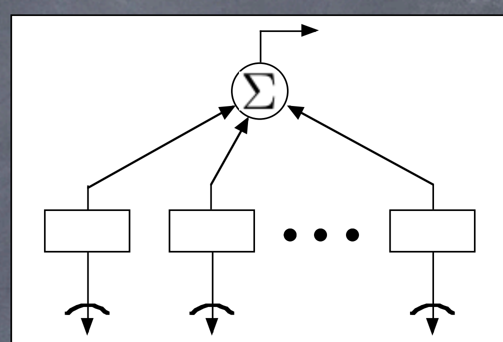
Session 20

Synthetic Arrays

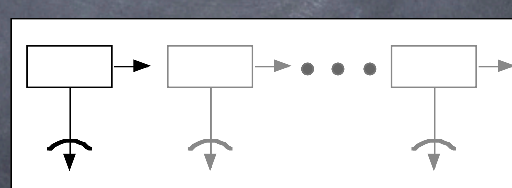
20.1

A real array:

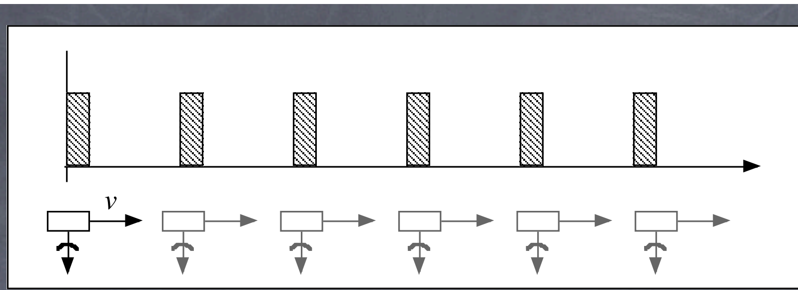


A synthetic array:

Another approach is to use a single element and move it between observations



Signal processing is used to synthesize an "equivalent" array.



The received signal is recorded with phase information.

We collect data at each position.

We apply proper phase shifts to the received data.

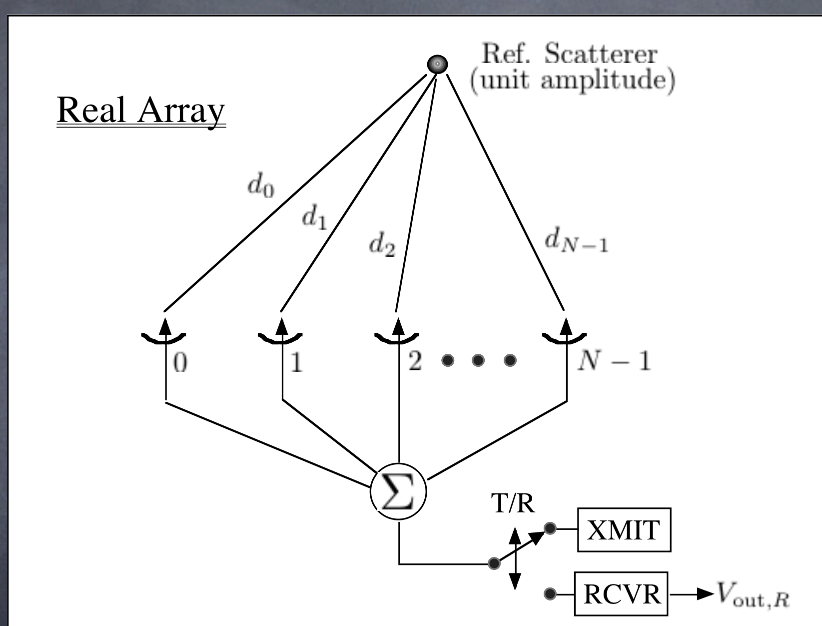
We sum to synthesize an array antenna.

This arrray—being sequentially generated—is a little different than a real array.

It does have high angular resolution like a real array.

This is the approach used in Synthetic Aperture Radar (SAR).

Comparison of Real and Synthetic Arrays



20.4

Real Array

The complex field at the n-th element is

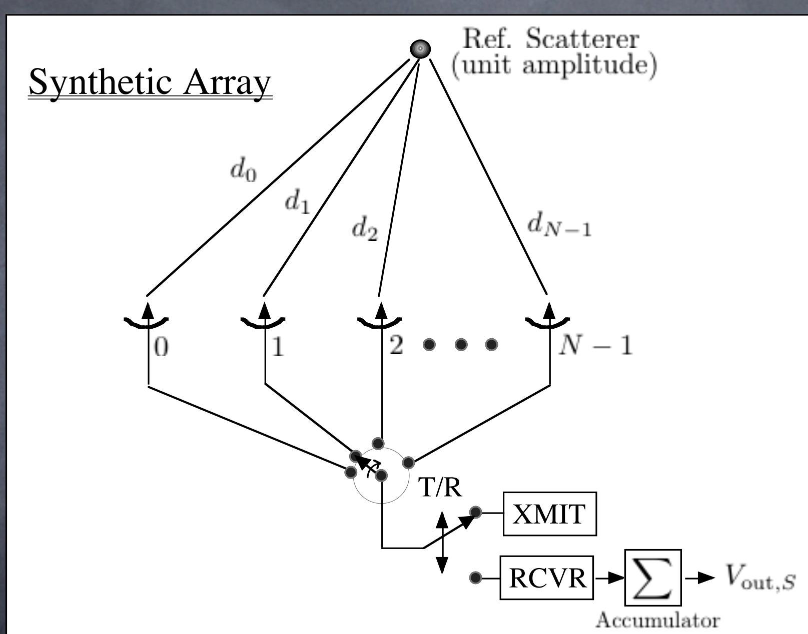
$$v_n = \sum_{m=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}(d_m + d_n)\right\}$$
$$= \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\} \sum_{m=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_m\right\},$$
$$m = 0, 1, \ldots, N-1.$$

