Session 17

Recall...

# Processing a Coherent Pulse Train

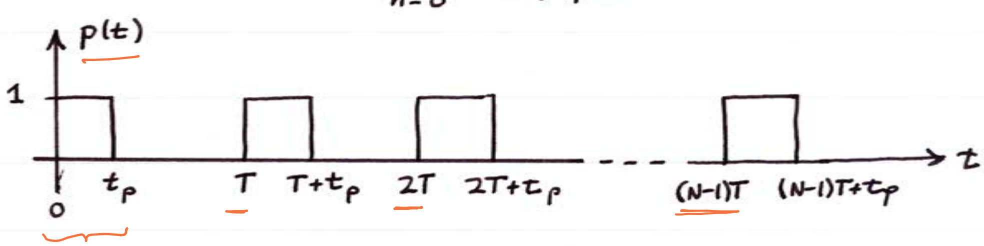


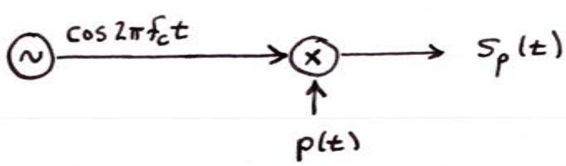
Assume a radar transmits a passband signal of the form

$$s_p(t) = p(t) \cos 2\pi f_c t$$

where

$$p(t) = \sum_{n=0}^{N-1} 1_{[0, t_p]}(t - nT).$$



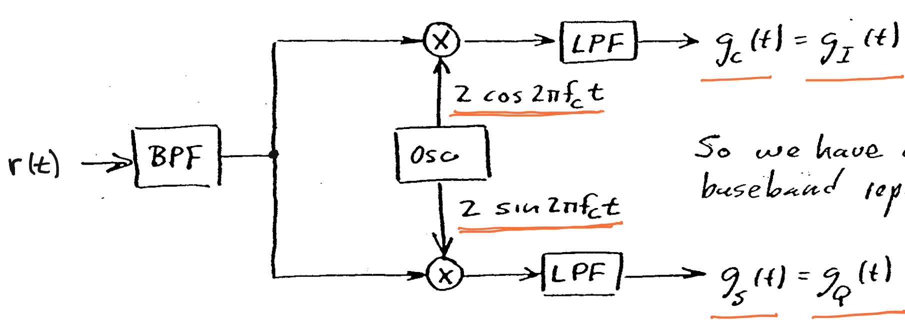



Recall...

17.2

We would like to find the complex baseband representation of the received signal *r(t)* via processing:

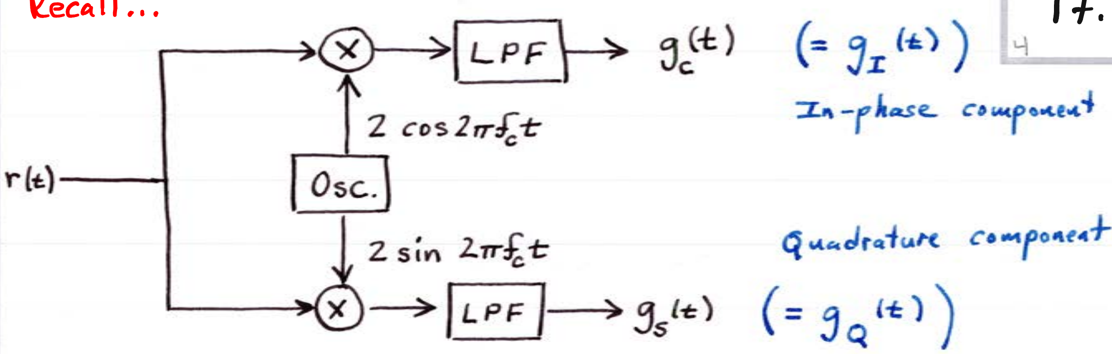
- Pass the signal through a bandpass filter that passes virtually all of the signal, but little outside of the signal's frequency band.
- Use an I/Q demodulator to get the in-phase and quadrature components.




So we have a complex baseband representation

Recall...

17.3



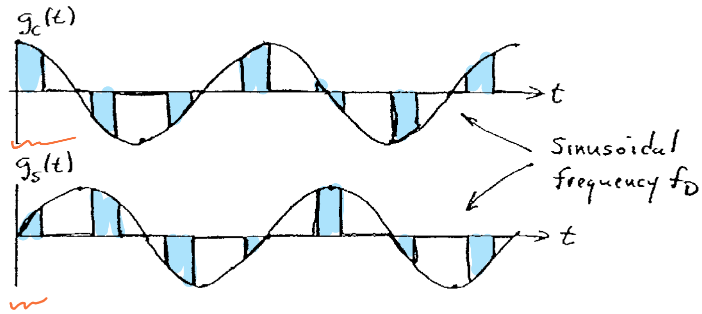

$$g_c(t) = p(t)\cos\phi(t)$$
$$g_s(t) = p(t)\sin\phi(t)$$
$$r(t) = p(t)\cos(2\pi f_c t + \phi(t))$$

Now consider the complex envelope function

$$u(t) = g_c(t) + i\,g_s(t) = p(t)\cos\phi(t) + i\,p(t)\sin\phi(t)$$
$$= p(t)\left[\cos\phi(t) + i\sin\phi(t)\right]$$
$$= p(t)\,e^{i\phi(t)}. \quad \text{(By Euler's identity)}$$

Recall...






Now we can form the complex envelope function

$$u(t) = g_c(t) + i g_s(t)$$

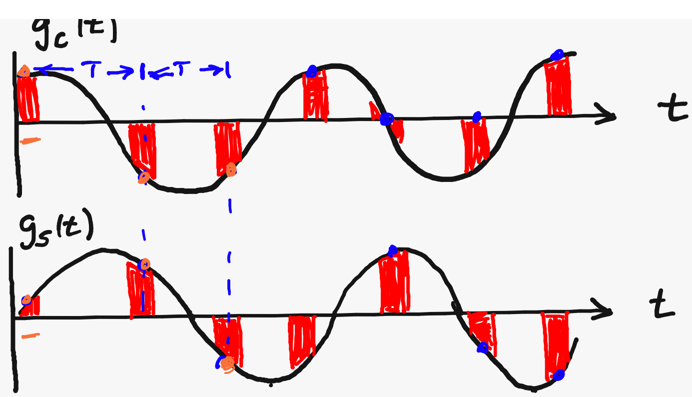
If we sample $u(t)$ at sampling instants $t_0, t_1, \ldots, t_{N-1}$ such that $t_n - t_{n-1} = T$ with $t_0$ selected within the first pulse $(0 < t_0 < t_p)$ we get one complex sample per transmitted pulse

$$\begin{aligned} u(t_n) &= g_c(t_n) + i g_s(t_n) \\ &= \rho(t_n) \exp\{i\phi(t_n)\} \\ &= \rho(t_n) \exp\{i 2\pi f_D t_n\} \\ &= \exp\{i 2\pi f_D t_n\} \quad \text{(single target case)} \end{aligned}$$

$, \; n = 0, 1, 2, \ldots, N-1$

Recall...






