Session 19

19.1

Phase Coded Waveforms

Phase-Coded Waveforms

If for a coded waveform

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t - nT) \exp\{i2\pi d_n t/T\} \exp\{j\phi_n\},$$

where

$$p(t) = 1_{[0,T]}(t),$$

we take

$$d_0 = d_1 = d_2 = \cdots = d_{N-1} = 0,$$

we get

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t - nT) \exp\{j\phi_n\}.$$

Such a signal is called a *phase-coded waveform.*

Such a waveform is characterized by the set of phases

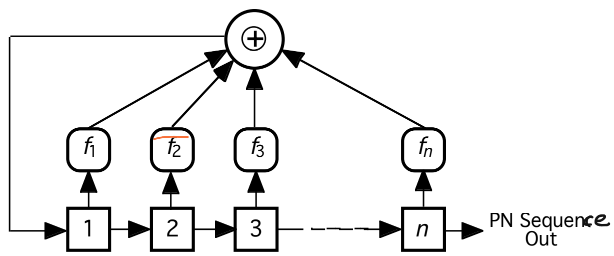
$$\{\phi_0, \phi_1, \phi_2, \ldots, \phi_{N-1}\}.$$

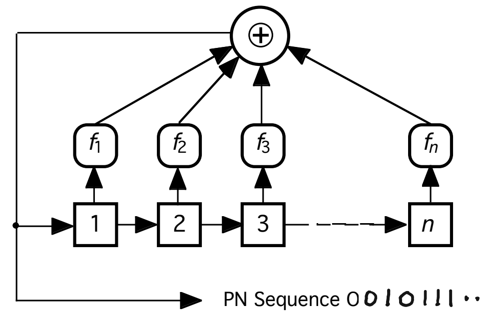
- There are a number of interesting Phase-Coded Waveforms. We will look at two:
  - Maximal Length *Linear Feedback Shift Register* (LFSR) sequences
  - Complementary Sequences

Maximum Length *Linear Feedback Shift Register* (LSRS) Sequences

Key references:

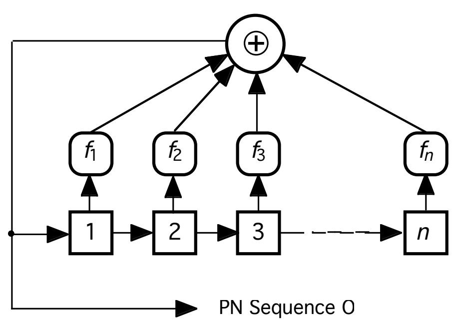
1. Solomon W. Golomb, *Shift Register Sequences*, Revised Edition, Aegian Park Press, 1982.
2. Robert J. McEliece, *Finite Fields for Computer Scientists and Engineers*, Kluwer, 1987.





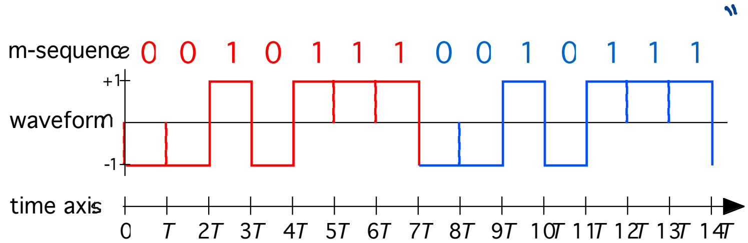
1. Registers hold values from $\{0, 1\}$.
2. Binary arithmetic (modulo-2 arithmetic).
3. Correct selection of binary coefficients $f_1, \ldots, f_n$ yields periodic sequences with period $N = 2^n - 1$.