The output of the entire array is the sum

$$V_{\text{out},R} = \sum_{n=0}^{N-1} v_n = \sum_{n=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\} \sum_{m=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_m\right\}$$
$$= \left[\sum_{n=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

20.5

Synthetic Array



The synthetic array is sequentially built, one element at a time.

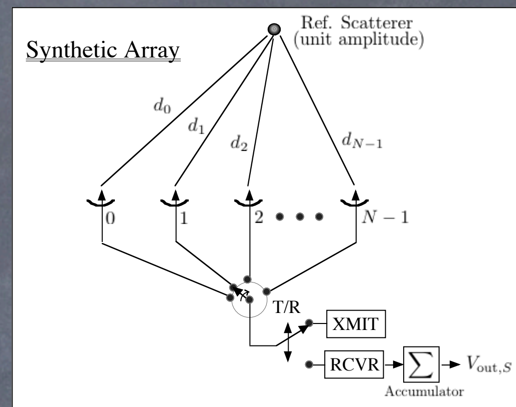
20.6

Synthetic Array Response

The response of the n-th array element is

$$v_n = \exp\left\{-i\frac{2\pi}{\lambda}(d_n + d_n)\right\}$$

$$= \exp\left\{-i\frac{4\pi}{\lambda}d_n\right\}$$



The synthetic array output response is

$$V_{\text{out},S} = \sum_{n=0}^{N-1} v_n = \sum_{n=0}^{N-1} \exp\left\{-i\frac{4\pi}{\lambda}d_n\right\} = \sum_{n=0}^{N-1}\left[\exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

20.7

Real Aperture:

$$V_{\text{out},R} = \left[\sum_{n=0}^{N-1} \exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

(Square of Sum)

cross terms!

Synthetic Aperture:

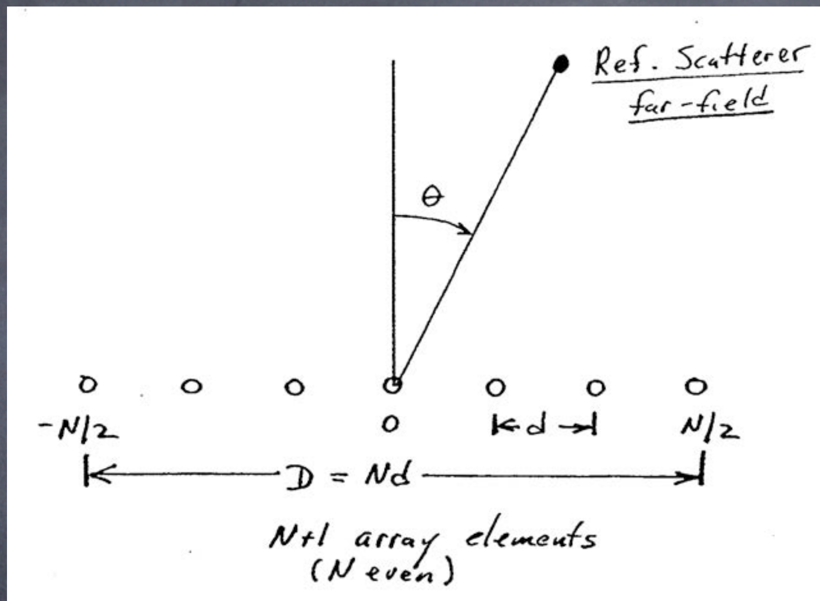
$$V_{\text{out},S} = \sum_{n=0}^{N-1}\left[\exp\left\{-i\frac{2\pi}{\lambda}d_n\right\}\right]^2$$

(Sum of Squares)

no cross terms!

20.8

Let's Compare with Identical Array Geometries



Both arrays have $N+1$ identical isotropic elements.

Elements distributed along line with separation d.

Reference scatterer in far-field.