Now we can form the complex envelope function

$$u(t) = g_c(t) + i g_s(t)$$

If we sample $u(t)$ at sampling instants $t_0, t_1, \ldots, t_{N-1}$ such that $t_n - t_{n-1} = T \;, n = 1, \ldots, N-1$ with $t_0$ selected within the first pulse $(0 < t_0 < t_p)$ we get one complex sample from each pulse return

Recall...



Note that if we define the complex sequence

$$u_n = u(t_n) = u(t_o + nT), \quad n = 0, 1, \dots, N-1$$

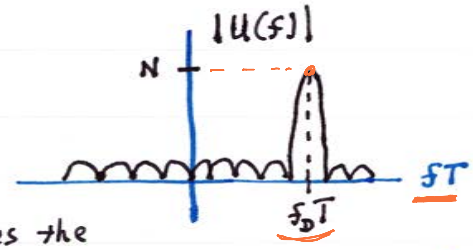
and compute the DTFT of $\{u_n\}$, we get

$$U(f) = \sum_{n=0}^{N-1} u_n e^{-i2\pi nTf} = \sum_{n=0}^{N-1} e^{i2\pi f_D (t_o + nT)} e^{-i2\pi nTf}$$

$$= e^{i2\pi f_D t_o} \sum_{n=0}^{N-1} e^{-i2\pi (f - f_D) nT}$$

For $f = f_D \Rightarrow |U(f)| = N$,

For $f \not\approx f_D \Rightarrow |U(f)| \cong 0$.




n.b where we pick $t_o$ determines the delay to the target. ($t_o$ = delay to first pulse return.)

Recall...



We could calculate the DFT of the sequence $u_n$:

$$U_k = \sum_{n=0}^{N-1} u_n \underbrace{e^{-i\frac{2\pi nk}{N}}}_{=1 \text{ for } k=0}, \quad k = 0, 1, \dots, N-1.$$

$u_n = e^{i2\pi f_D T n}$

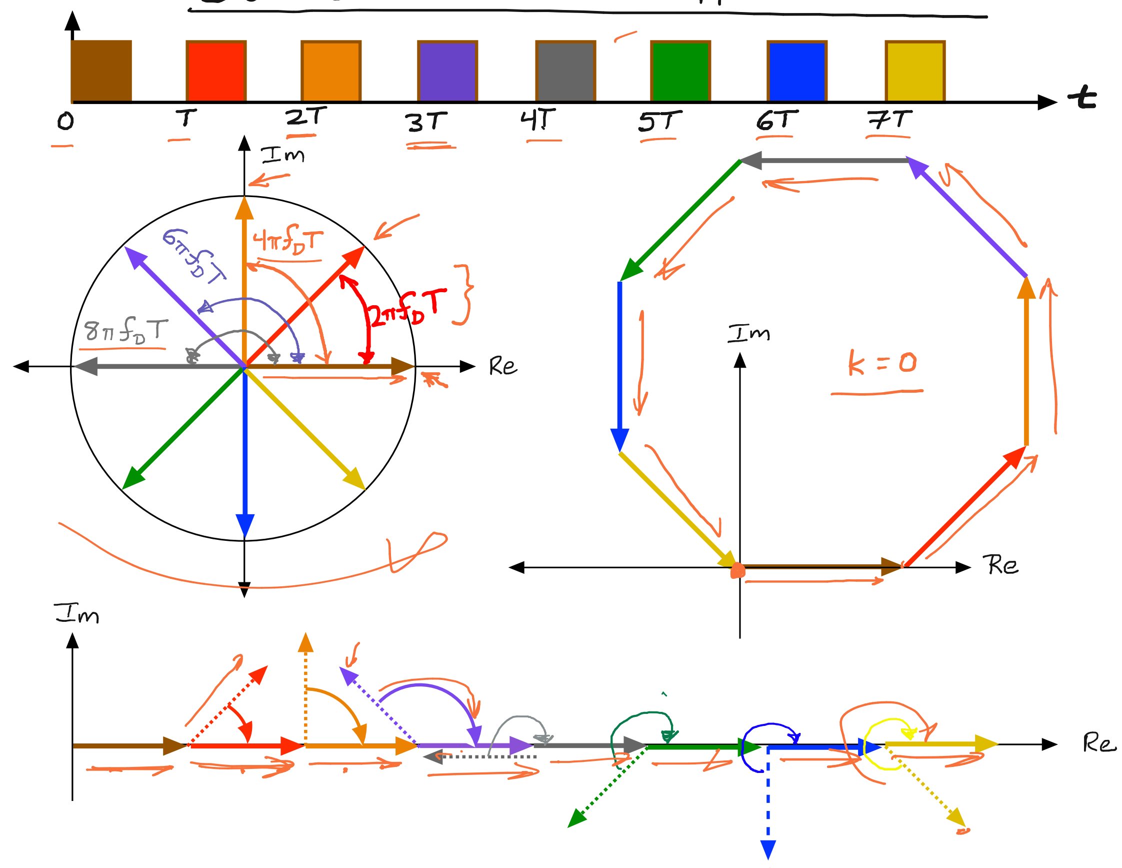
suppose $\frac{2\pi k}{N} \cong 2\pi f_D T$

Here again, we would get a large magnitude when $\frac{k}{N} \simeq T f_D$:

$\frac{k}{N} \simeq f_D T$

$$U_k \cong \begin{cases} N \cdot e^{i2\pi f_D t_o}, & k \cong N T f_D \\ 0, & \text{elsewhere.} \end{cases}$$

