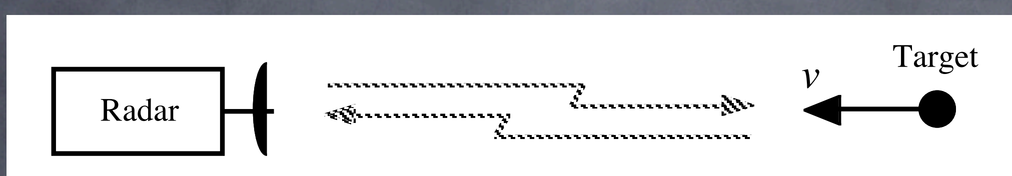
Session 2

Recall ...

The Doppler Effect



> The radar transmits waveform s(t).

> The received waveform is of the form

$$r(t) \;=\; \sqrt{\alpha}\cdot s(\alpha t - \tau) \;=\; \sqrt{\alpha}\cdot s(\alpha(t-\tau')),$$

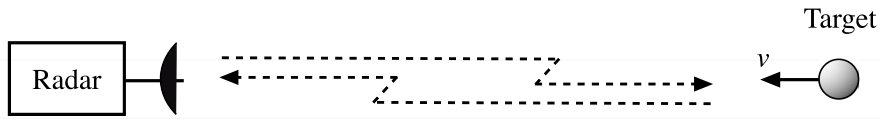
where

$$\boxed{\alpha = \frac{1+v/c}{1-v/c}.}$$

Let's derive this!

is the Doppler compression factor.

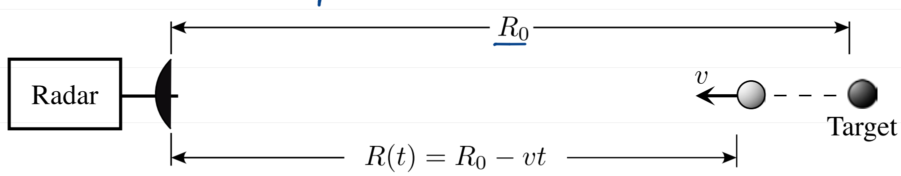
2.2



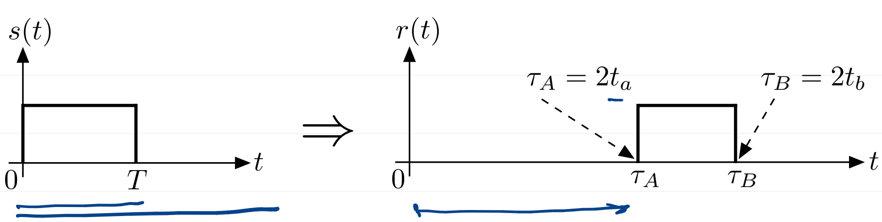
- Consider a target approaching the radar with speed v.
- Assume both the radar and the target are moving with constant velocity (inertial reference frames) and the relative closing velocity is v.
- At time $t = 0$, the target has range R_0 as seen by the radar.
- Thus at time t, the target is at range

$$R(t) = R_0 - vt$$

as viewed by the radar.



2.3



- Assume that as viewed at the radar, the leading edge of the pulse hits the target at time t_a
- Then at time t_a, the waveform has traveled a distance ct_a as seen by the radar.
- At this point in time, the range of the target from the radar is

$$R(t_a) = R_0 - vt_a$$

as seen by the radar.

2.4

Thus it follows that

$$c t_a = R(t_a) = R_0 - v t_a$$

$$\Rightarrow R_0 = (c+v) t_a$$

$$\Rightarrow t_a = \frac{R_0}{c+v}$$

Thus the range to the target at this time can be written as

$$R(t_a) = R_0 - v t_a = R_0 - v\left(\frac{R_0}{c+v}\right)$$

$$= R_0\left(1 - \frac{v}{c+v}\right)$$

$$= \frac{R_0 c}{c+v}.$$

2.5

- Once the front edge of the waveforms hits the target, it is reflected and travels the same distance $c t_a$ back to the radar, for a total travel time of $2 t_a$ from the time of transmission of the leading edge to the time of receiving the leading edge of the reflected pulse. (ASBTR)*.
- Thus the delay from transmission of the leading edge to reception of the leading edge is $\tau_A = 2 t_a = \dfrac{2R_0}{c+v}$.

