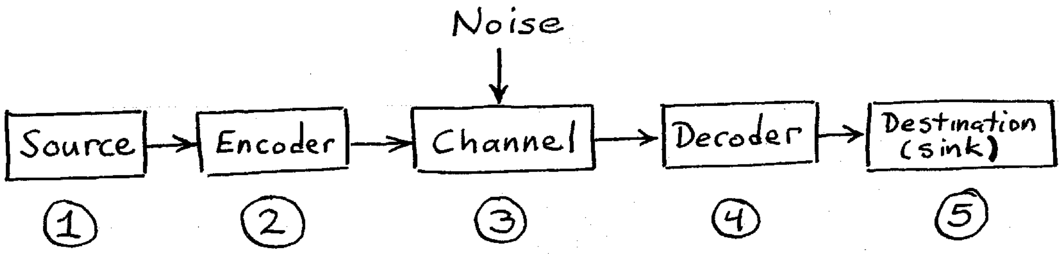
A Typical Communication System

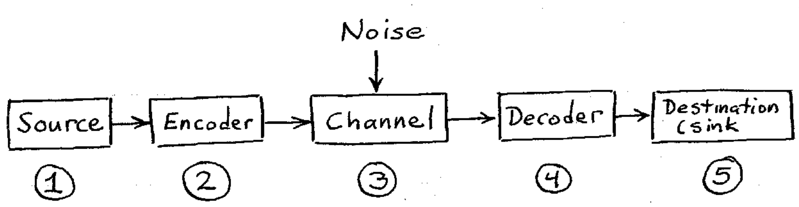
2.1



① **Source**: Origin of information to be communicated

(e.g., voice, images, computer data, DNA sequences, physical measurements, etc.)

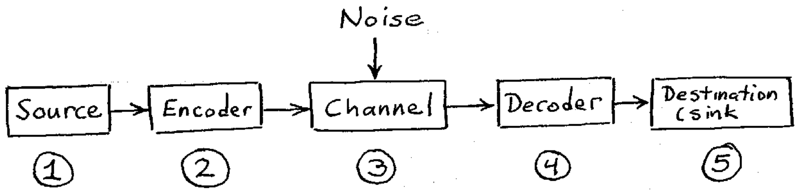
2.2



③ **Channel**: Physical Medium across which information must be transfered.

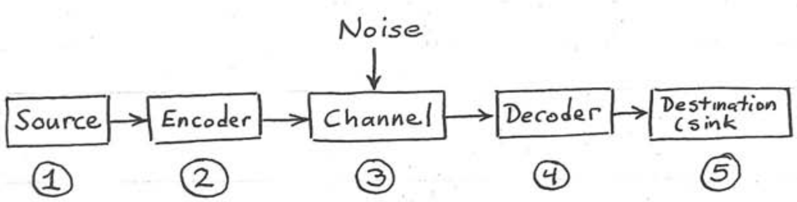
(e.g., microwave link, optical fiber, free-space (mw or optical), magnetic storage medium (disk or tape) optical storage medium (CD or DVD) phonograph record, telephone line, tin-can telephone string, etc.)

2.3



5 **Destination (sink, user)**: This is where the information is used. (Could be CRT/display, human visual system, human ear, computer system, etc.)

2.4



2 **Encoder**: Provides necessary processing (coding) of source output before transmission through channel.

"Matches source to channel"

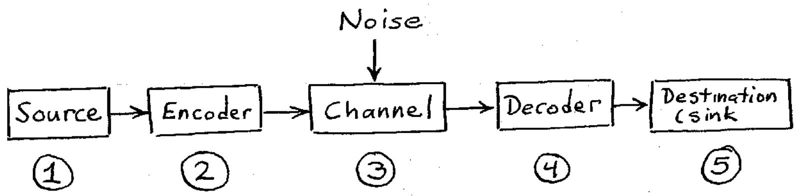
Example: Suppose you have a source that produces a message using a 4-letter alphabet and you want to send it over binary channel!

