# Asynchronous Multicarrier DS-CDMA Using Mutually Orthogonal Complementary Sets of Sequences

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***Abstract*—The use of sets of multiple spreading sequences per user in multicarrier code-division multiple-access (CDMA) is investigated. Each user is assumed to have a distinct set of spreading sequences, with a different spreading sequence for each carrier in each user's set. We show that when these sets of sequences are chosen to be the mutually orthogonal (MO) complementary sets of sequences, multiple-access interference is minimal on a nonfading channel. As a result of the autocorrelation sidelobe cancellation properties of the MO complementary sequences, it is possible to pack symbols more closely together on the nonfading channel, resulting in a higher data rate than in multicarrier CDMA using the same spreading sequence for each carrier. The resulting communication system scheme results in an easily parallelized receiver architecture that may be useful in nonfading coherent channels, such as the optical fiber channel or the Rician channel with a strong line-of-sight component. On the Rayleigh fading channel, the performance of the system is identical to that of multicarrier CDMA employing a single spreading sequence per user, with only a minimal increase in receiver complexity.**

***Index Terms*—Code-division multiple access, complementary sequences, multicarrier modulation, multiple-access interference.**

## I. Introduction

RECENTLY, there has been great interest in applying multicarrier modulation techniques to obtain diversity in communications systems. One such technique is multicarrier direct-sequence code-division multiple access (DS-CDMA) [1], in which each of the $M$ carriers in a multicarrier system is multiplied by a spreading sequence unique to each user. This technique has a number of desirable features, including narrow-band interference suppression and a lower required chip rate than that of a single-carrier system occupying the same total bandwidth. The lower required chip rate is a result of the fact that the entire bandwidth is divided equally among $M$ frequency bands. This also allows the receiver to incorporate parallelized signal processing, with each of the $M$ parallel branches having a much lower computational load than that of a single serial processor for a single-carrier system occupying the same bandwidth. In addition, it is easier to implement the parallel receiver architecture of a number of carriers than a larger order RAKE [2].

One question that immediately arises in considering this type of multicarrier DS-CDMA system is whether or not there is any advantage to having each user use a set of $M$ spreading sequences—applying a distinct sequence from the family to each of the $M$ carriers—instead of applying a single spreading sequence to each of the $M$ carriers. If there are any advantages to this approach, what are they, and what family or families of sequences should be used?

In this paper, we describe one approach to assigning a unique set of spreading sequences to each user in a multicarrier DS-CDMA system. Each of the $M$ sequences assigned to an individual user (one sequence per carrier) is distinct. So each user has a unique set of spreading sequences, and each of the spreading sequences in a user's set is different. We describe one such family of sequences that eliminates multiple-access interference (MAI) in asynchronous multicarrier DS-CDMA systems when compared to systems employing a single spreading sequence to each carrier for a particular user. The reduction in MAI reduces the effect of the near–far problem, as well as other MAI-induced errors. Therefore, the proposed system can support more users for a fixed-error probability constraint. Furthermore, the autocorrelation sidelobes are canceled. We may pack information symbols more closely together and hence increase the data rate achievable by a single user.

The proposed system does have some disadvantages when compared to the system in [1]. One disadvantage is that the system is not as resistant to frequency-selective fading, as all frequency components are important for effective reduction of autocorrelation sidelobes, and it is this reduction that allows for individual users to signal at a higher data rate. However, even with this disadvantage, the system appears well suited to certain types of communication channels, such as fiber optical channels, which are relatively stable phase-coherent channels, or Rician channels with a strong line-of-sight (LOS) path, where the effects of frequency-selective fading are minimal.

In a DS-CDMA system, we want the autocorrelation of spreading sequences to be zero for all nonzero shifts. While it is not possible to construct a single binary sequence of values $\{+1, -1\}$ having an (aperiodic) autocorrelation function equal to zero for all nonzero shifts, it is possible to construct two such sequences whose autocorrelation functions, when coherently added, result in a function having value zero for nonzero shifts. An example of such a pair of sequences is the Golay sequences

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[3]. This fact suggests the possible advantage of using multiple sequences per user in a multicarrier DS-CDMA system.

MAI in a DS-CDMA system mainly results from nonzero cross correlation between the intended user's spreading sequence and an unintended user's spreading sequence when matched filtering is used. In fact, for maximal connected sets of $m$-sequences or Gold sequences of length $N = 2^n - 1$, the peak cross correlation magnitude $\theta_c$ is $t(n) = 1 + 2^{\lfloor (n+2)/2 \rfloor}$ [4, p. 604]. This too points to the possibility of reducing MAI using a set of multiple spreading sequences per user in the multicarrier DS-CDMA system.

In this paper, we introduce a technique for multicarrier DS-CDMA that employs a set of spreading sequences for each user. Each user applies a different sequence to each of his subcarriers. By selecting these sequences to be mutually orthogonal (MO) complementary sequences, MAI can be eliminated in the ideal phase-coherent channel, and either data rate or capacity can be increased.

