m=1}^{M}\sum_{j=1}^{J} a_{mj}^* P^*(f) e^{i2\pi f((m-1)T_r+(j-1)T)}}{S_{nn}(f)} e^{-i2\pi f M T_r} \\
&= \frac{P^*(f)}{S_{nn}(f)} e^{-i2\pi f T_r} \cdot \sum_{m=1}^{M}\sum_{j=1}^{J} a_{mj}^* e^{-i2\pi f(M-m)T_r} e^{-i2\pi f(J-j)T} \\
&= \frac{P^*(f)}{S_{nn}(f)} e^{-i2\pi f T_r} \left[\sum_{j=1}^{J} e^{-i2\pi f(J-j)T} \left(\sum_{m=1}^{M} a_{mj}^* e^{-i2\pi f(M-m)T_r}\right)\right]
\end{aligned}
$$

This tells us we can process each of the J pulse "phases" as a separate uniform pulse train and add the results to get the complete matchd filter response for the multiphase pulse train.

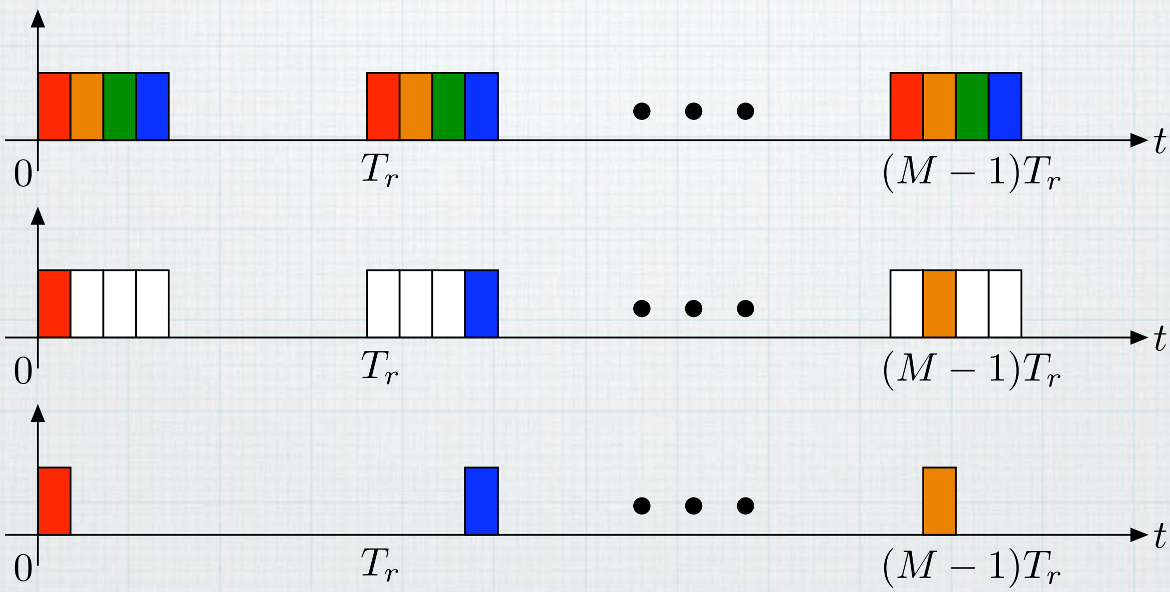
27.11

Processing Nonuniform Pulse Trains



Nonuniform PPM Pulse Trains

So how do we generate nonuniform PPM pulse trains? For each $m = 1, \ldots, M$, we set all but one a_{mj} equal to zero:



PPM Firing Sequence and Matrix

- The PPM firing sequence can be represented by the $M \times J$ array of amplitude coefficients. Here for each row m there is only one nonzero coefficient in the j-th column indicating that the j-th PPM position is occupied in the m-th pulse.
- Here is a Costas firing array for $M = J = 16$:

$$A = (a_{mj}) = \begin{pmatrix}
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}$$