Source → Encoder →

$a_1 \mapsto 0$ $a_3 \mapsto 110$

$a_2 \mapsto 10$ $a_4 \mapsto 111$

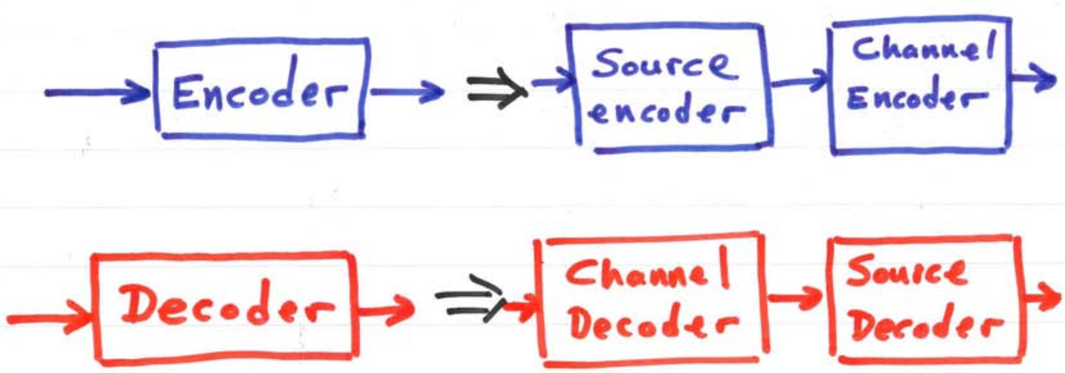
2.5



4 **Decoder**: Processes the channel output with the goal of producing an "acceptable" replica of the source output.

2.6

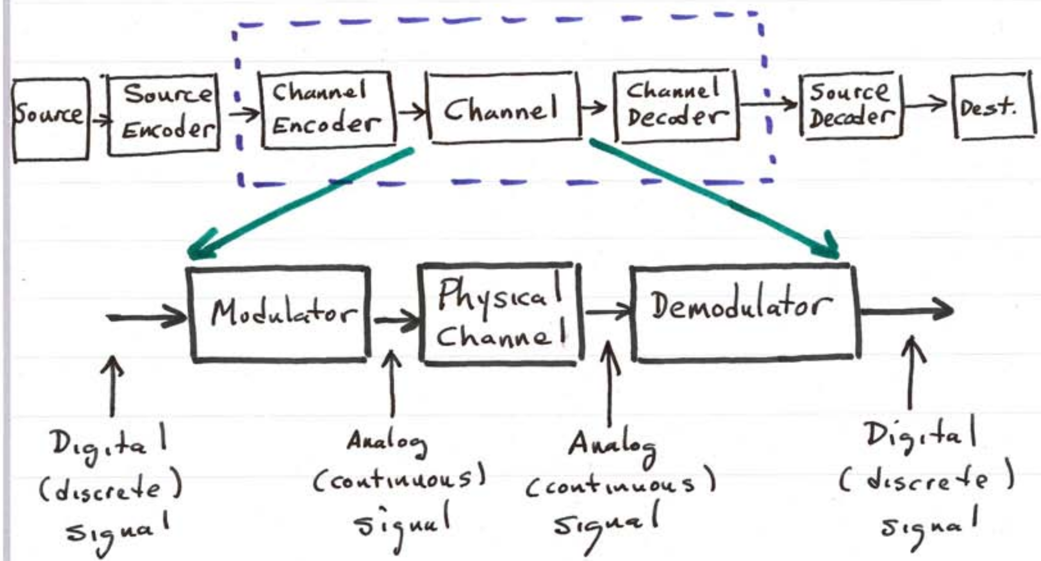
Sometimes the encoder and decoder are broken into two parts (each)



We will see that we can always do this decomposition and still achieve optimal performance if we encode in large blocks (asymptotically).