Despreading in the receiver is accomplished on a carrier-by-carrier basis using a set of matched filters matched to the spreading sequences applied to the respective carriers. Hence, excluding MAI and noise, the output of the matched filter corresponding to a particular carrier channel is just the autocorrelation function of the corresponding spreading sequence. The MAI part of the output of the matched filter is the summation of cross correlation functions between the intended user's and unintended users' spreading sequences. After adding the output of all matched filters, the autocorrelation sidelobes and MAI are zero by the defining property of MO complementary sets of sequences.

This paper is organized as follows. In Section II, we briefly discuss the MO complementary sets of sequences. In Section III, we introduce the proposed multicarrier DS-CDMA model. In Section IV, we analyze the system performance in the additive white Gaussian channel (AWGN) channel. In Section V, we present the performance for the Rayleigh fading channel. In Section VI, we discuss the implications of the system performance analysis to the use of the multicarrier DS-CDMA scheme in communication systems.

## II. MO Complementary Sets of Sequences

Here, we briefly describe MO complementary sets of sequences. For a more detailed discussion, see [5].

Let $\theta_{A_iA_i}$ denote the autocorrelation function of the sequences $A_i$, and let $\theta_{A_iA_i}(l) \triangleq \sum_n A_i(n)A_i(n-l)$ denote the $l$th element in the sequence $\theta_{A_iA_i}$. A set of sequences $(A_i, 1 \le i \le M)$ is a complementary set of sequences if and only if

$$\sum_{i=1}^{M} \theta_{A_iA_i}(l) = 0 \qquad \forall\, l \ne 0.$$

Let $\theta_{A_iB_i}$ denote the cross correlation function of the sequences $A_i$ and $B_i$, and let $\theta_{A_iB_i}(l) \triangleq \sum_n A_i(n)B_i(n-l)$ denothe the $l$th element in the sequence $\theta_{A_iB_i}$. A set of sequences $(B_i, 1 \le i \le M)$ is said to be a mate of the set $(A_i)$ if the following is true:

1) the length of $A_i$ is equal to the length of $B_i$, for $1 \le i \le M$;
2) the set $(B_i, 1 \le i \le M)$ is a complementary set; and
3) $\sum_{i=1}^{M} \theta_{A_iB_i}(l) = 0,\ \forall\, l.$

Complementary sets of sequences are said to be *mutually orthogonal complementary sets of sequences* (for brevity, we call them MO complementary sets of sequences) if any two are mates to each other. Binary complementary sets of sequences are discussed in [3], binary MO complementary sets of more than two sequences are discussed in [5], and multiphase MO complementary sets of sequences are discussed in [6]. Complementary sets of sequences have been used in radar to eliminate range sidelobes, allowing the detection and resolution of objects that would otherwise be hidden in the range sidelobes of large nearby scatterers. However, only a set of two complementary sequences is normally used in radar applications [7], although, recently, the use of related waveform sets employing larger numbers of waveforms has been considered for reducing both range and Doppler sidelobes in radar [8], [9].

One way to think about MO complementary sequences uses Hadamard matrices. To do so, we replace the elements of a Hadamard matrix by sequences, and we replace multiplication of matrix elements by a correlation of the substituted sequences. We can generate longer sequences and a larger number of sets of sequences using the following theorem [5].

*Theorem 1:* Let $A$ be a matrix of sequences whose columns are MO complementary sets. Let

$$B = \begin{bmatrix} A \otimes A & -A \otimes A \\ -A \otimes A & A \otimes A \end{bmatrix} \tag{1}$$

where $\otimes$ denotes interleaving. Then, columns of $B$ are also MO complementary sets. The interleaving of two sequences $\hat{A} = \{a_1, a_2, \cdots\}$ and $\hat{B} = \{b_1, b_2, \cdots\}$ is given by

$$\hat{A} \otimes \hat{B} = \{a_1, b_1, a_2, b_2, \cdots\}.$$

The interleaving of two matrices of sequences is done on a component-by-component basis (each component of a matrix of sequences is a sequence, not a number).

The following example shows how to use this theorem.