20.9

$$V_{\text{out},R}(\theta) = \left[\sum_{n=-N/2}^{N/2} \exp\left\{ +i\frac{2\pi}{\lambda} nd\sin\theta \right\} \right]^2$$

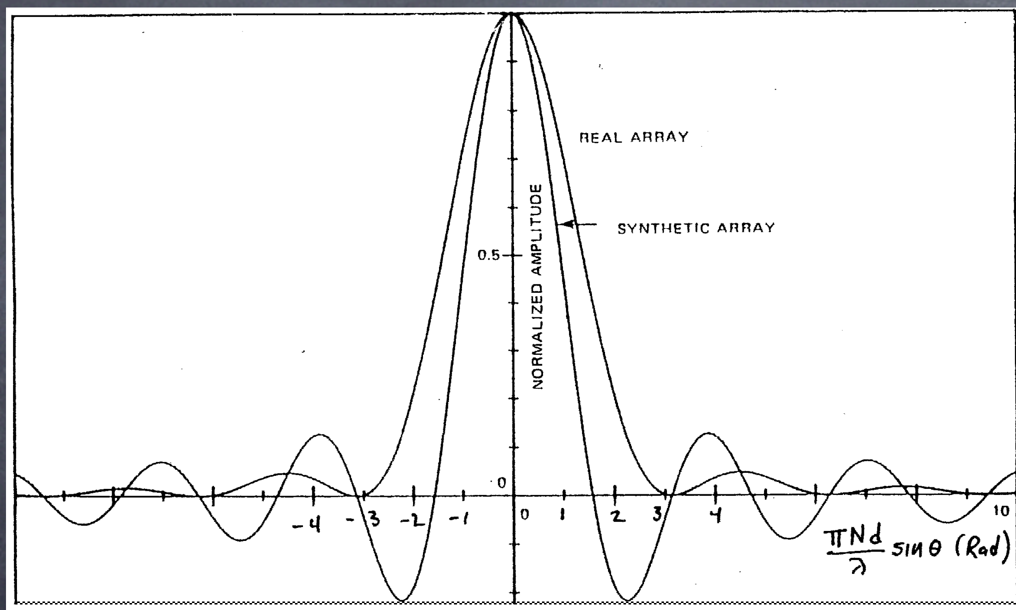
$$= \boxed{\left[\frac{\sin\left[\frac{\pi d}{\lambda}(N+1)\sin\theta\right]}{\sin\left[\frac{\pi d}{\lambda}\sin\theta\right]} \right]^2}$$

$$V_{\text{out},s}(\theta) = \sum_{n=-N/2}^{N/2} \exp\left\{ i\frac{4\pi}{\lambda} nd\sin\theta \right\}$$

$$= \boxed{\frac{\sin\left[(N+1)\frac{2\pi d\sin\theta}{\lambda}\right]}{\sin\left[\frac{2\pi d\sin\theta}{\lambda}\right]}}$$

20.10

A Plot of the Responses



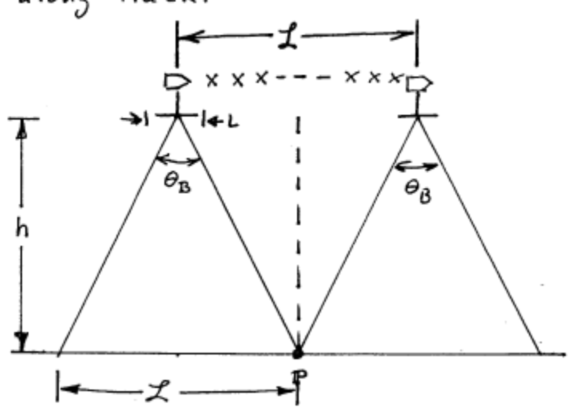
The synthetic array has higher resolution, but also higher sidelobes.

20.11

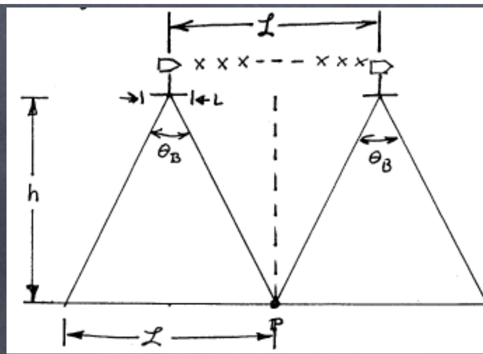
So it appears we can make a very (perhaps arbitrarily) long aperture using the synthetic aperture technique.

There are, however, some limitations.

Consider a SAR having physical antenna of size L along track.



20.12



The beamwidth of the physical antenna is

$$\theta_B \simeq \frac{\lambda}{L}$$

If the radar is at height h, what is the path length $\mathcal{L}$ illuminated on the ground?

$$\mathcal{L} = h\,\theta_B = \frac{h\lambda}{L}$$

Now if we image a point P starting when it enters the beam and ending when it leaves the beam, we get a synthetic aperture of size $\mathcal{L}$.

This synthetic aperture has has a beamwidth

$$\theta_s = \frac{\lambda}{2\mathcal{L}} = \frac{\lambda}{2\frac{h\lambda}{L}} = \frac{L}{2h}$$

20.13

Thus the azimuth resolution on the ground is

$$X_a = h\,\theta_s = h\left(\frac{L}{2h}\right) = \frac{L}{2} \Big\} \text{ Half the antenna length.}$$

⇒ Small physical antenna gives high resolution.