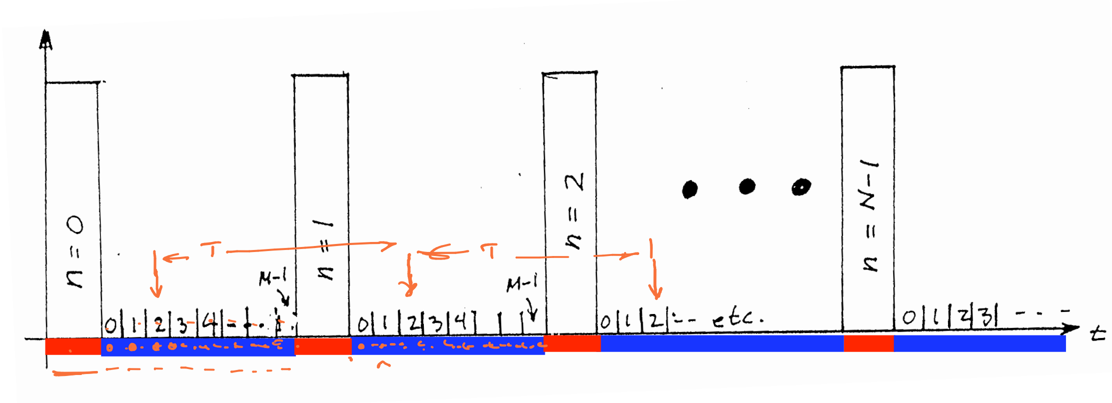
## Successive Pulses with Doppler Shift $f_D$.



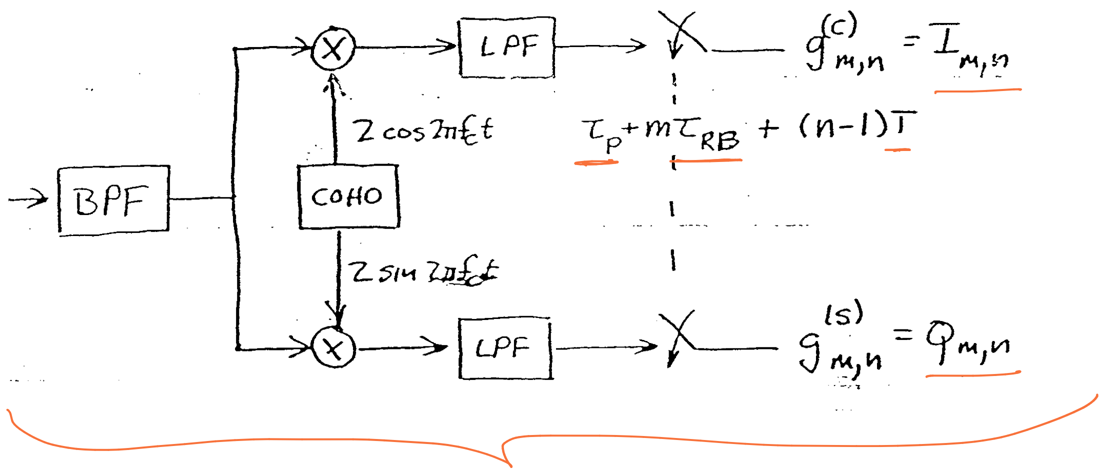



So we will divide the listening period into time bins of width $T_{RB}$ and sample the signal once per bin for each range bin of interest.

Label the range bins $0, 1, \ldots, M-1$.

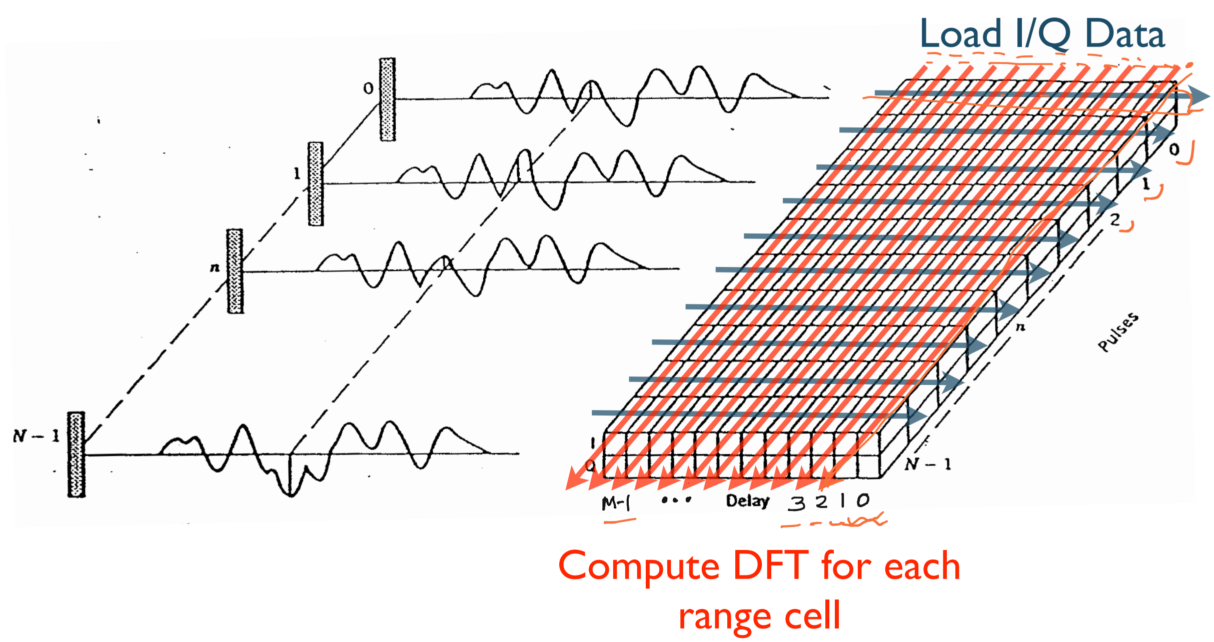


We can get the sample returns from each of the range bins (both I and Q components to represent the complex baseband signal) by processing the received signal as follows:




*n.b.*, $\tau_p$ is the transmitted pulse duration.