Maximum Length *Linear Feedback Shift Register* (LSRS) Sequences



For $n = 3, f_1 = 0, f_2 = f_3 = 1$, we get the following length 7 periodic sequence when the initial state is not all zeros:

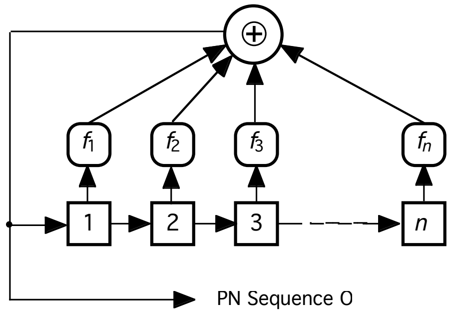
"0" → -1
"1" → +1



Here we map

- "0" → "−1", or phase $\phi_n = \pi$,
- "1" → "+1", or phase $\phi_n = 0$.

Maximum Length *Linear Feedback Shift Register* (LSRS) Sequences



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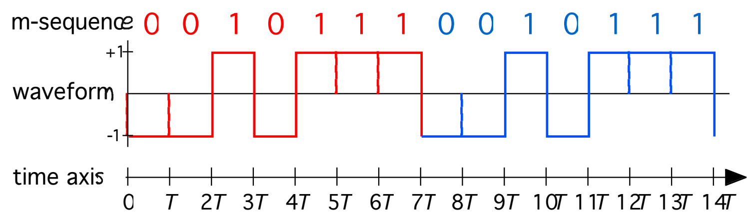
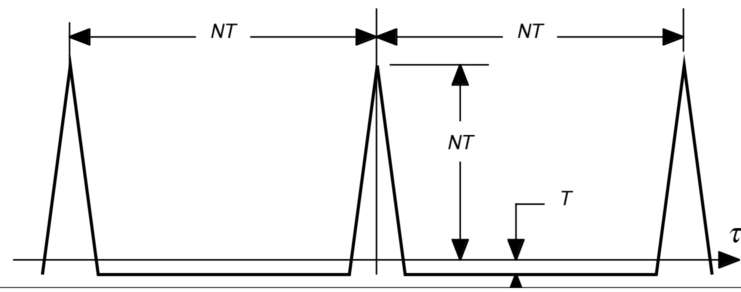
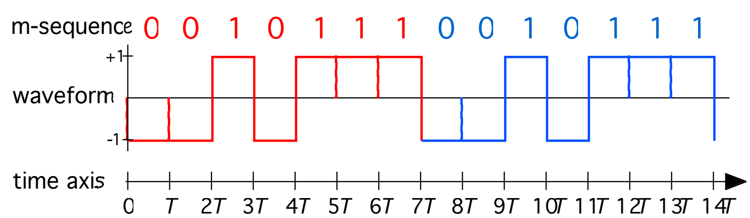


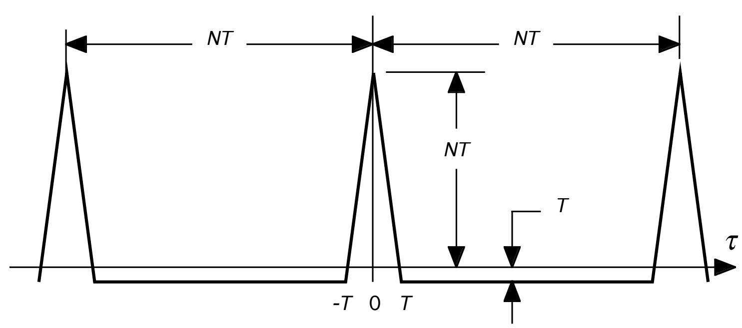




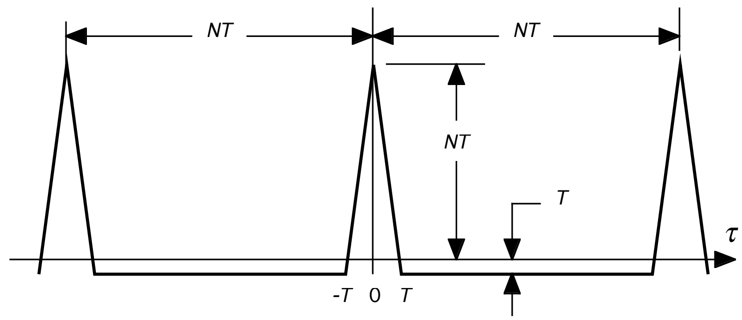
The time-autocorrelation function

$$R_s(\tau) = \int_{t_0}^{t_0+NT} s(t)s^*(t-\tau)\,dt$$

appears as follows:



$$R_s(\tau) = \int_{t_0}^{t_0+NT} s(t)s^*(t-\tau)\,dt$$



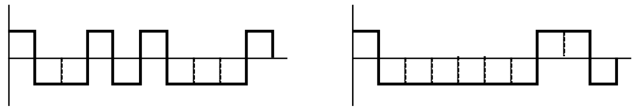
- With proper selection of *N* and *T*, periodic waveforms for high range resolution CW radar can be achieved.
- These waveforms are used in direct-sequence spread-spectrum systems as "pseudo-noise spreading sequences".
- Can be used for "spread spectrum radar systems".
- Multi-access characteristics of families of these waveforms useful for multistatic radar.
- Useful for Low Probability of Intercept (LPI) radar.
- Doppler ambiguity characteristics can be problematic.

Golay Complementary Sequences

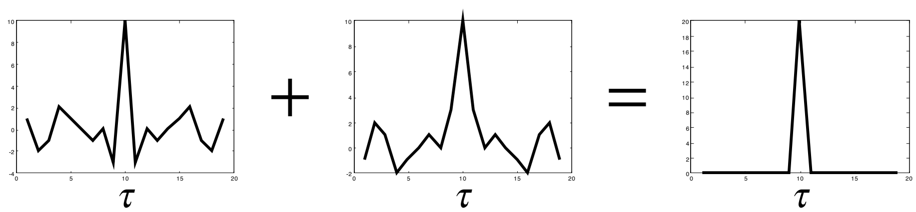
- *Complementary Sequences* are two (or more) phase coded sequences that can be used together to yield good range resolution results.
- Initially introduced by Marcel Golay for the design of optical spectrometers (see Harwit and Sloane, *Hadamard Transform Optics.)*
- One of the first examples of diversity waveform techniques.

Coherent Imaging Combining and Golay Sequences

≤ The Golay sequences provide an example of diversity waveform delay-only imaging.



1. Two complementary sequences are separated in time.
2. Their respective matched filters are time gated to range of interest.
3. Matched filter outputs with appropriate delay are coherently added; the sidelobes are canceled in the process.



Similarly, we will coherently add the individual delay-Doppler images obtained from the individual waveform measurements.

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LLNL 99-7

There are many approaches to constructing Golay complementary pairs. For lengths $N = 2^n$, for $n = 1, 2, 3, \ldots$, the following approach works:

Let $S_0 = [+]$

$$S_1 = [S_0\ S_0] = [++]$$
$$\hat{S}_1 = [S_0\ \bar{S}_0] = [+-]$$
Comp. Pair

(n.b. $\bar{S}_0 = -[S_0]$)

$$S_2 = [S_1\ \hat{S}_1] = [+++-]$$
$$\hat{S}_2 = [S_1\ \bar{\hat{S}}_1] = [++-+]$$
Comp. Pair

$\vdots \qquad \vdots$

$$S_n = [S_{n-1}\ \hat{S}_{n-1}]$$
$$\hat{S}_n = [S_{n-1}\ \bar{\hat{S}}_{n-1}]$$
Comp. Pair

- Complementary sequences have been effectively used in a number of radar measurement problems where there is not significant motion between measurements. (e.g., sounding of the ionosphere.)
- For non-zero Dopplers, there can be significant sidelobes present (why?)
- Careful time-gating is required to make this work.
- Just separating the two waveforms and running a matched filter for the composite ambiguity function yields the following response:

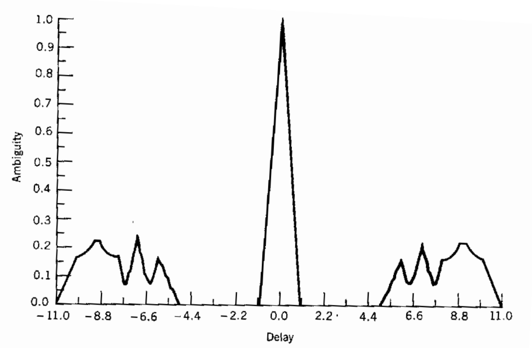

Figure 8.10 A zero-Doppler cut of the ambiguity function of the complementary phase-coded pair [+ + − ; + j +].

Figures from Levanon, *Radar Principles*, Chapter 8.

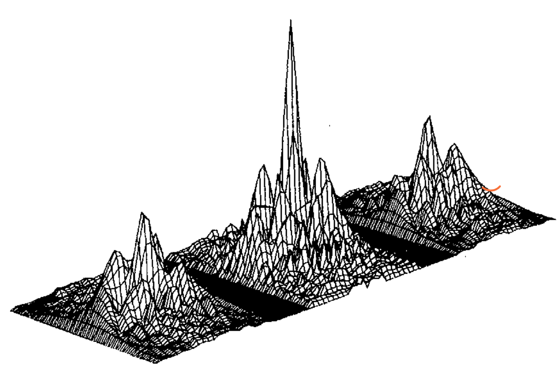

Figure 8.11 The ambiguity function of the complementary phase-coded pair [+ + − ; + j +].

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Synthetic Aperture Radar

We will look at SAR from the point-of-view of Antenna Arrays

- Large antennas can be formed by arraying a number of smaller antennas.
- Three main benefits:
    1. Increased Gain
    2. Increased Directivity
    3. Electrically Steerable (Big advantage!)

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Real Antenna Arrays

- Arrays can be formed in 1, 2, or 3 dimensions.
- An antenna aperture generated in this way is called an array antenna.
- The individual antennas making up the array are called array elements.

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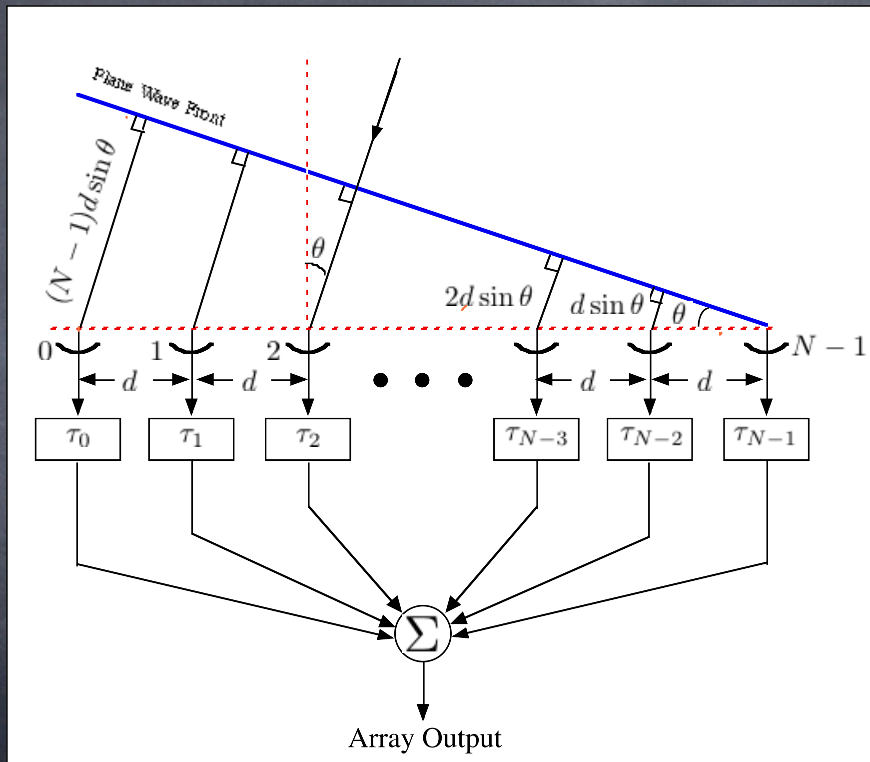
One-Dimensional Uniform Array