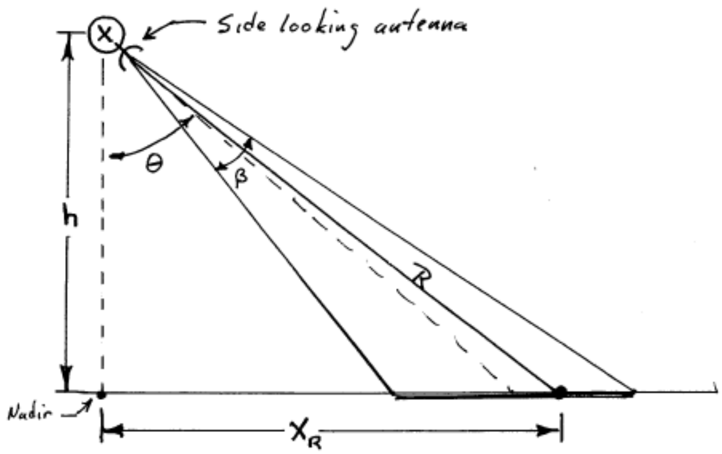
⇒ Ultimate resolution X_a is <u>not</u> a function of distance from surface.

<u>Bottom Line</u>: In order to generate a large synthetic aperture, you must have a broad illumination pattern.

- The farther radar is from surface, the larger the footprint on the ground.
  - ⇒ Larger synthetic aperture
  - ⇒ finer synthetic beamwidth

  This exactly counterbalances the increase in distance h, giving X_a independent of h.

20.14



The distance X_R can be determined by noting that the range R to the point can be related to X_r by

$$\frac{X_R}{R} \simeq \sin\theta \quad (\beta << \theta)$$

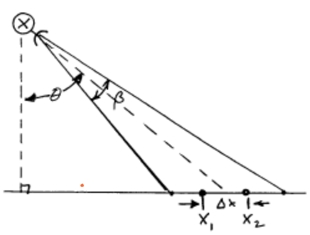
Then R can be determined by the time delay required to receive the signal

$$2R = c\tau \Rightarrow R = \frac{c\tau}{2}$$

$$\therefore X_R \simeq R\sin\theta = \frac{c\tau}{2}\sin\theta \quad (\beta << \theta).$$

20.15

What about the ability to resolve two targets closely spaced in range?



If two points are separated by Δx in "cross track" dimension, then there echos will be separated by time difference

$$\Delta t \cong \frac{2\Delta x}{c}\sin\theta \Rightarrow \Delta x \simeq \frac{c\Delta t}{2\sin\theta}$$

If $\Delta\tau$ is the smallest time delay difference that can be resolved (can be determined from ambiguity function), then

$$\Delta\tau \cong \frac{1}{B} \qquad B = \text{signal bandwidth (one-sided)}$$

Thus if two points are resolved, $\Delta t \geq \Delta\tau$, and hence

$$\Delta x_{min} = \frac{c\Delta\tau}{2\sin\theta} = \frac{c}{2B\sin\theta}$$

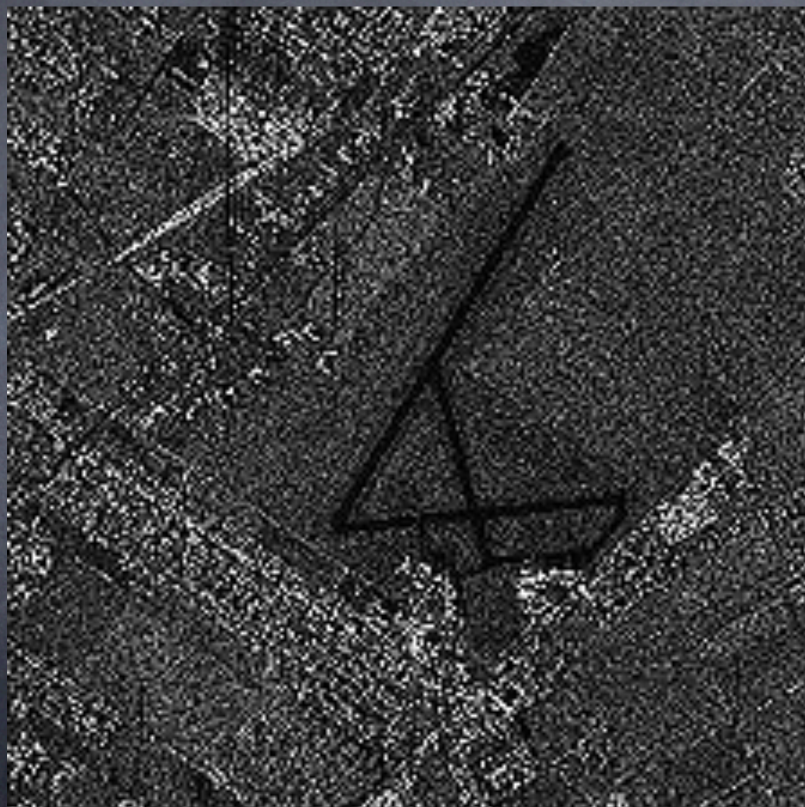
e.g. $B = 20\,MHz$ and $\theta = 20°$

$$\Rightarrow \Delta x_{min} = \frac{3\times10^8}{2(20MHz)\sin 20°} \simeq 22m$$

at $\theta = 45° \Rightarrow \Delta x_{min} \cong 10m$.