* ASBTR ≜ "As seen by the radar", or as measured in the radar's intertial reference frame.

2.6

- Now consider the trailing edge of the transmitted waveform.
- The trailing edge leaves the radar at time T (ASBTR).
- The trailing edge hits the target at some time t_b (ASBTR)
- At this time t_b, the trailing edge has traveled a distance $c(t_b - T)$ (ASBTR)
- At this time t_b, the target is at range $R(t_b) = R_0 - v(t_b)$

(ASBTR).

2.7

Thus it follows that

$$c(t_b - T) = R_0 - v t_b$$

and so

$$(c+v)t_b = R_0 + cT$$

from which it follows that

$$t_b = \frac{R_0 + cT}{c+v}$$

2.8

The range to the target at this is

$$R(t_b) = R_0 - vt_b = R_0\left(\frac{R_0 + cT}{c+v}\right) - \frac{vcT}{c+v}$$

$$= R_0\left(1 - \frac{v}{c+v}\right) - \frac{v_c T}{c+v}$$

$$= \frac{R_0 c}{c+v} - \frac{vcT}{c+v}$$

- Once the trailing edge is reflected, it travels the same distance back to radar at speed c.

2.9

- It arrives back at the radar at time

$$\tau_B = t_b + \frac{R(t_b)}{c}$$

$$= \frac{R_0 + cT}{c+v} + \frac{R_0}{c+v} - \frac{vT}{c+v}$$

$$= \frac{2R_0}{c+v} + \frac{(c-v)T}{c+v}$$

(ASBTR)

- The received reflected signal from the target appears at the radar in the time interval $[\tau_A, \tau_B]$.

2.10

- The duration of the received signal is

$$\tilde{T} = \tau_B - \tau_A$$

$$= \frac{2R_0}{c+v} + \frac{(c-v)T}{c+v} - \frac{2R_0}{c+v}$$

$$= \frac{(c-v)T}{c+v} \qquad \text{(ASBTR)}$$

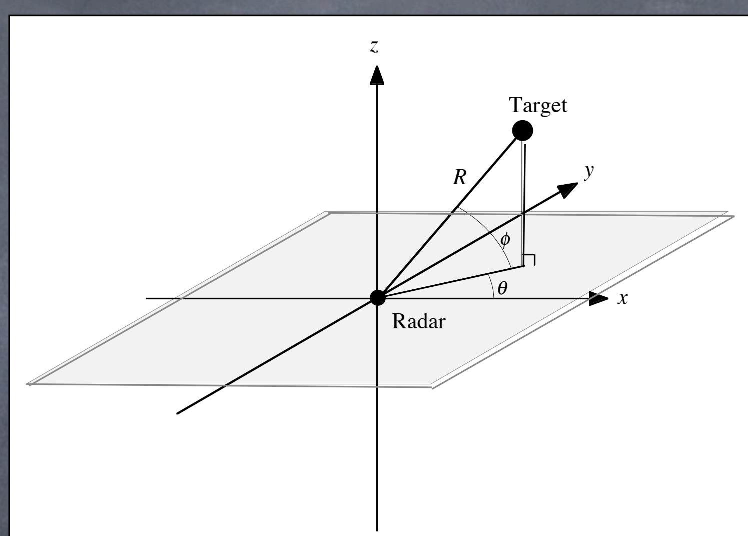
- Thus the Doppler compression factor is

$$\alpha = \frac{\tilde{T}}{T} = \frac{c-v}{c+v} = \frac{1-v/c}{1+v/c}$$

2.11

Azimuth and Elevation Angle Measurement

Radar measures azimuth and elevation angles using antenna



ϕ = elevation angle

θ = azimuth angle

R = range to target

$(R, \theta, \phi) \sim$ spherical coordinates

$(x, y, z) \sim$ cartesian coordinates

We can easily translate:

$$\Longrightarrow \quad \boxed{\begin{aligned} x &= R\cos\theta\cos\phi \\ y &= R\sin\theta\cos\phi \\ z &= R\sin\phi \end{aligned}}$$