*Example 1:* Let

$$A = \begin{bmatrix} ++ & -+ \\ +- & -- \end{bmatrix} \triangleq \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{2}$$

where + is 1 and – is –1. Note that $A$ is a 2 × 2 matrix of sequences. The first column of $A$, $[{}^{++}_{+-}]$, is a complementary set of sequences because

$$\theta_{A_{11}A_{11}} + \theta_{A_{21}A_{21}} = \{0, 4, 0\}.$$

The second column of $A$, $[{}^{-+}_{--}]$, is also a complementary set of sequences because

$$\theta_{A_{12}A_{12}} + \theta_{A_{22}A_{22}} = \{0, 4, 0\}.$$

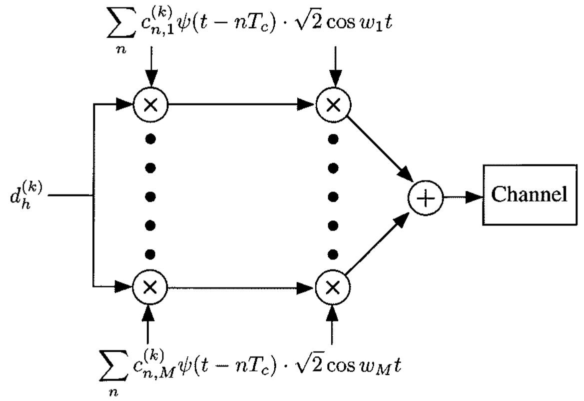


Fig. 1. Transmitter model.

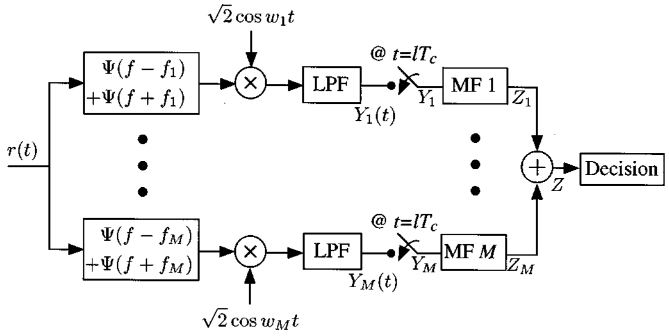


Fig. 2. Receiver model.

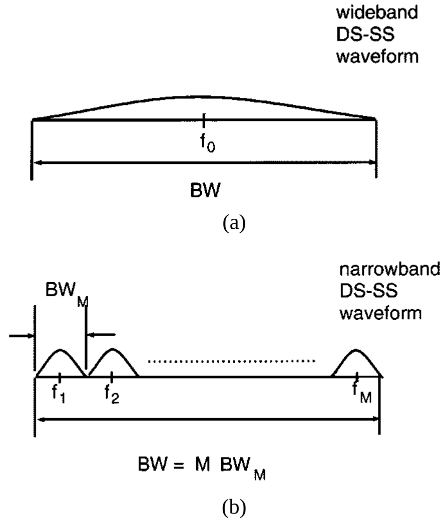


Fig. 3. The spectra of the transmitted signal. (a) Single-carrier system. (b) Multicarrier system.

One complementary set of sequences is a mate of the other, and they are MO complementary sets, because the sum of cross correlation functions is zero

$$C^{(2)} \triangleq \theta_{A_{11}A_{12}} + \theta_{A_{21}A_{22}} \tag{3}$$
$$= \{0, 0, 0\}. \tag{4}$$

Now, we use the theorem. First, we compute

$$A \otimes A = \begin{bmatrix} ++\otimes++ & -+\otimes-+ \\ +-\otimes+- & --\otimes-- \end{bmatrix} \tag{5}$$

and

$$-A \otimes A = \begin{bmatrix} --\otimes++ & +-\otimes-+ \\ -+\otimes+- & ++\otimes-- \end{bmatrix}. \tag{6}$$

It follows that $B$ is defined in (7), shown at the bottom of the page. Therefore, the sum of cross correlation functions of the first and $k$th columns of $B$ is

$$C^{(k)} \triangleq \theta_{B_{11}B_{1k}} + \theta_{B_{21}B_{2k}} + \theta_{B_{31}B_{3k}} + \theta_{B_{41}B_{4k}} \tag{8}$$
$$= \{0, 0, 0, 0, 0, 0, 0\}, \qquad k = 2, 3, 4. \tag{9}$$

Multiphase MO complementary sequences exist [6] and are more bandwidth-efficient than binary MO complementary sequences when used as spreading sequences. However, to illustrate the basic ideas of the proposed system and to simplify the analysis, we only consider binary MO complementary sets in the proposed system.

## III. System Model

The transmitter and receiver models are shown in Figs. 1 and 2. The spectrum of the transmitted signal for a single-carrier and multicarrier system is shown in Fig. 3. The total bandwidth is given by

$$BW = MBW_M \tag{10}$$

where $M$ is the number of carriers, and we assume a strictly band-limited chip waveform with bandwidth $BW_M$. The symbol duration is $T = NT_c$, where the chip period $T_c$ is $M$ times larger than that of single-carrier systems, and $N$ is the length of spreading sequences. Similarly, $E = NE_c$ is the bit energy, where $E_c$ is the chip energy. Without loss of generality, we assume user $k = 1$ is the intended user. The phase and delay of user $k = 1$ are assumed to be zero without loss of generality. Users $k = 2, \cdots, K$ are interfering users. They are assumed to be independent.