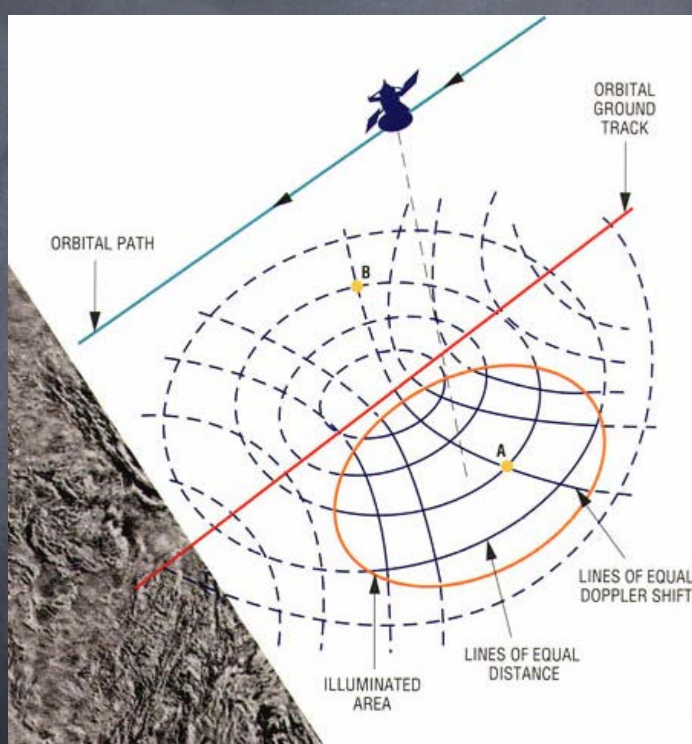
20.16

SAR Image of Airfield



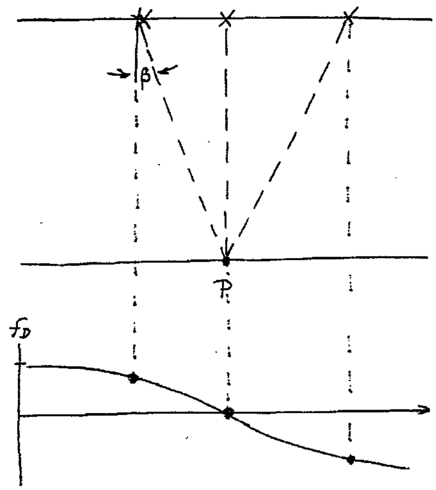
20.17

Doppler Synthesis Approach to SAR



The Doppler Synthesis Approach to SAR

20.18



As a radar flies over a target P, the Doppler shift in the signal reflected from P will change at various positions along the path.

The received signal will have a frequency $f_R = f_c + f_D$.

$$f_D = \frac{2v}{\lambda} \cos\left(\frac{\pi}{2} - \beta\right) = \frac{2v}{\lambda} \sin \beta.$$

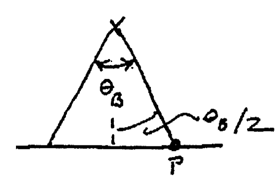
20.19

The entire range of received frequencies we would expect to see over the entire synthetic aperture is

$$f_c \pm f_{D,\max},$$

where

$$f_{D,\max} = \frac{2v}{\lambda} \sin\left(\frac{\theta_B}{2}\right) \approx \frac{v\theta}{\lambda} = \frac{v}{L},$$

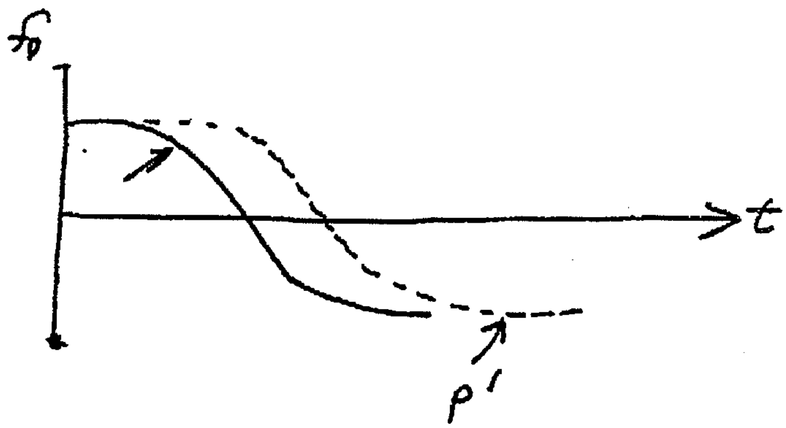


where L is the physical length of the antenna along track.

If a neghboring point P' is displaced a distance x_a further along track from P, the Doppler history of P' will be the same as P, but will be delayed by

$$t = \frac{x_a}{v}.$$

($n.b.$ v is the radar's speed.)