Antennas and the Transmission of EM Energy

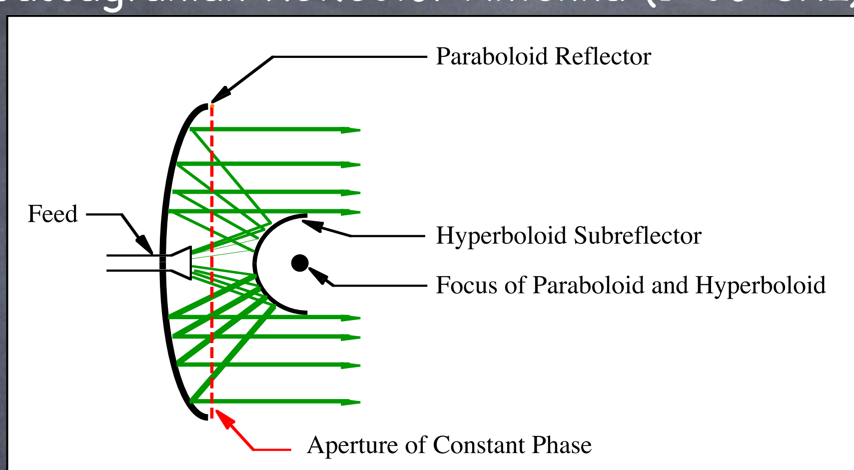
2.12

- Ability of a radar to make useful measurements is ultimately limited by the amount of energy scattered and collected
- This is a function of
  - Transmitted Energy
  - Target Scattering Characteristics
  - Transmit and Receive Antenna Characteristics

Typical Microwave Antenna

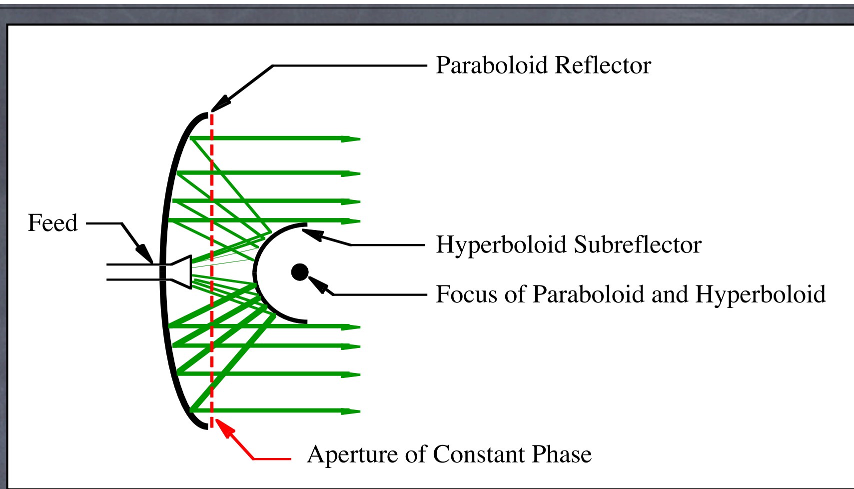
2.13

Cassagranian Reflector Antenna (1-30 GHz):



- Path length from Feed to aperture is constant regardless of particular path.
- Distance to distant point on axis to feed is constant regardless of particular path.

2.14



- Outgoing field crossing the aperture of the paraboloid is in phase at all points on the aperture.