### A. Transmitter Model

Let $d_h^{(k)}$ be the data stream for the $k$th user, and let $\{c_i^{(k)}, i = 1, 2, \cdots, M\}$ be one of the MO complementary sets of se-

$$B = \begin{bmatrix} ++++ & --++ & -+-+ & +--+ \\ ++-- & ---- & -++- & +-+- \\ -+-+ & +--+ & ++++ & --++ \\ -++- & +-+- & ++-- & ---- \end{bmatrix} \triangleq \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix} \tag{7}$$

quences of length $N$ for the user $k$. The transmitted signal of user $k$ in the $i$th branch is given by

$$TS_i^{(k)}(t) = \sum_{n=-\infty}^{\infty} d_h^{(k)} c_{n,i}^{(k)} \psi\left(t - nT_c - \tau^{(k)}\right), \quad k = 1, \cdots, K, \quad i = 1, \cdots, M \tag{11}$$

where $h = \lfloor n/N \rfloor$, $\tau^{(k)}$ is the delay for user $k$, and the chip waveform $\psi(t)$ has Fourier transform $\Psi(f)$. We assume $\Psi(f)$ is strictly band-limited.

### B. Channel Models

We consider two channels in this paper. First, we assume the system operates over an AWGN channel with zero mean and two-sided power spectral density $\eta_0/2$. We also assume perfect carrier-phase coherence. The fiber optic channel is well modeled by such a channel, because the photon rate is usually sufficiently high to justify the Gaussian approximation for the optical shot noise, and the receiver's electronic noise—which is usually dominant—is well modeled by additive Gaussian noise [10]. Second, we analyze the system's performance in the Rayleigh slow fading channel. For user $k$, the impulse response of the $i$th frequency band is $\alpha_{k,i} e^{j\beta_{k,i}}$, where the $\alpha_{k,i}$ are independently, identically distributed (i.i.d.) Rayleigh random variables with unit second moment (so average received signal power is equal to transmitted signal power), and the $\beta_{k,i}$ are i.i.d. uniform random variables over $[0, 2\pi)$. We also assume $d_h^{(k)}$, $\tau^{(k)}$, $\alpha_{k,i}$, and $\beta_{k,i}$ are independent.

### C. Receiver Model

The chip-matched filters $\Psi^*(f - f_i) + \Psi^*(f + f_i)$, $i = 1, \cdots, M$ (i.e., ideal bandpass filters) are used to separate the $M$ multicarrier frequency bands. Matched filters matched to $c_i^{(k)}$, $i = 1, 2, \cdots, M$ are then used for despreading in the receiver. The $M$ matched filter outputs are then summed and sampled. Note that it is natural to use a radix-2 fast Fourier transform to perform this processing, because the complementary sequences generated using the procedure in [5] always have length of $2^p$, where $p$ is an integer.

## IV. Performance Analysis in AWGN Channel

We compare our system to a single-carrier DS-CDMA operating on an AWGN channel [11]. Note that in neither case is the Gaussian approximation used. As we defined it before, $\tau^{(k)}$ is the delay of user $k$ and $T_c$ is the chip duration. Let $\lambda^{(k)} = \lfloor \tau^{(k)}/T_c \rfloor$, and $\delta^{(k)} = \tau^{(k)} - \lambda^{(k)} T_c$. The MAI from interferer $k$ is given by [11]

$$B_k = \frac{1}{E_c}\left[A_k\left(d_h^{(k)}, \lambda^{(k)}, \Delta^{(k)}\right) \cos\left(\beta_{k,1}\right)\right]. \tag{12}$$

We express $A_k$ as

$$A_k = \begin{cases} d_h^{(k)} \tilde{A}_k, & \text{if } d_h^{(k)} = d_{h-1}^{(k)} \\ d_h^{(k)} \hat{A}_k, & \text{if } d_h^{(k)} = -d_{h-1}^{(k)} \end{cases}$$

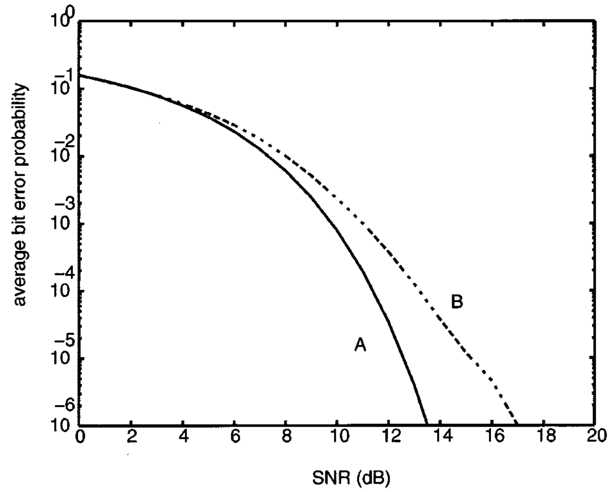


Fig. 4. The average error probability in AWGN. (a) The proposed system with complementary sequences of length 8, 8 carriers. (b) The single-carrier DS-CDMA, AO/LSE $m$-sequence of length 63, sinusoidal chip waveform, $K = 3$.