Autocorrelation Functions of Pulse Trains

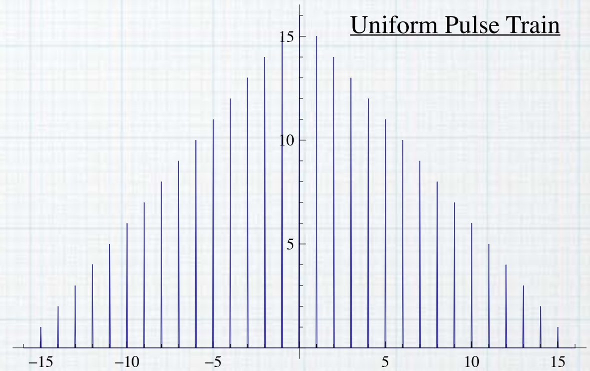
Waveform Parameters:

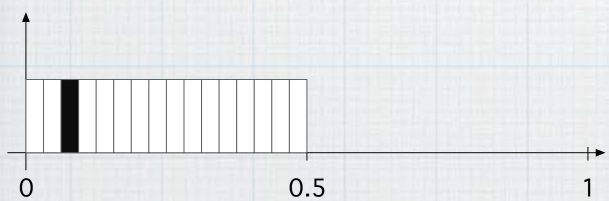
$M = 16$

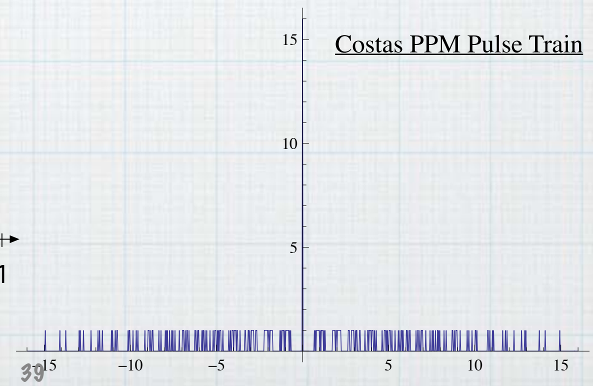
$T_r = 1.0$

$T = 1/32 = 0.03125$

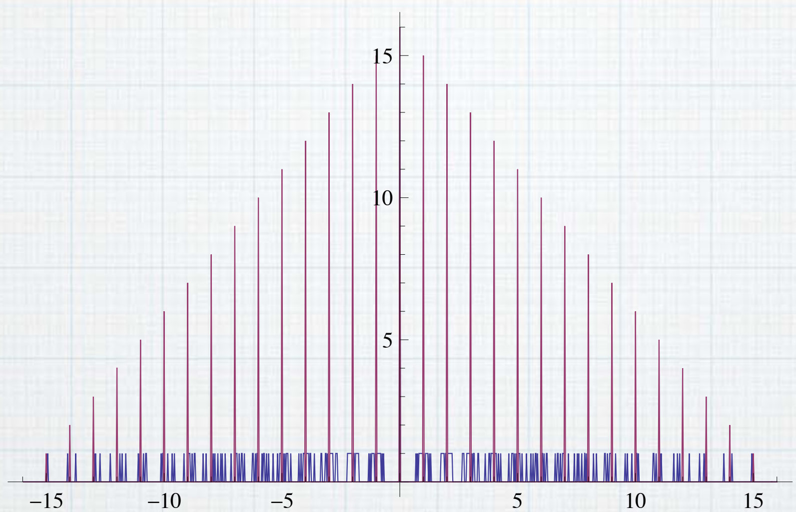
$J = \text{No. PPM Slots} = 16$







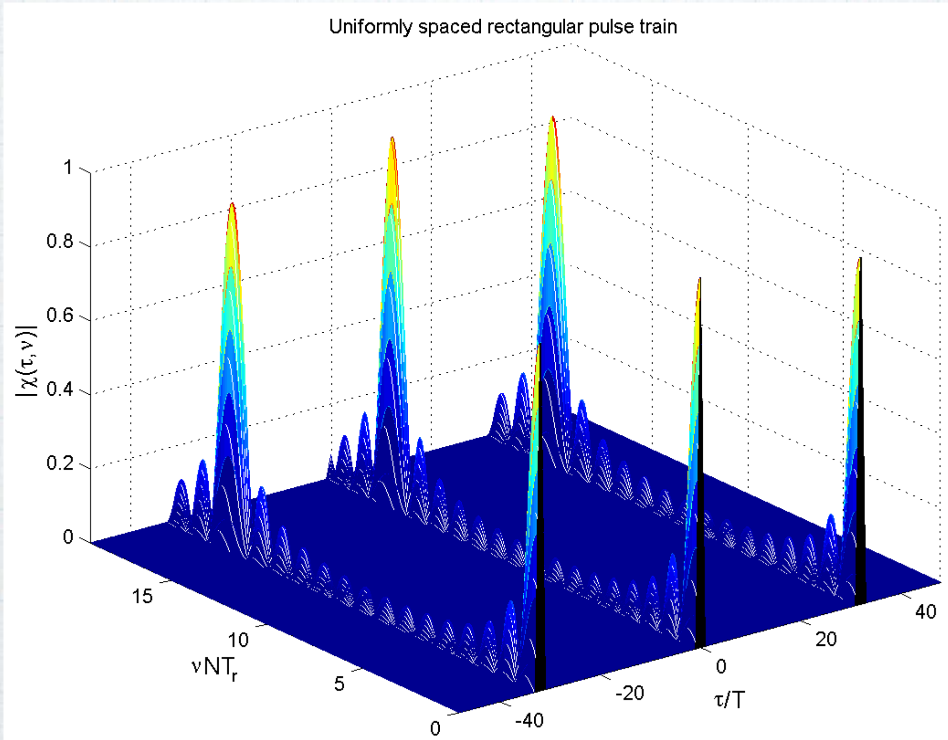
Comparison of Autocorrelation Functions of Pulse Trains