- This is the condition required for focusing energy in a narrow beam (Fourier Tran -

2.15

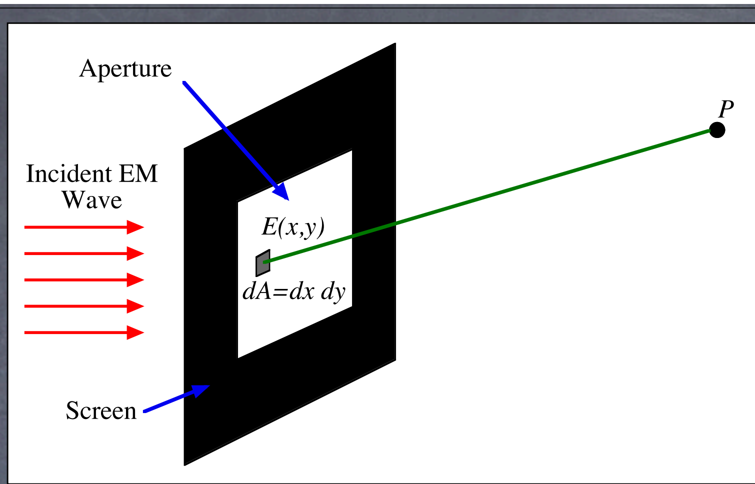
- In General, for Apertures that are In-Phase

- On Transmit: Greater the concentration of power, and the larger the power density.

- On Receive: Greater the collection area and thus the greater the power collected.

- Reciprocity Theorem states that these two facts are basically the same thing.

2.16

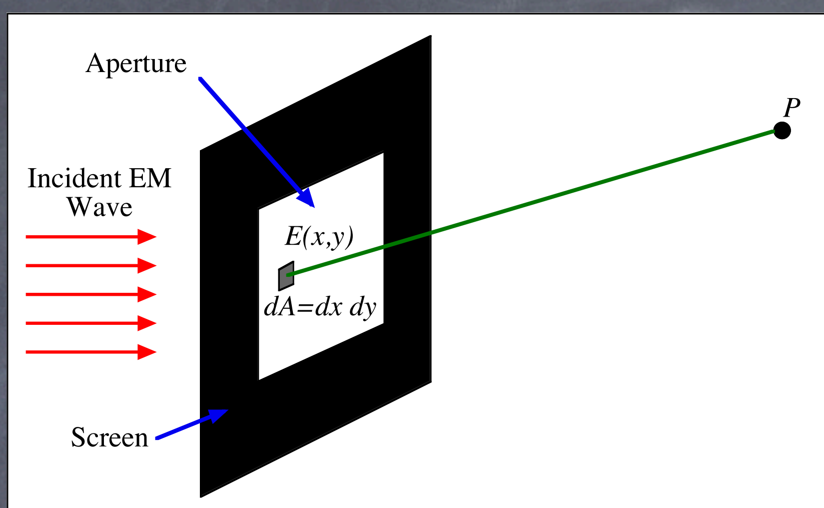


> Consider a plane wave normally incident on an aperture A

> Assume Electric field (for some fixed polarization) is E(x,y)

> We want field F(P) at point P.

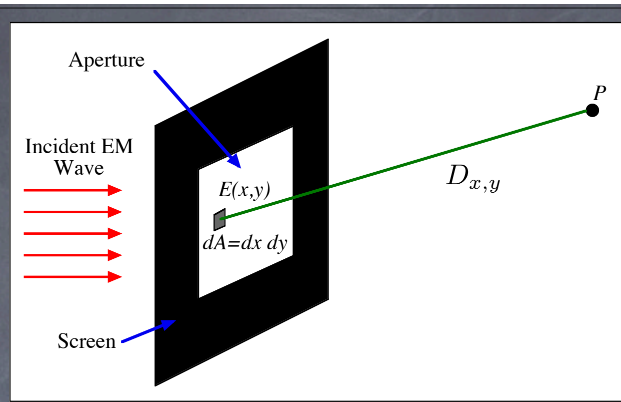
2.17



> Huygens's Principle states that we sum the contributions from each differential element dA of the aperture to get the total field at P.