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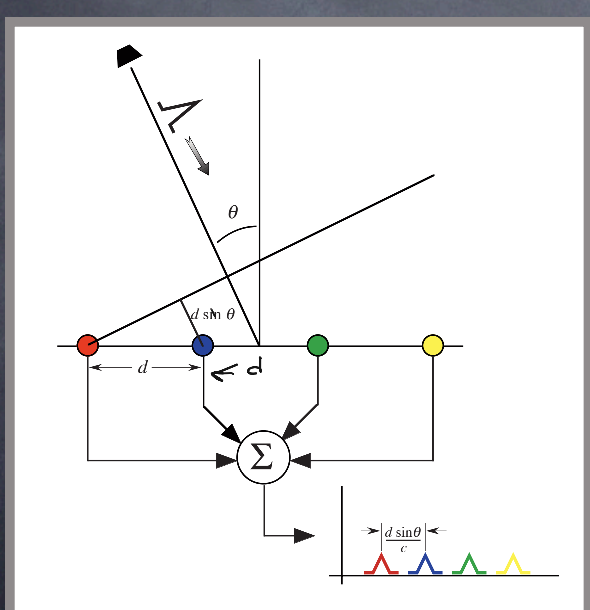
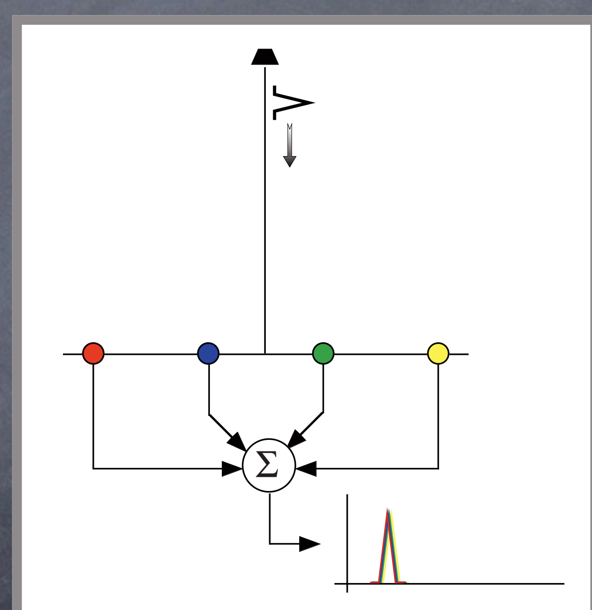
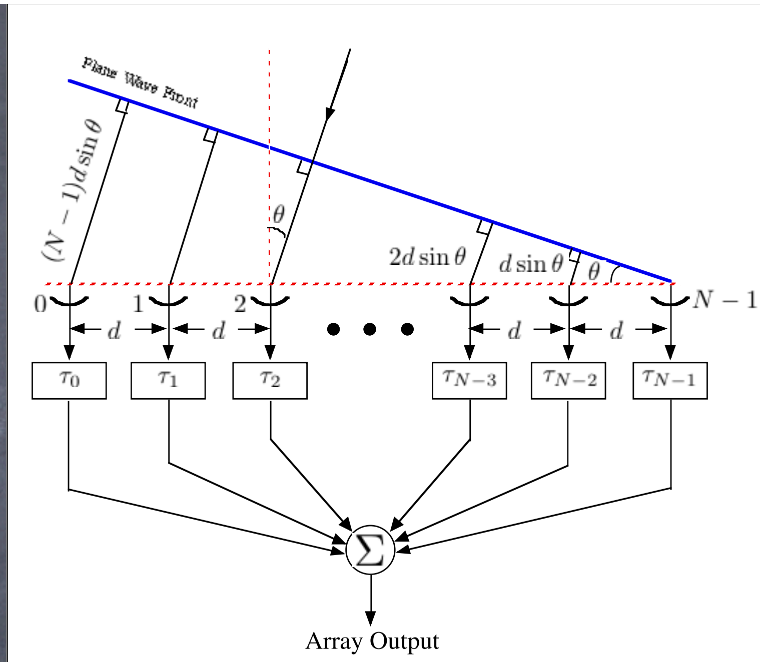
Linear Array Principle of Operation