The shortest time displacement that can be measured after processing a signal with spectral bandwidth

$$B_D = 2f_D$$

is

$$t_m = \frac{1}{B_D} = \frac{1}{2f_D} = \frac{L}{2v}.$$

(Rayleigh Criterion)

Thus the finest possible resolution is

$$x_a = vt_m = \boxed{\frac{L}{2}}$$

Same as the synthetic array approach

When we use SAR to image a surface, we use pulses instead of a continuous signal to measure the Doppler shift and delay of the returns.

The processed pulses give us a sampled version of the Doppler shifted signal we would get from the continuous-time carrier.

From the Nyquist criterion, we know that the sampling rate (the pulse repetition frequency) must be greater than $2 f_{D,max}$:

$$PRF \geq 2 f_{D,max} = \frac{2v}{L} .$$

If we assume the carrier is downconverted to baseband, then

The downconverted Doppler Spectrum becomes Low-Pass.

20.24

n.b. $PRF \geq \frac{2v}{L}$

$\Rightarrow$ The sensor should take one sample (i.e., transmit a pulse) each time the sensor moves half an antenna length.

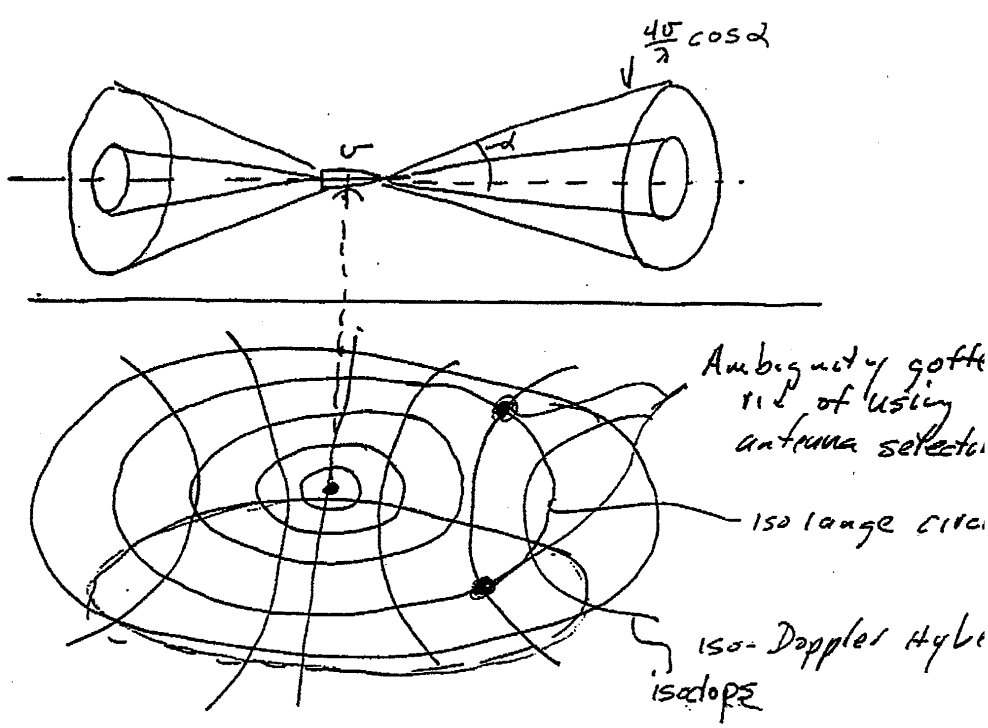
e.g. Low-Earth orbit: $v = 7$ km/sec, $L = 10$ m

$$PRF \geq \frac{2 \cdot 7 \text{ km/sec}}{10 \text{ m}} = 1.4 \text{ KHz}$$

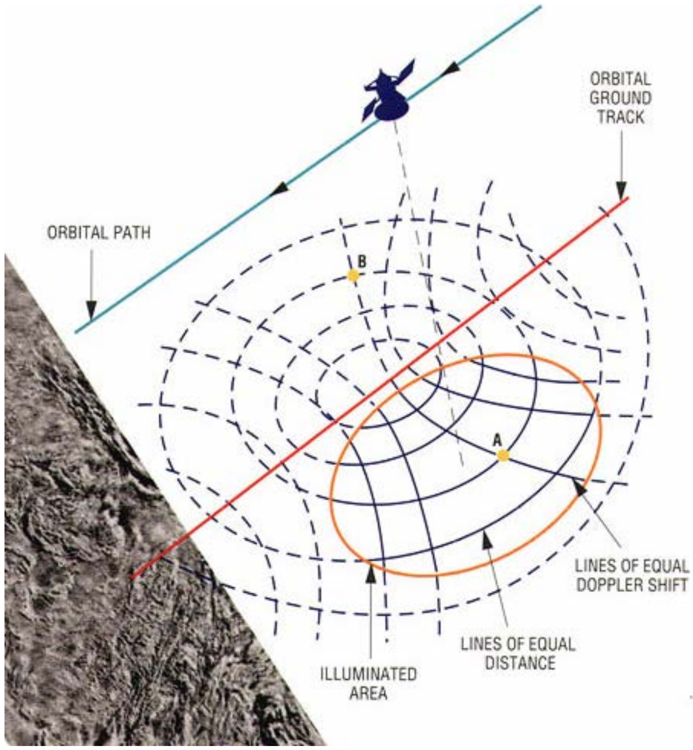
$$IPP = \frac{1}{PRF} \leq 0.714 \text{ mS} = 714 \,\mu\text{S}.$$