2.18

Translating into Mathematics:

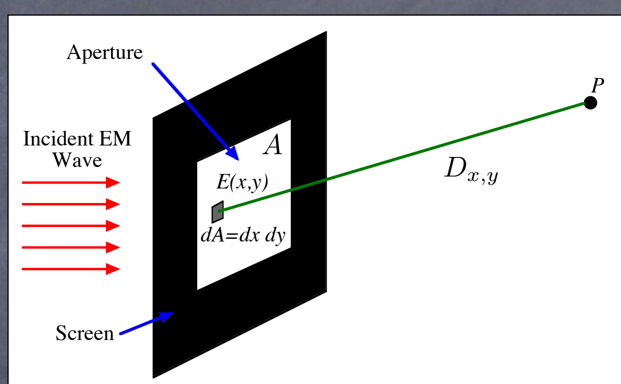


$$dF_{x,y}(P) = \frac{C}{D_{x,y}(P)} \exp\left\{-i\frac{2\pi}{\lambda}D_{x,y}(P)\right\} dA$$

$$C = \text{a constant}$$

2.19

The total field F(p) at P is obtained by integrating over the aperture A



$$F(P) = \int\!\!\int_A \frac{C}{D_{x,y}(P)} \exp\left\{-i\frac{2\pi}{\lambda}D_{x,y}(P)\right\} dx\, dy$$

Scalar Diffraction Theory

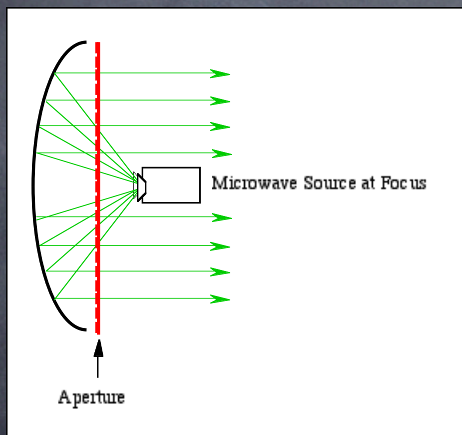
- This approach to calculating a scalar component of a electric field is called scalar diffraction theory.
- Useful because a linearly polarized wave traveling through free-space retains its polarization.
- This allows us to treat EM fields as scalar quantities instead of vector fields.
- Quite accurate for propagation of waves from large apertures at small diffraction angles.

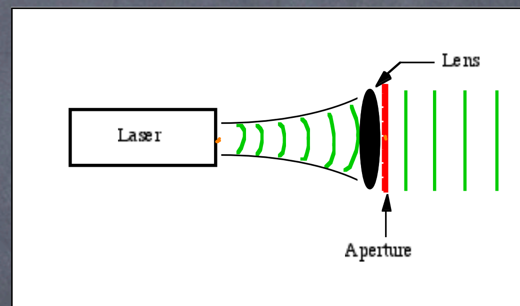
Scalar Diffraction

- Using Scalar diffraction, we can characterize the field arising at a point in space from the the field across the aperture that gave rise to it.
- References on Scalar Diffraction:
- Joseph Goodman, Fourier Optics
- M. Born and E. Wolf, Principles of Optics

Antenna Apertures

An antenna aperture is a surface of constant phase near the "face" of the antenna.



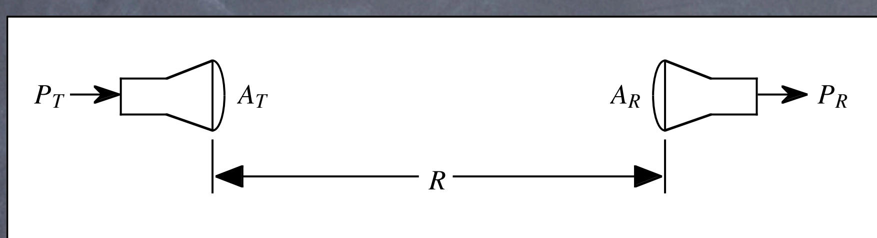


The aperture of an antenna has an area A.

This area characterizes the antenna's behavior.

The Friis Equation

> Suppose we have two antennas "pointing at each other" a large distance R apart.



If

$$P_T = \text{transmitted power}$$
$$P_R = \text{received power}$$

then

$$\frac{P_R}{P_T} = \frac{A_T A_R}{\lambda^2 R^2}. \qquad \text{(Friis Equation)}$$