19.18



Setting the delays to

$$\tau_m = \frac{[(N-1)-m]d\sin\theta}{c}, \quad m = 0, \ldots, N-1$$

causes the received signals to add up "in-sync."

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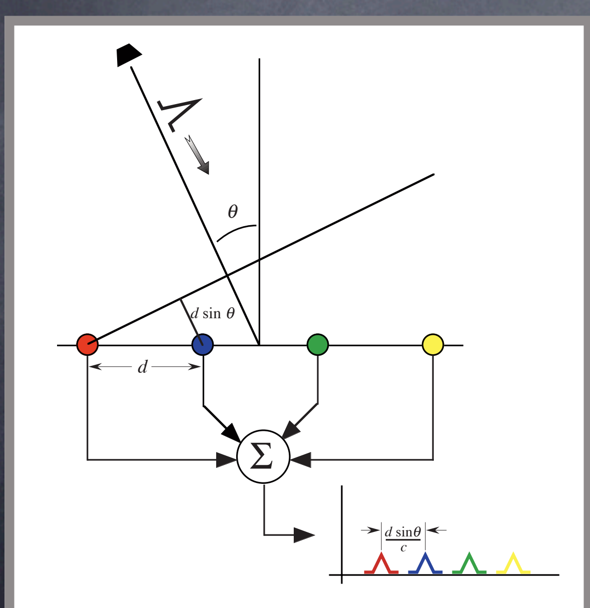
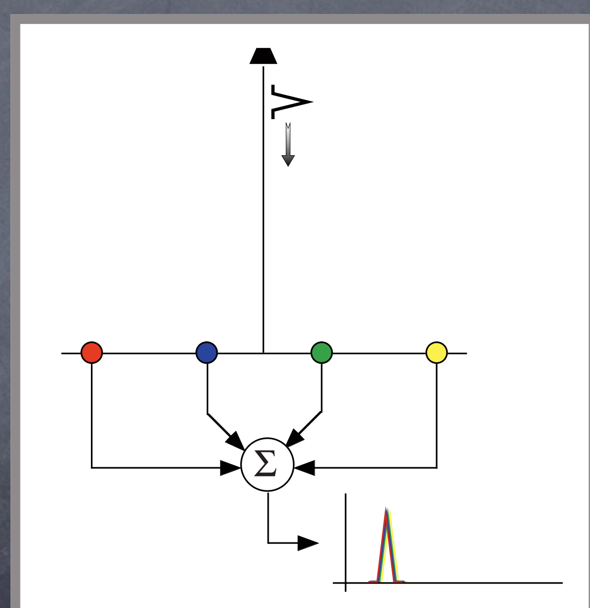
Signals adding in-sync result in a stronger version of the original signal.