17.11

We can span the whole frequency range $[-W, W]$ or equivalently $[0, 2W]$ by sampling $N$ different frequency points

$$f_k = k \cdot \frac{2W}{N} = \frac{k}{NT}, \quad k = 0, 1, \ldots, N-1.$$

Thus we have that

$$\tilde{g}(f_k) = G_k = \sum_{n=0}^{N-1} g_n e^{-i2\pi\left(\frac{k}{NT}\right)nT} = \sum_{n=0}^{N-1} g_n e^{-i2\pi\frac{kn}{N}}$$

n.b. This is just the DFT of $\{g_n\}_{n=0}^{N-1}$, since

$$\mathrm{DFT}_N^{-1}(\{G_k\}) = \{g_n\}$$

$$\mathrm{DFT}_N(\{g_n\}) = G_k$$

where

$$G_k = \sum_{n=0}^{N-1} g_n e^{-i2\pi kn/N} \quad \text{and} \quad g_n = \frac{1}{N}\sum_{k=0}^{N-1} G_k e^{i2\pi kn/N}.$$

for $k = 0, 1, 2, \ldots, N-1$   for $n = 0, 1, \ldots, N-1$

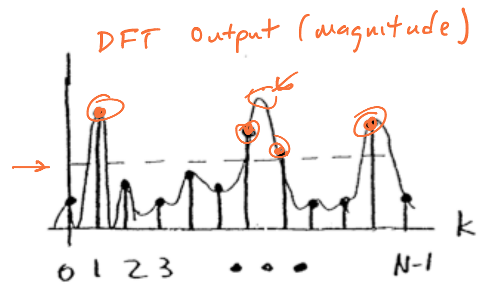
17.12

As we saw before in the DTFT, if a signal $g(t)$ [LIU] has Doppler shift $f_D$ that is near $\frac{k}{NT}$ for some integer $k$, then

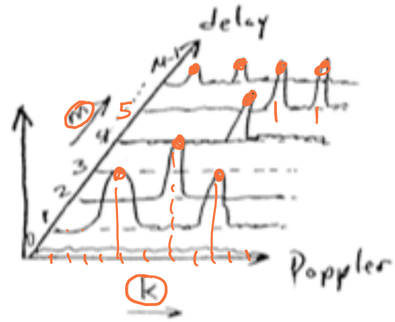
$$G_k = \tilde{g}(f_k) \text{ will have a large response.}$$

→ One proportional to target $\sqrt{\sigma}$.

If we have a number of different targets with Doppler frequencies $f_{D_1}, \ldots, f_{D_M}$, we will have a peak in the Doppler spectrum corresponding to each one.



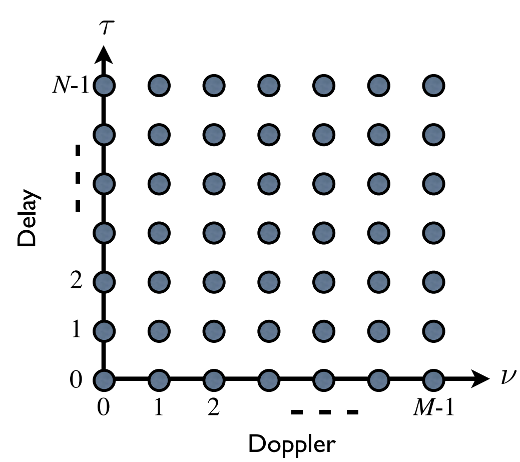
If we process in this way for each range cell (each m), we can construct a "Delay-Doppler* spectrum"

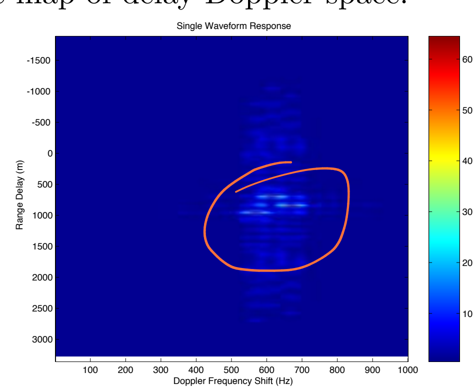


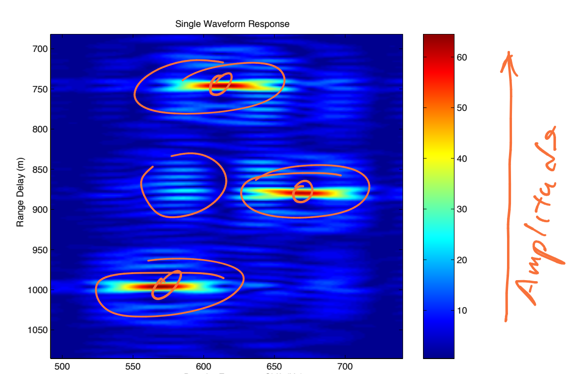

From such a Delay-Doppler image one can detect where the targets are in Delay and Doppler ($\tau$ and $\nu$).

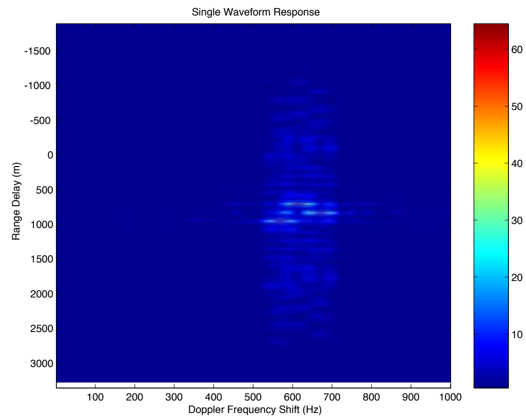


Actually, because delay range bins are discrete and the Doppler response is computed using a DFT, we get a discrete map of delay-Doppler space:






You can analyze a sequence of these images and "track" targets in delay and Doppler.

Actually, you can cut up angular space using a directional antenna to make measurements and track targets in $\theta, \phi, \tau$ and $\nu$.

This is how tracking radars work. Tracking is usually done with a "tracking filter"

e.g. Kalman Filter.