20.25

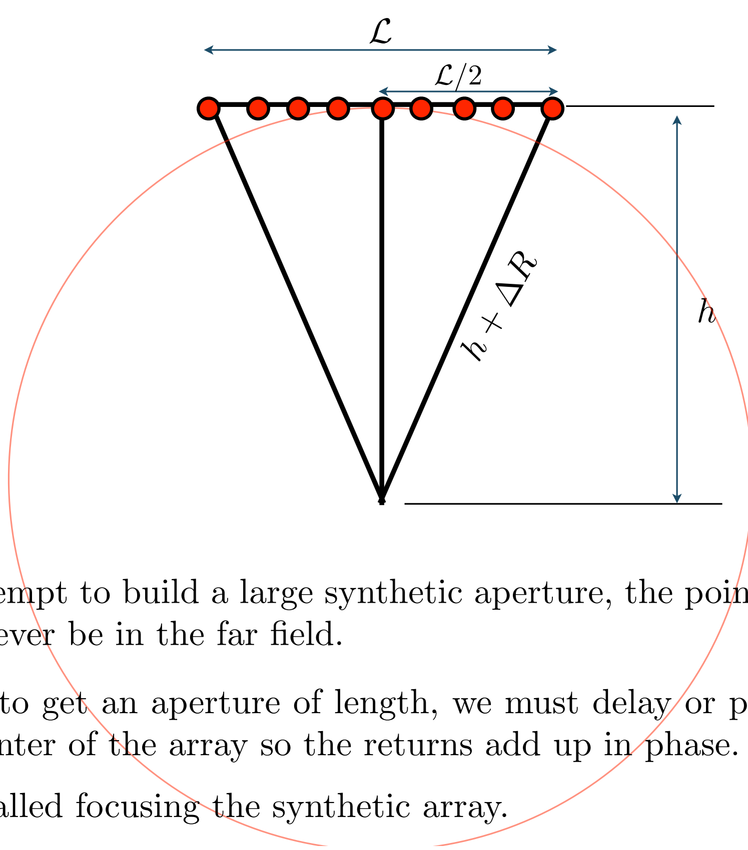
The Doppler point of view of SAR makes it easy to see how SAR imaging works



Doppler Synthesis Approach to SAR



Unfocused and Focused SAR



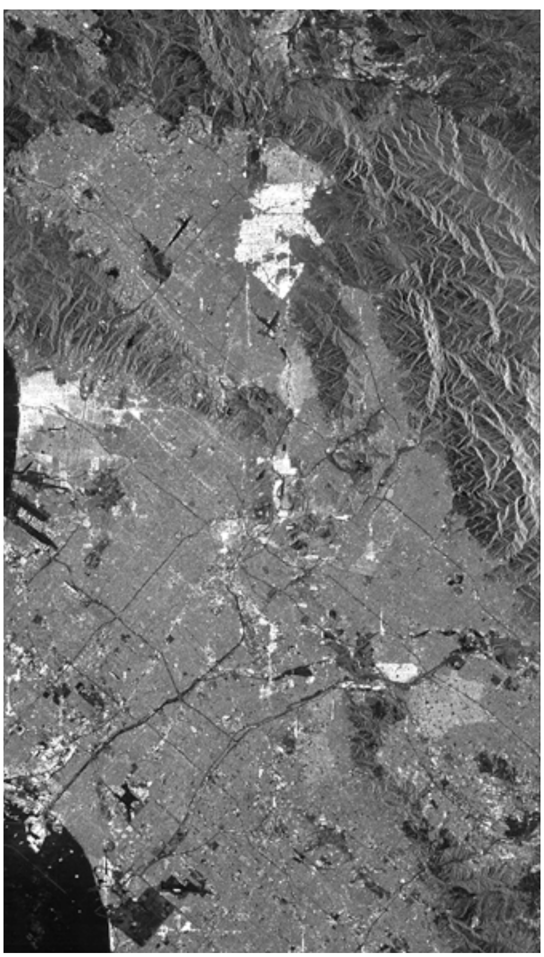
If we attempt to build a large synthetic aperture, the point we are imaging will almost never be in the far field.

In order to get an aperture of length, we must delay or phase shift the signals at the center of the array so the returns add up in phase.

This is called focusing the synthetic array.

- Without focusing, the effective length of the array becomes smaller than its true length, because the signals being added do not add in phase.
- From simple geometry we see that the correction (delay) is a function of the range to the object.
- This means there are different corrections for each range cell.
- This results in increased computational complexity of SAR processing.
- Almost all current SAR systems perform focusing.