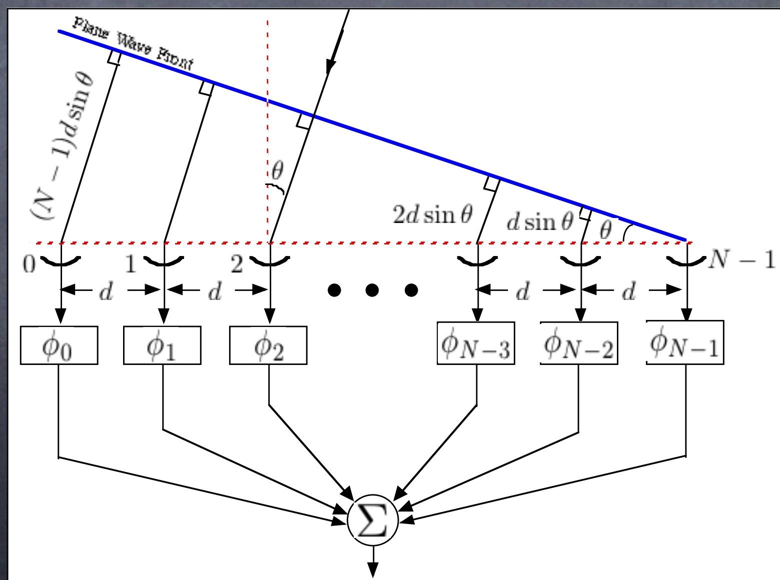
19.20

By the time shift theorem of Fourier transforms, we have

$$s(t) \overset{\mathcal{F}}{\leftrightarrow} S(f) \quad \Rightarrow \quad s(t-\tau) \overset{\mathcal{F}}{\leftrightarrow} S(f)\, e^{-i2\pi f\tau}.$$

So for a sinusoidal or narrowband signal at frequency f_0, we can replace the delay τ_m by phase shift

$$\phi_m = 2\pi f_0 \tau_m.$$



19.21

Assuming narrowband waves and phase shifters with

$$\phi_0 = \phi_1 = \phi_2 = \cdots = \phi_{N-1} = 0$$

and N identical elements with effective area $A_e(\theta)$ (gain $G_e(\theta)$) for a wave from direction θ, it can be shown the effective area of the array is

$$A(\theta) = A_e(\theta) \cdot \frac{1}{N} \left| \sum_{n=0}^{N-1} e^{i2\pi n(d/\lambda)\sin\theta} \right|^2$$

or equivalently

$$G(\theta) = G_e(\theta) \cdot \frac{1}{N} \left| \frac{\sin\left[N\pi(d/\lambda)\sin\theta\right]}{\sin\left[\pi(d/\lambda)\sin\theta\right]} \right|^2$$

$$\text{Array Length} = (N-1)d$$

Larger d implies higer resolution, but there is a price to pay.

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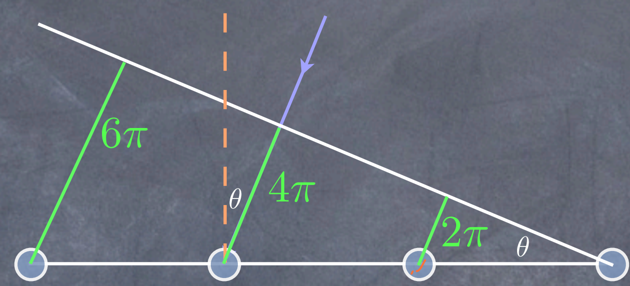
If $d > \lambda/2$, we get grating lobes due to constructive interference at Bragg angles:



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In order to reduce grating lobes, you must have $d \lesssim \lambda/2$.

You can also

1. Use nonuniform spacing of elements;
2. Use an $A_e(\theta)$ that reduces the most problematic grating lobes. (elements may be large)

Radio astronomy arrays often have severe grating lobes.

In radar they can be more problematic. Usually take $d \approx \frac{\lambda}{2}$

In radar they can be more problematic.

Usually take $d \approx \lambda/2$.