Of course the DFT response $G_K$ will be maximum for a target Doppler $f_D$ such that

$$f_D = \frac{k}{nT} + m \cdot \frac{1}{T}$$

however, for other $f_D \simeq \frac{k}{NT}$, DFT response will also be large.

n.b $\quad G_k \equiv 0 \iff f_D = \frac{j}{NT}, \ j \neq k.$

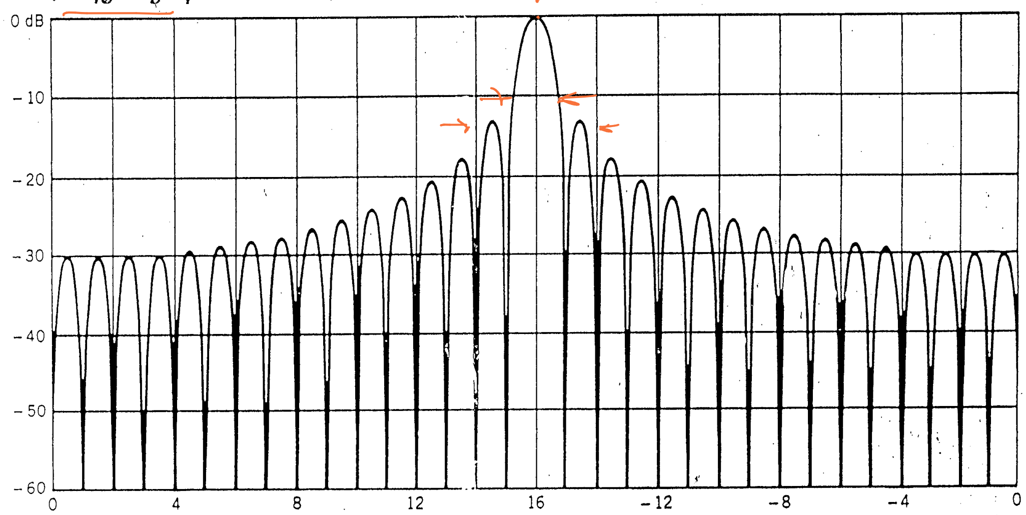
It can be shown that the normalized response of the k-th Doppler filter $G_k$ to a target with Doppler frequency $f_D$ is

$$|\hat{G}_k(f_D)| = \frac{1}{N} \left| \frac{\sin[\pi N (f_D T - k/N)]}{\sin[\pi (f_D T - k/N)]} \right|$$

17.17

$$|\hat{G}_k(f_D)| = \frac{1}{N}\left|\frac{\sin[\pi N(f_D T - k/N)]}{\sin[\pi(f_D T - k/N)]}\right|$$

$|\hat{G}_{16}(f_D)|$ for $N = 32$

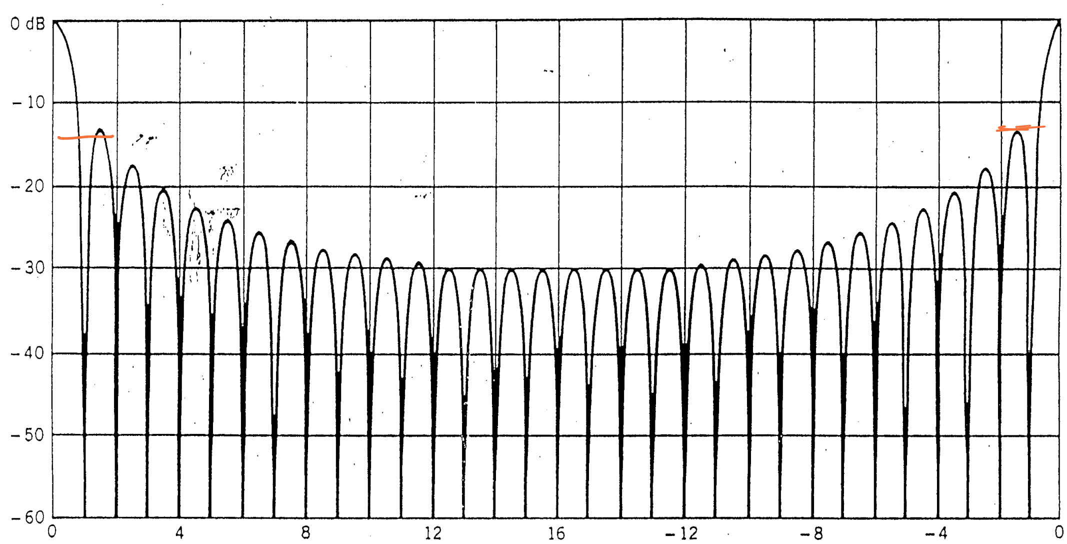



$Mf_DT$

*Figure 10.6* Frequency response of the 16th filter in an $N = M = 32$ DFT with a rectangular weighting.

17.18

$|\hat{G}_0(f_D)|$ for $N = 32$




$Mf_DT$

*Figure 10.5* Frequency response of the zeroth filter in an $N = M = 32$ DFT with a rectangular weighting.

# Sidelobes can be reduced with windowing before taking the DFT

$$\sum_{n=0}^{N-1} u_n \cdot w_n e^{-i 2\pi f_D T}$$

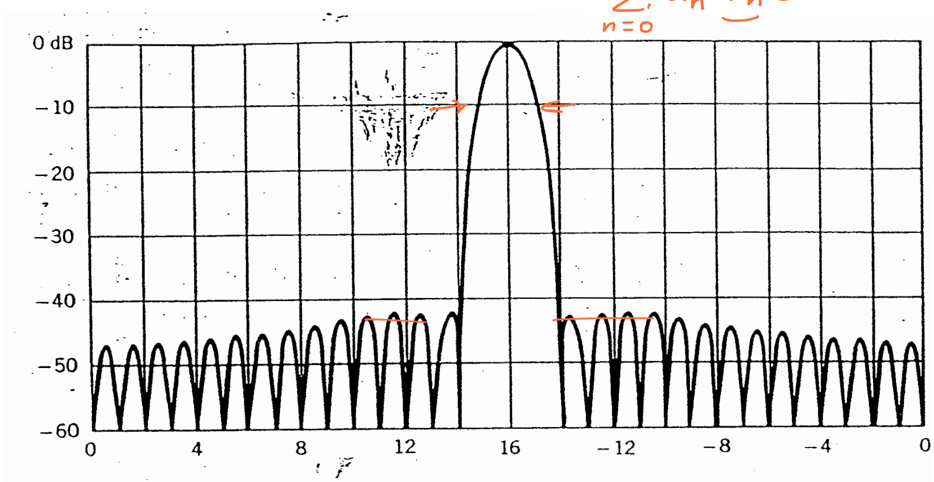
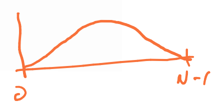



*Figure 10.7* Frequency response of an $N = M = 32$ DFT, Hamming weighting.

Hamming Window: $w_n = a_0 - a_1 \cos\left(\frac{2\pi n}{N}\right)$

$a_0 = 0.53836$

$a_1 = 0.46164$






# Sidelobes can be reduced with windowing before taking the DFT

Hann (Julius Von Hann) Window: Same as Hamming, but $a_0 = a_1 = 1/2$.