- Significant reduction in ambiguous response (ambiguity volume) of nonuniform PPM pulse train along the zero-Doppler axis.
- Significant reduction in range ambiguity of nonuniform PPM pulse train when compared to uniform waveform.
- Woodward will not be mocked—the ambiguity volume must be shifted elsewhere in the delay-Doppler plane.
- The ambiguity volume *can* be shifted elsewhere in the delay-Doppler plane.

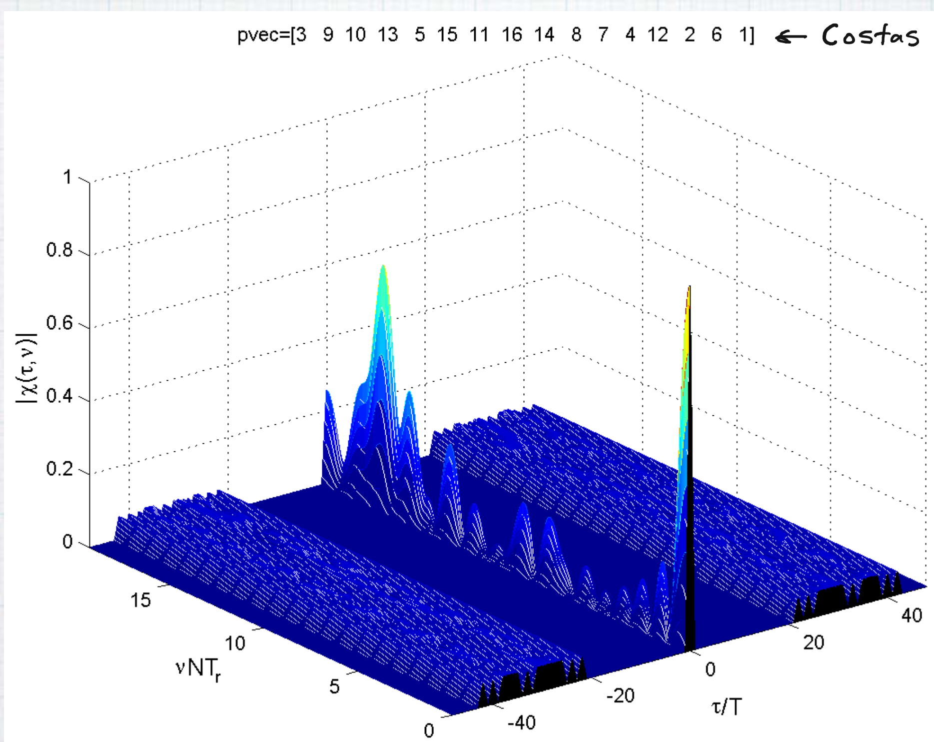
Ambiguity Function of Uniform Pulse Train