where

$$\tilde{A}_k = \frac{1}{T_c}\left[\theta^{(k)}(\lambda^{(k)})\hat{R}_\psi(\Delta^{(k)}) + \theta^{(k)}(\lambda^{(k)}+1)R_\psi(\Delta^{(k)})\right] \tag{13}$$

$$\hat{A}_k = \frac{1}{T_c}\left[\hat{\theta}^{(k)}(\lambda^{(k)})\hat{R}_\psi(\Delta^{(k)}) + \hat{\theta}^{(k)}(\lambda^{(k)}+1)R_\psi(\Delta^{(k)})\right]. \tag{14}$$

Here, $\theta^{(k)}(\lambda) = C^{(k)}(\lambda) + C^{(k)}(\lambda - N)$ and $\hat{\theta}^{(k)}(\lambda) = C^{(k)}(\lambda) - C^{(k)}(\lambda - N)$where $C^{(k)} = \sum_{m=1}^{M} C_m^{(k)}$ (defined in Section II) is the sum of cross correlation functions of $c_m^{(k)}$ and $c_m^{(1)}$. $\hat{R}_\psi(\Delta)$ and $R_\psi(\Delta)$ are partial autocorrelation functions of the chip waveform and are given by [11]

$$\hat{R}_\psi(\Delta) = \int_\Delta^{T_c} \psi(t)\psi(t-\Delta)\,dt \tag{15}$$

$$R_\psi(\Delta) = \int_0^\Delta \psi(t)\psi(t+T_c-\Delta)\,dt. \tag{16}$$

Note that $C^{(k)}(\lambda)$ is an all-zero sequence (as stated in Section II); so are $\theta^{(k)}(\lambda^{(k)})$ and $\hat{\theta}^{(k)}(\lambda^{(k)})$, $k = 2, \cdots, K$. Therefore, the total $\mathrm{MAI} = \sum_{k=2}^{K} B_k$ is zero no matter what data bits the other users are transmitting. In addition, the MAI is independent of the chip waveform.

The receiver output signal for user 1 has a very narrow peak at $t = T$. The range of the peak is $[T - T_c, T + T_c]$. At all other times, the output is zero. The error probability is

$$P_e = Q\left(\sqrt{\mathrm{SNR}}\right) \tag{17}$$

where the signal-to-noise ratio (SNR) is $E/(\eta_0/2)$, and

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{z^2}{2}\right)dz. \tag{18}$$

The bit-error probability remains the same as the number of interfering users increases, assuming perfect carrier synchronization. Of course, the number of interfering users cannot increase arbitrarily, as there must be MO complementary sets of sequences to accomplish them.

The average error probability as a function of SNR is shown in Fig. 4. We observe that our system outperforms the single-channel system analyzed in [11] in AWGN. The sequences for single-carrier DS-CDMA are the auto-optimal/least sidelobe energy (AO-LSE) $m$-sequences [12] of length 63. Note that there is little difference in bit-error probability in the single-carrier CDMA/AWGN channel when different chip waveforms are used [11].

## V. Performance in Rayleigh Fading Channels

We anticipate that our system has similar performance to that in [1] in the Rayleigh fading channel with equal-gain combining (EGC) or maximum-ratio combining (MRC). There are reasons we use EGC instead of MRC, although MRC is optimal in Rayleigh fading [1]. First, the Rayleigh fading channel is completely phase-noncoherent, so it destroys the properties of MO complementary sets of sequences. Therefore, it is intuitive that our scheme has no performance gain over that in [1] whenever EGC or MRC is used. Second, we do not know the cross correlation properties of MO complementary sets of sequences, but we do know the properties of the sum of the cross correlation. Therefore, we use EGC to obtain analytic results with deterministic sequences (MO complementary sets of sequences). We cannot do that for MRC with deterministic sequences.

We assume the bit rate is the same for our system and the system described in [1]. We assume perfect carrier, symbol, and chip synchronization for user $k = 1$, and we evaluate the performance of the first user. It is standard to assume $d_h^{(k)}$, $k = 2, \cdots, K$, are independent, so $d_h^{(k)} c_{n,i}^{(k)}$, $k = 2, \cdots, K$, are independent sequences as well. We first look at the $i$th branch of the receiver for user $k = 1$.