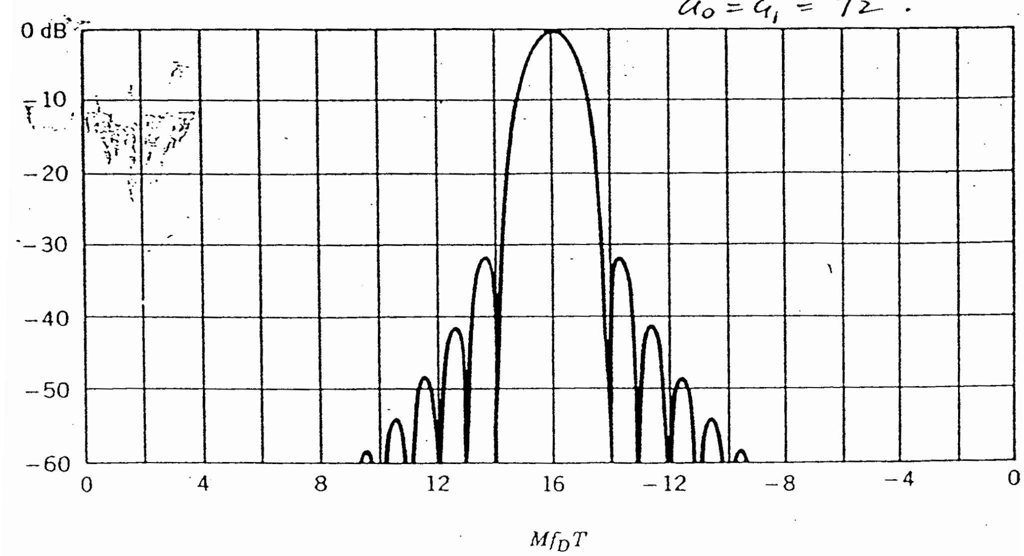



Figure 10.8 Frequency response of an $N = M = 32$ DFT, Hann weighting.

# Comparison of Windows

$|\hat{G}_{16}(f_D)|$ for $N = 32$

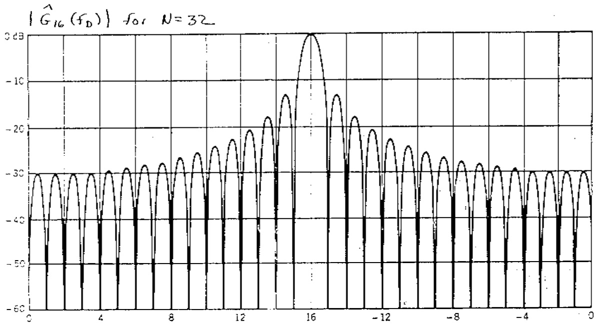



Figure 10.6 Frequency response of the 16th filter in an N = M = 32 DFT with a rectangular weighting.

Hann (Julius Von Hann) Window: Same as Hamming, but $a_0 = a_1 = 1/2$.

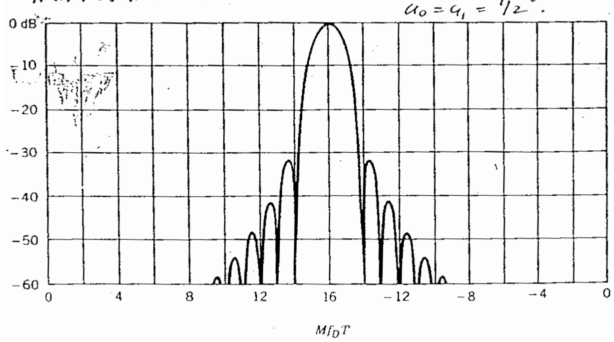



Figure 10.8 Frequency response of an N = M = 32 DFT, Hann weighting.

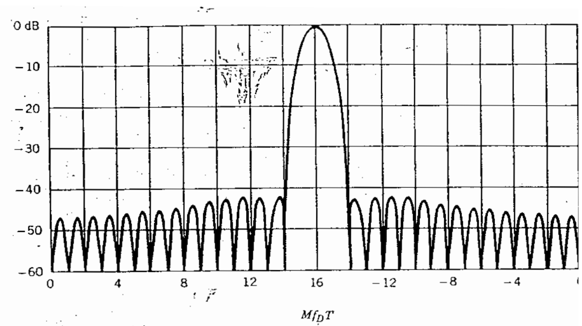



Figure 10.7 Frequency response of an N = M = 32 DFT, Hamming weighting.

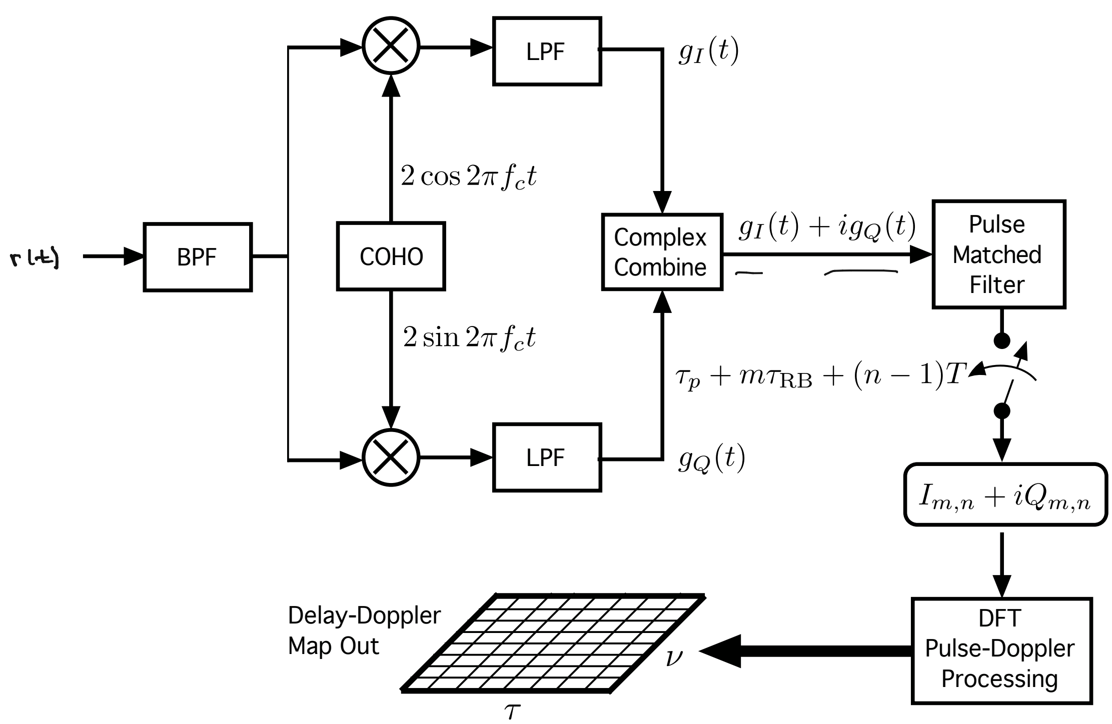
Hamming Window: $W_n = a_0 - a_1 \cos\left(\frac{2\pi n}{N}\right)$