- Some Synthetic Aperture Radar Images

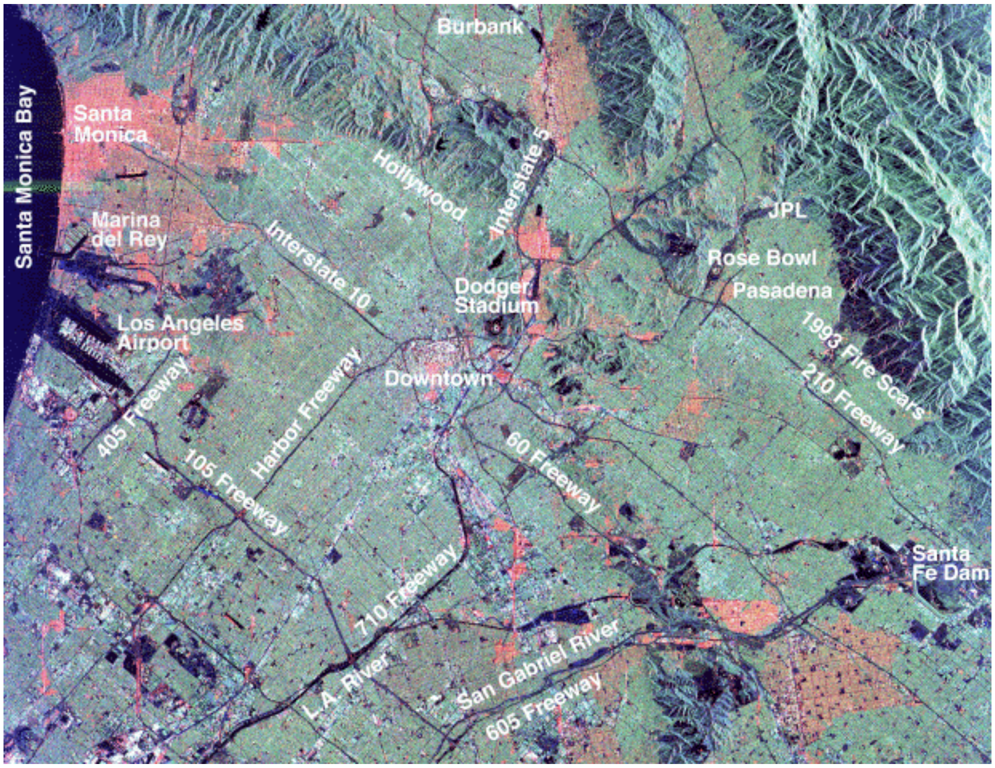


20.31

SEASAT SAR Image of Los Angeles Basin

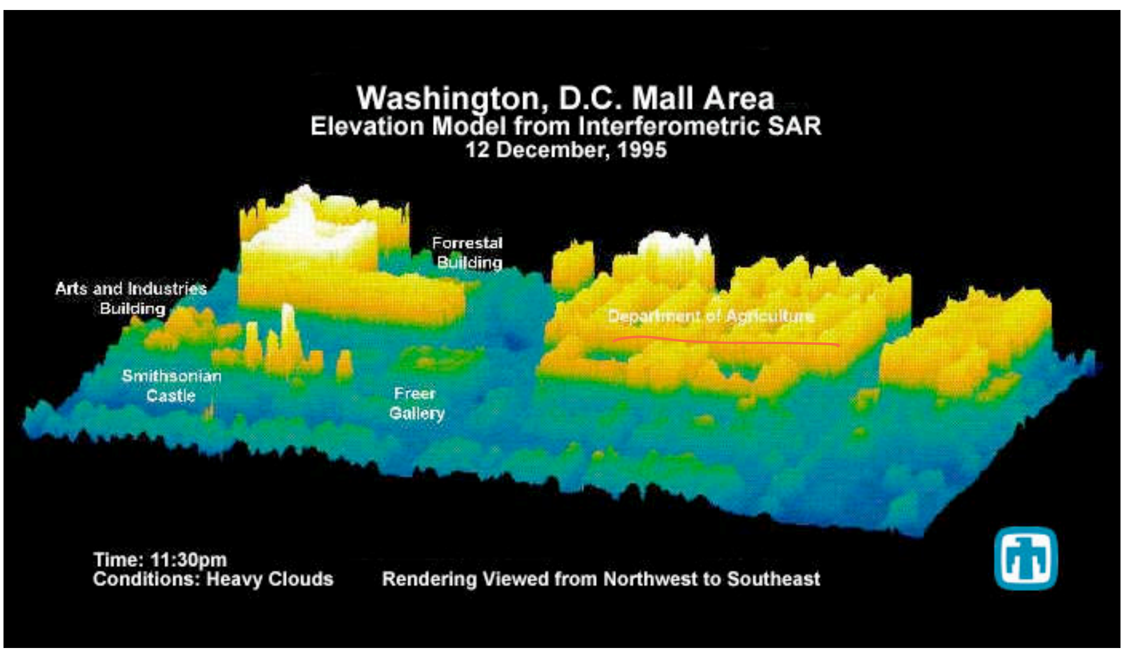
20.32

SIR-C/SAR-X (Shuttle Imaging Radar)



20.33

InSAR Image



20.34

InSAR Image
Cochiti Mesa, New Mexico