### A. Chip-Matched Filter Output

The chip-matched filter output is given by

$$Y_i(t) = S_{y_i}(t) + I_{y_i}(t) + N_{y_i}(t) \tag{19}$$

where

$$S_{y_i}(t) = \sqrt{E_c}\alpha_{1,i} \sum_{n=-\infty}^{\infty} d_h^{(1)} c_{n,i}^{(1)} x(t - nT_c) \tag{20}$$

$$I_{y_i}(t) = \sum_{k=2}^{K} \sqrt{E_c}\zeta_{k,i} \sum_{n=-\infty}^{\infty} d_h^{(k)} c_{n,i}^{(k)} x(t - nT_c - \tau^{(k)}). \tag{21}$$

Here, $S_{y_i}(t)$ is the signal component, $I_{y_i}(t)$ is the MAI term, and $N_{y_i}(t)$ is the the noise passing through bandpass and lowpass filters, where $\zeta_{k,i} \equiv \alpha_{k,i} \cos(\beta_{k,i} - \beta_{1,i})$, and $x(t)$ has Fourier transform $X(f) \equiv |\Psi(f)|^2$. We assume $X(f)$ satisfies the Nyquist constraint and $\int_{-\infty}^{\infty} X(f)\, df = 1$.

### B. Decision Statistics

After matched filtering and sampling, we have decision statistic

$$Z_i = S_{Z_i} + I_{Z_i} + N_{Z_i} \tag{22}$$

where

$$S_{Z_i} = \sum_{n'=0}^{N-1} c_{n',i}^{(1)} S_{y_i}(n'T_c) \tag{23}$$

$$I_{Z_i} = \sum_{n'=0}^{N-1} c_{n',i}^{(1)} I_{y_i}(n'T_c) \tag{24}$$

$$N_{Z_i} = \sum_{n'=0}^{N-1} c_{n',i}^{(1)} N_{y_i}(n'T_c). \tag{25}$$

Without loss of generality, assume $d_h^{(1)} = 1$. The mean of decision statistic can be written as

$$E[Z_i|\alpha_{1,i}] = N\sqrt{E_c}\alpha_{1,i} \tag{26}$$

because $|\Psi(f)|^2$ satisfies the Nyquist criterion. Furthermore

$$\mathrm{var}[Z_i|\alpha_{1,i}] = \sigma_i^2 \tag{27}$$

$$= \mathrm{var}[I_{Z_i}] + \mathrm{var}[N_{Z_i}] \tag{28}$$

because the noise term $N_{Z_i}$ is independent of $I_{Z_i}$ and $N_{Z_i}$ are independent of $\alpha_{1,i}$.

We use a band-limited chip waveform to minimize self-interference. The allocated spectrum is divided equally into $M$ frequency bands, as shown in Fig. 3. If $I_{Z_i}$ is Gaussian, then $I_{Z_i}$ and $I_{Z_j}$, $i \neq j$, are independent, because there is no spectral overlap, and hence they are uncorrelated. This Gaussian approximation is good for large $N$ and $K$ and is commonly used in the analysis of multicarrier CDMA systems [1], [2], [13]. It is possible to compute the probability density function (pdf) of MAI without Gaussian approximation in the AWGN channel [11]. However, it is difficult to apply the result to multicarrier systems because the independence of $I_{Z_i}$ and $I_{Z_j}$, $i \neq j$, is required to make the problem analytically tractable.

We use EGC techniques at the receiver, i.e.,

$$Z = \sum_{i=1}^{M} Z_i. \tag{29}$$

Denote $\overline{\alpha} = (\alpha_{1,i}, \cdots, \alpha_{1,M})$. Then

$$E[Z|\overline{\alpha}] = N\sqrt{E_c} \sum_{i=1}^{M} \alpha_{1,i} \tag{30}$$

$$\mathrm{var}[N_Z] = \sum_{i=1}^{M} \mathrm{var}[N_{Z_i}]$$

$$= MN\eta_0/2 \tag{31}$$

and

$$\mathrm{var}[I_Z] = \sum_{i=1}^{M} \mathrm{var}[I_{Z_i}] \tag{32}$$

$$= \sum_{i=1}^{M} \left[ NR_{Ii}(0) + 2\sum_{l=1}^{N-1} R_{Ii}(lT_c) \sum_{n'=l}^{N-1} c_{n',i}^{(1)} c_{n'-l,i}^{(1)} \right] \tag{33}$$

where $R_{I_i}(\tau)$ is the autocorrelation function of $I_{y_i}(t)$. It has the Fourier transform $S_{I_i}(f)$, which can be written as

$$S_{I_i}(f) = \frac{E_c}{T_c} |\Psi(f)|^4 \sum_{k=2}^{K} E\left[\zeta_{k,i}^2\right] \sum_{l=-\infty}^{\infty} c_{n,i}^{(k)} c_{n+l,i}^{(k)} e^{-j2\pi f l T_c}. \tag{34}$$