27.16



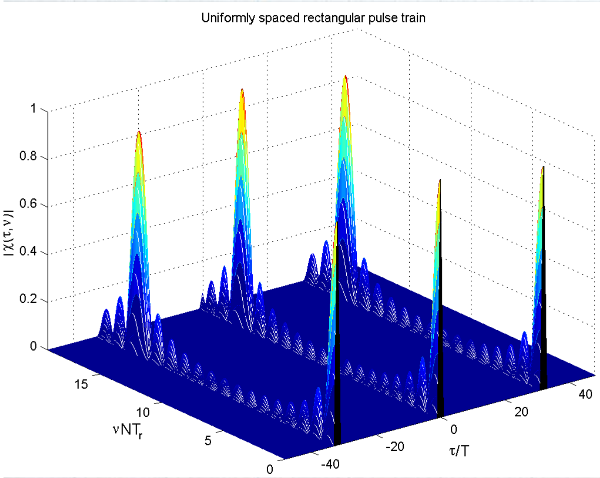
Ambiguity Function of Nonuniform Pulse Train

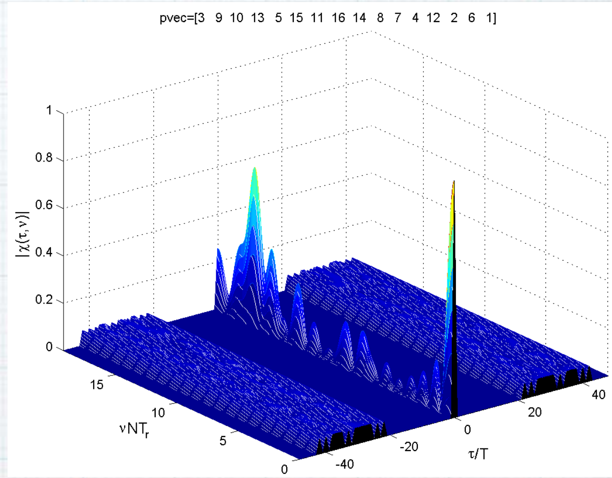
27.17



Comparison of Ambiguity Functions of Pulse Trains

27.18





Ambiguity Function of Nonuniform Pulse Train

27.19

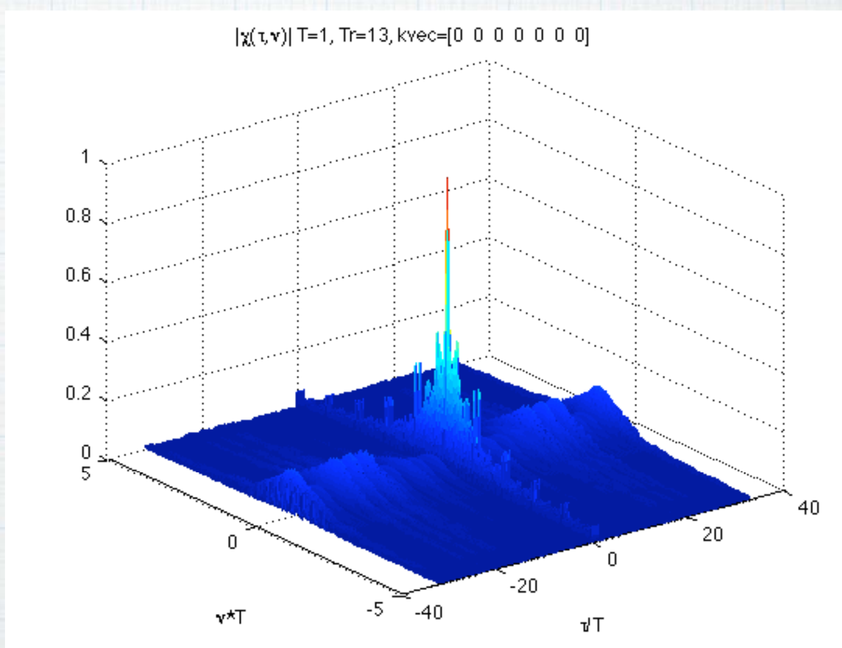
Waveform Parameters:

$M = 13$

$T_r = 13$

$T = 1$

No. PPM Slots $= 13$



27.20

Ambiguity Function of Nonuniform Pulse Train

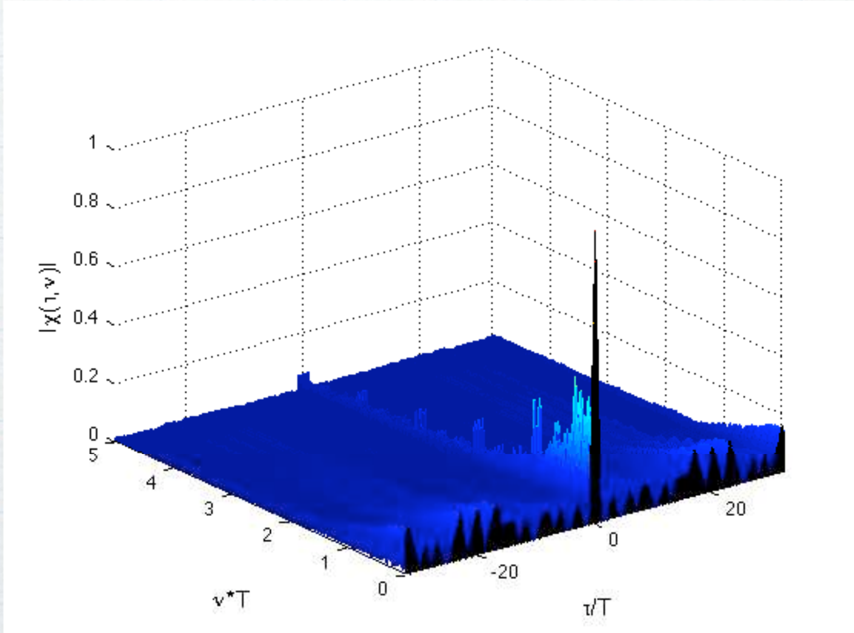
Waveform Parameters:

$M = 13$

$T_r = 13$

$T = 1$

No. PPM Slots = 13



27.21

What about non-Costas firing sequences $A = [a_{mj}]$?

In general, there are J^M possible waveforms we could transmit with our PPM scheme

If we wanted a unique slot in each pulse, there are

$$J(J-1)(J-2)\cdots(J-M+1) = \frac{J!}{(J-M)!}$$

if $J \geq M$.

We have not yet found any particularly good non-Costas firing sequences, but with so many to choose from, there must be some good ones.