$a_0 = 0.53836$

$a_1 = 0.46164$



# Including the Matched Filter



# Coded Radar Signals

Reference: N. Levanon and E. Mozeson, ***Radar Signals***, Wiley, 2004 (ISBN 0-471-47378-2)

- We will consider coded radar "pulses" as opposed to pulse trains.
- We have already seen that modulation of a radar pulse (e.g. a "chirp") increases the range resolution of the signal through bandwidth expansion.
- Modulation of a radar pulse can significantly modify its ambiguity function (e.g. the "sheared" ambiguity function generated by a chirp)
- We want to consider intelligent approaches to modulating waveforms
- Coded waveforms provide a structured approach to designing waveforms.

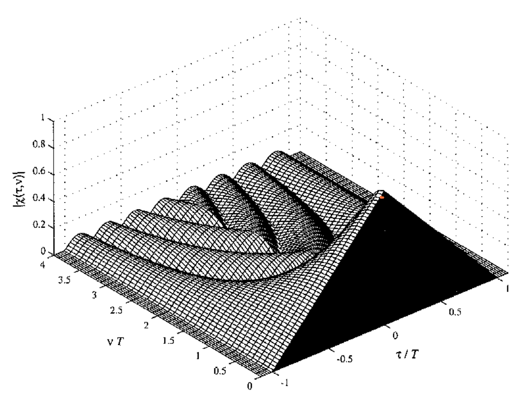


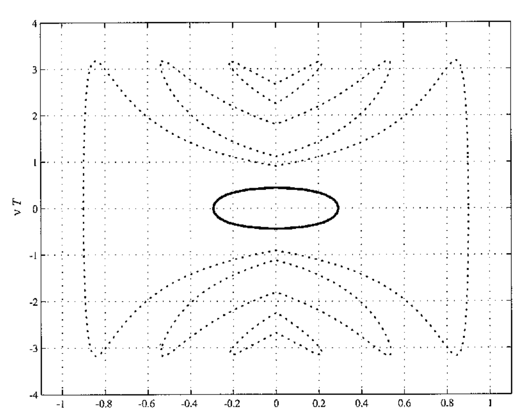
# Rectangular Pulse

$$s(t) = 1_{[0,T]}(t).$$

$$\beta_s(\tau,\nu) = e^{-i\pi(T+\tau)}(T-|\tau|)\frac{\sin \pi\nu(T-|\tau|)}{\pi\nu(T-|\tau|)} \cdot 1_{[-T,T]}(\tau$$

A plot of $|\beta_s(\tau,\nu)|$ for a pulse of duration $T = 1$ appear as follows



# Linear FM Chirp Pulse

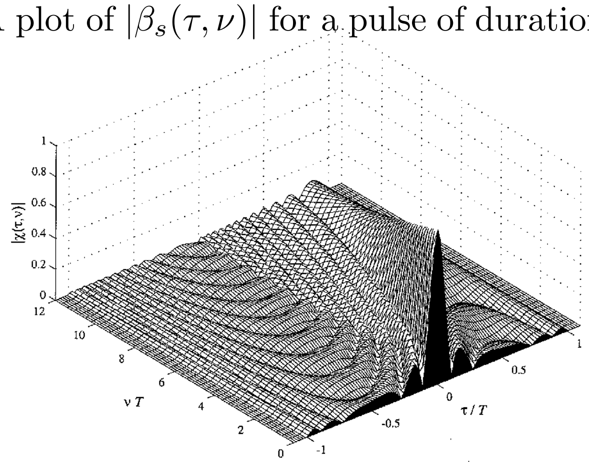
17.24

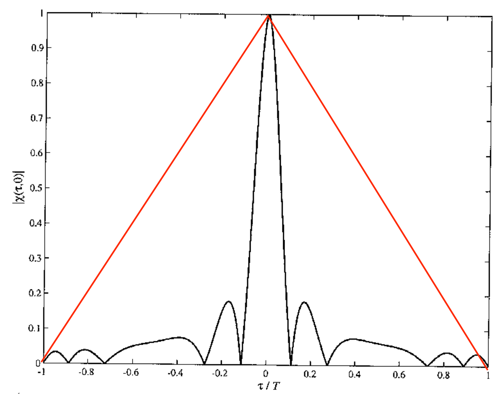
$$v(t) = e^{i\pi\alpha t^2} \cdot 1_{[0,T]}(t),$$

where $\alpha \in \mathbf{R}$.
Then

$$\beta_v(\tau,\nu) = e^{-i\pi(T+\tau)}(T-|\tau|)\frac{\sin \pi(\nu-\alpha\tau)(T-|\tau|)}{\pi(\nu-\alpha\tau)(T-|\tau|)} \cdot 1_{[-T,T]}(\tau).$$

A plot of $|\beta_s(\tau,\nu)|$ for a pulse of duration $T = 1$ with $\alpha = 10$ appears as follows:




# Coded Radar Signals

17.25

A coded waveform $s(t)$ is a signal of the form

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t-nT)\exp\{j2\pi d_n t/T\}\exp\{j\phi_n\},$$

Unit Energy

where

$$p(t) = 1_{[0,T]}(t),$$

$\{d_n\}_{n=0}^{N-1}$ = a sequence of integer frequency modulating indices,

$\{\phi_n\}_{n=0}^{N-1}$ = a sequence of real valued phases,

$T$ = duration of a single waveform "chip,"

$NT$ = total duration of the coded waveform.
We will initially take

$$\phi_0 = \phi_1 = \phi_2 = \cdots = \phi_{N-1} = 0,$$

which will give us a *frequency-coded signal*

$$s(t) = \frac{1}{\sqrt{NT}} \sum_{n=0}^{N-1} p(t-nT)\exp\left\{j2\pi\left(\frac{d_n}{T}\right)t\right\}.$$