Note that $d_h^{(k)}$ are i.i.d. random variables and $E[d_h^{(k)} d_h^{(l)}] = \delta(k-l)$, so there are no cross terms containing different $k$. Note that $\zeta_{k,\,i}$ are independent random variables, and $c_{n,\,i}^{(k)}$, $c_{n+m,\,i}^{(k)}$ are deterministic. Let $\phi_{k,\,i} \equiv \beta_{k,\,i} - \beta_{1,\,i}$. Then

$$E[\zeta_{k,\,i}^2] = E\left[\alpha_{k,\,i}^2 \cos^2(\phi_{k,\,i})\right] \tag{35}$$

$$= E\left[\alpha_{k,\,i}^2\right] E\left[\frac{1+\cos(2\phi_{k,\,i})}{2}\right] \tag{36}$$

$$= \frac{1}{2}. \tag{37}$$

By the correlation properties of complementary sequences, $\sum_{i=1}^{M} c_{n,\,i}^{(k)} c_{n+l,\,i}^{(k)} = M\delta(l)$, and

$$\sum_{i=1}^{M} NS_{Ii}(f) = N\frac{E_c}{2T_c}|\Psi(f)|^4 \cdot \sum_{k=2}^{K} \sum_{l=-\infty}^{\infty} \sum_{i=1}^{M} c_{n,\,i}^{(k)} c_{n+l,\,i}^{(k)} e^{-j2\pi f l T_c} \tag{38}$$

$$= (K-1)MN\frac{E_c}{2T_c}|\Psi(f)|^4. \tag{39}$$

When a sinc function with unit energy ($E_c = 1$) is used as the chip waveform [i.e., $\Psi(f) = \sqrt{T_c}\sqcap(fT_c)$], the conditional SNR can then be written as

$$\mathrm{SNR}_c = \frac{E^2[Z|\overline{\alpha}]}{\mathrm{var}[Z|\overline{\alpha}]} \tag{40}$$

$$= \frac{E^2[Z|\overline{\alpha}]}{\mathrm{var}[I_Z] + \mathrm{var}[N_Z]} \tag{41}$$

$$= \frac{N^2 E_c \left(\sum_{i=1}^{M} \alpha_{1,\,i}\right)^2}{MN(K-1)E_c/2 + MN\eta_0/2} \tag{42}$$

$$\geq \frac{u}{(K-1)M/(2N) + M/\mathrm{SNR}} \tag{43}$$

$$\equiv \mathrm{SNR}_{c_B}(u) \tag{44}$$

where $u = \sum_{i=1}^{M} \alpha_{1,i}^2$ is a chi-square random variable with $2M$ degrees of freedom. The above inequality is obtained by the inequality $(\sum_{i=1}^{M} \alpha_{1,\,i})^2 \geq \sum_{i=1}^{M} \alpha_{1,\,i}^2$ $(\alpha_{1,\,i} \leq 0)$. Because the $Q$-function is monotonically decreasing, an upper bound of average error probability is given by

$$P_e \leq \int_0^{\infty} Q\left(\sqrt{\mathrm{SNR}_{c_B}(u)}\right) f_u(u)\, du \tag{45}$$

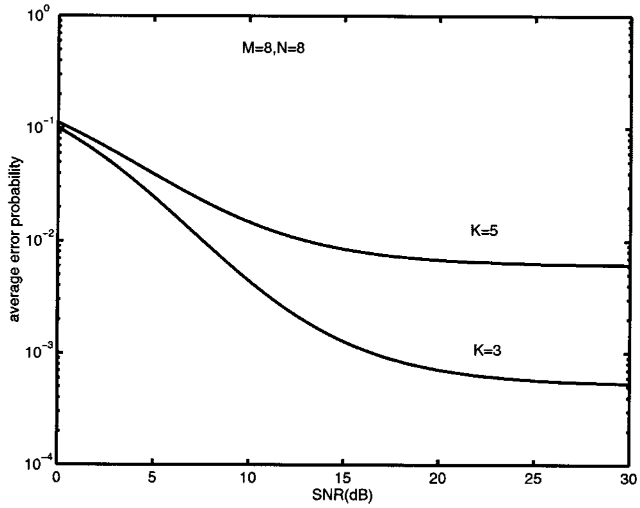


Fig. 5. The average error probability for the Rayleigh fading channel.

where $f_u$ denotes the pdf of $u$ and is given by [14]

$$f_u(u) = \frac{1}{2^M (M-1)!} u^{M-1} e^{-u/2} U(u) \tag{46}$$

where $U(z)$ is a unit step function.

Now, we compare the performance of our system to [1], assuming the same symbol interval and EGC is used for diversity reception in both systems. *It is important to point out the difference in the notation of the chip rate: $T_c$ in our system, but $MT_c$ in [1].* Note that the Fourier transform of $\mathrm{var}[I_Z(t)]$ is equal to $((MN(K-1)E_c)/2T_c)|\Psi(f)|^4$ in [1] and our system. Therefore, our system has the same $\mathrm{SNR}_c$ and $P_e$ as theirs. The intuitive explanation is as follows. Because a user has different sequences in $M$ branches, frequency diversity is lost, but "sequence diversity" is introduced. There is still diversity reception to reduce the effect of fading. In addition, the Rayleigh fading channel's phase response is completely random, therefore, which particular sequences are used does not matter. Fig. 5 shows the average error probability versus the SNR for both systems.

## VI. Discussion

The proposed system is well suited to a fiber optical channel or Rician channels with a strong LOS path. In such channels, the data rate can be increased for each user. Actually, for the ideal AWGN channel, the data rate per user can be as much as $M$ times higher than that of [1] without significant increase in system complexity. The MAI is reduced, so our system can support more users for a given average error probability constraint. It is important to point out that although severe constraints on the existence of a set of $M$ spreading sequences instead of a single spreading sequence for each user, the number of users this system can support (capacity) is comparable to their system assigning a single Gold sequence per user. In fact, there are $N+2$ Gold sequences of length $N = 2^n - 1$ [4, p. 605]; there are $N$ MO complementary sets of sequences of length $N = 2^n$ [5, p. 649]. One plausible explanation is that observing matrix B in Example I of Section II, there is significant redundancy present. Namely, both (1,1) entry and (3,3) entry of B is the sequence

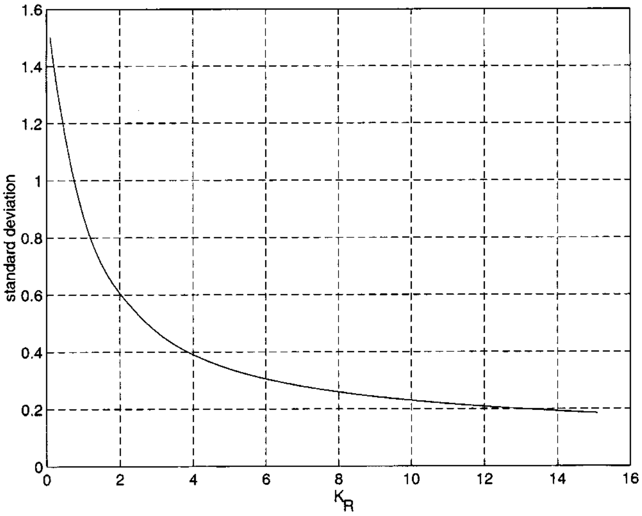


Fig. 6. The standard deviation of the Rician fading channel phases response $\sigma_\beta$ versus Rice factor $K_R$.

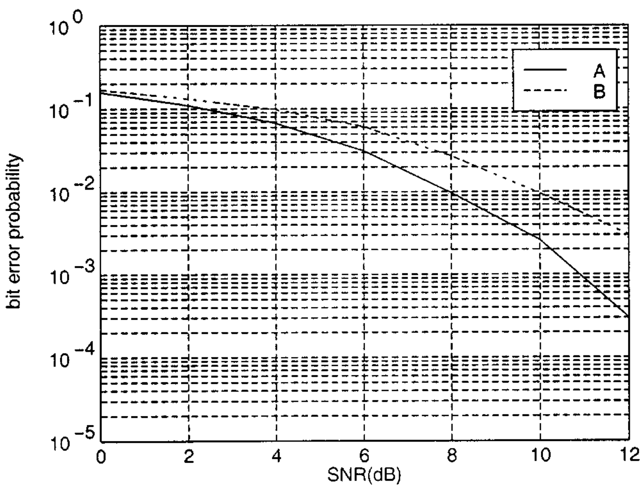


Fig. 7. The simulation results of the multicarrier CDMA systems in Rician fading channels, $K_R = 10$, $M = N = K = 4$. The solid line denotes the proposed system, different spreading sequences for each carrier, EGC. The dotted line denotes the same spreading sequence for each carrier, MRC.

$\{1111\}$. One shortcoming of our scheme is that it is vulnerable to frequency-selective fading, and our analysis concludes it performs the same as the system described in [1] in Rayleigh frequency-selective fading.

An example of a Rician fading channel with a strong LOS path is the radio channel in factories [15]. The empirical data indicates that fading in some factories is Rician with a Rice factor of $K_R = 10$ dB, where the Rice factor is the ratio between LOS path power and scattered component power [15]. Fig. 6 shows that the standard deviation of the phase variation is small for such a channel (the pdf of the phase is given in [16]). Thus, we anticipate that our system will outperform the single-sequence-per-user system described in [1] for such a channel. The simulation results are given in Fig. 7. Note that when the SNR increases, the difference between the two systems also increases, because MAI dominates thermal noise in high SNR.

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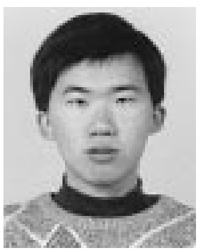

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