17.26

# Frequency-Coded Waveforms

A frequency coded waveform $s(t)$ is a signal of the form

$$s(t) = \sum_{l=0}^{N-1} p(t - lT) e^{-j2\pi\Omega_l t},$$

where

$$T = \textit{chip duration},$$

$$p(t) = 1_{[0,T]}(t) \qquad \text{(chip waveform)},$$

and

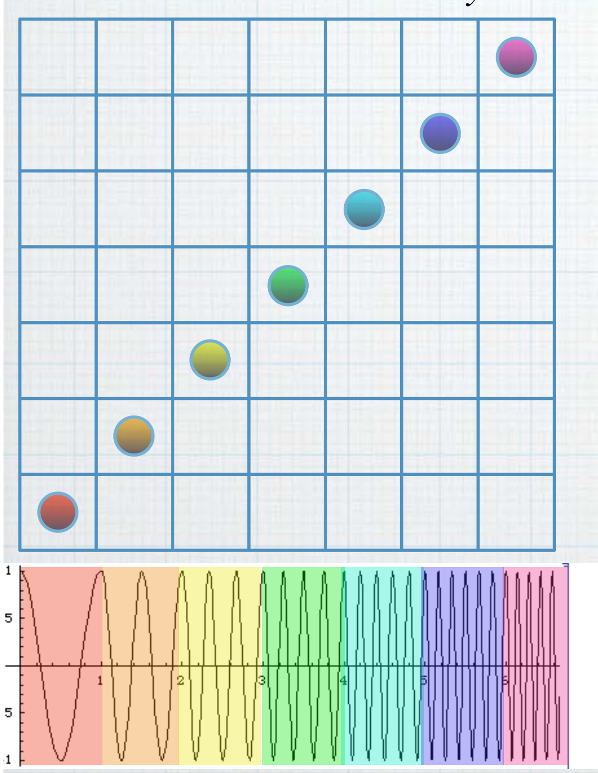
$$\Omega_l = d_l/T, \quad l = 1, 2, \ldots N,$$

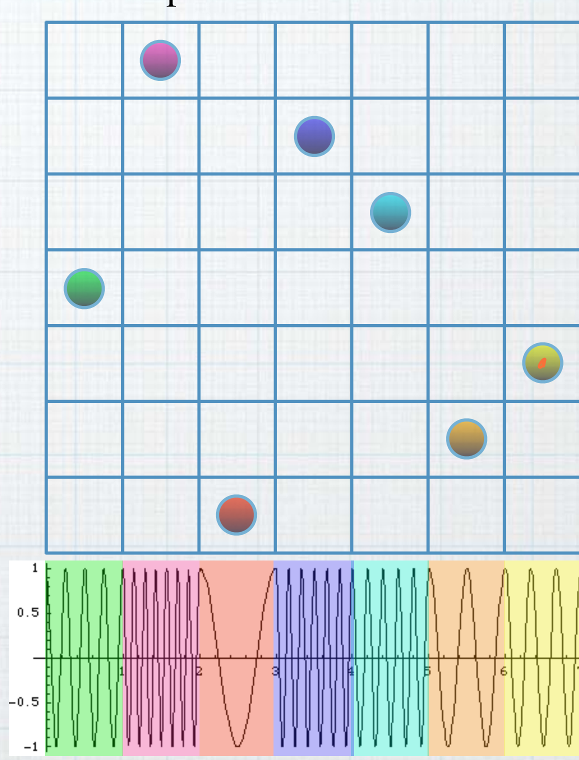
where $\{d_l\}$ is a permutation of the integers $1, 2, \ldots N$.

17.27

# Frequency-Coded Waveforms

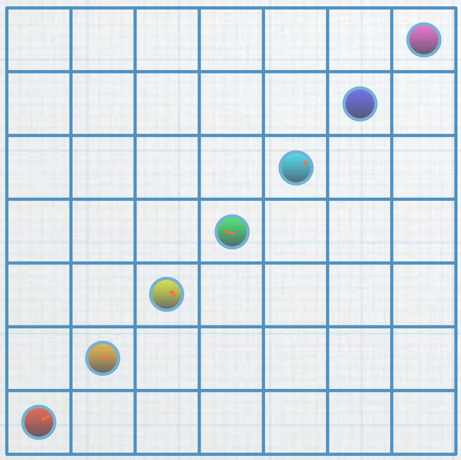
*Geometric Array* or *Binary Matrix* Representation



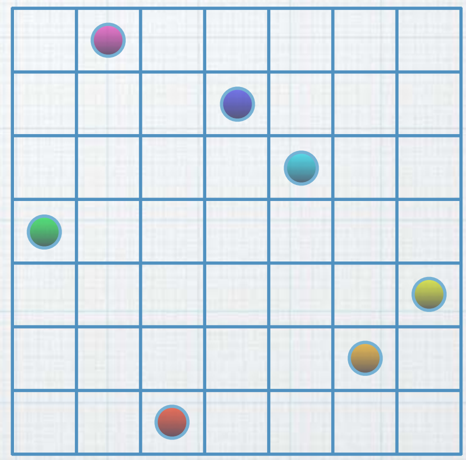


17.28

# Frequency Coding Matrices for Frequency-Coded Signals



**Stepped Frequency Approximation to a Chirp (LFM)**



**Costas Sequence**

17.29

# The Ambiguity Function of Frequency-Coded Waveforms

The ambiguity function of $s(t) = \sum_{l=0}^{N-1} p(t - lT)e^{-j2\pi\Omega_l t}$ is

$$\chi_s(\tau, \nu) = \chi_s^{(1)}(\tau, \nu) + \chi_s^{(2)}(\tau, \nu),$$

where $\chi_s(\tau,\nu) \triangleq \int_{-\infty}^{\infty} s(t)\, s^*(t-\tau)\, e^{+j2\pi\nu t}\, dt$

$$\chi_s^{(1)}(\tau, \nu) = \sum_{m=0}^{N-1} e^{-j2\pi m\nu T} e^{-j2\pi\Omega_m\tau} \chi_p(\tau, \nu),$$

and

$$\chi_s^{(2)}(\tau, \nu) = \sum_{m=0}^{N-1} \sum_{n=0, n\neq m}^{N-1} e^{-j\pi(\Omega_m+\Omega_n)\tau} e^{-j\pi(m+n)T} \boxed{\cdot \chi_p(\tau + (m-n)T, \nu + (\Omega_n - \Omega_m))}$$

n.b. $\beta_s(\tau,\nu) = \chi_s(\tau,-\nu)$.