2.7

When this decomposition is done, the communication system appears as follows:



2.8

In light of this...

1. Think about how a digital audio CD system would fit into this framework

2. Think about how a deep-space probe (e.g. Voyager) sending images back to earth fits into this framework.

As we will see, typically

channel decoding

and

source encoding

are the most complex and difficult parts of the system

2.9

Mathematical Preliminaries

2.10

Probability:

Defn: A probability space $(\mathcal{S}, \mathcal{F}, P)$ is a triple consisting of

$\mathcal{S}$ = sample space

$\mathcal{F}$ = event space (σ-field)

P = probability measure

2.11

- $\mathcal{S}$ = set of all outcomes
  (one and only one outcome occurs when the random experiment is performed.)

- An <u>event</u> $A \subset \mathcal{S}$. We say that an event A occurs if outcome $\omega \in A$.

- The <u>event space</u> $\mathcal{F}$ is a collection of events satisfying closure properties:
  1. If $A \in \mathcal{F} \Rightarrow \overline{A} \in \mathcal{F}$
  2. If $A_1, \ldots, A_n \in \mathcal{F}$, $\Rightarrow \bigcup_{i=1}^{n} A_i \in \mathcal{F}$
  3. If $A_1, \ldots, A_n, \ldots \in \mathcal{F}$
     $\Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$.

2.12

It follows that all countable sequences of set operations ($\cup, \cap, \overline{}$) on elements of $\mathcal{F}$ yield elements of $\mathcal{F}$.

- A <u>probability measure</u> $P : \mathcal{F} \to \mathbb{R}$ assigns a real number $P(A)$ to each $A \in \mathcal{F}$ such that $P(\cdot)$ satisfies the <u>Axioms of Probability</u>.

Axioms of Probability

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1. $P(A) \geq 0, \quad \forall A \in \mathcal{F}$.

2. $P(\mathcal{S}) = 1$.

3. If $A_1, \ldots, A_n \in \mathcal{F}$ are disjoint,

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

4. If $A_1, A_2, \ldots, A_n, \ldots \in \mathcal{F}$ are disjoint,

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i).$$

Conditional Probability:

2.14

Given $(\mathcal{S}, \mathcal{F}, P)$, let $B \in \mathcal{F}$ s.t. $P(B) > 0$. Then the conditional probability measure conditioned on B, $P(\cdot|B)$ is

$$P(A|B) \triangleq \frac{P(A \cap B)}{P(B)}, \quad \forall A \in \mathcal{F}.$$

n.b We can easily show that $P(\cdot|B)$ is a valid probability measure and thus $(\mathcal{S}, \mathcal{F}, P(\cdot|B))$ is a valid probability space.

Independent Events:

2.15

Given $(\mathcal{S}, \mathcal{F}, P)$ and $A_1, A_2 \in \mathcal{F}$, A_1 and A_2 are <u>statistically independent</u> iff

$$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2).$$

$A_1, A_2, \ldots, A_n \in \mathcal{F}$ are <u>statistically independent</u> iff

$$P(A_{j_1} \cap A_{j_2} \cap \cdots \cap A_{j_m}) = P(A_{j_1})\, P(A_{j_2}) \cdots P(A_{j_m})$$

for all combinations of 2 through n events.

2.16

Defn: A <u>random variable</u> X defined on $(\mathcal{S}, \mathcal{F}, P)$ is a function $X : \mathcal{S} \to \mathbb{R}$ mapping $\mathcal{S}$ to the real numbers.

(Technically, its a Borel measurable function so that for any Borel set $A \in B(\mathbb{R})$ of the real line

$$X^{-1}(A) \in \mathcal{F}, \quad \forall A \in B(\mathbb{R}).$$

This allows us to compute the probability that RV X takes on a value in set A.)

2.17

I will assume you know how to characterize random variables using

pdfs — continuous RVs
pmfs — discrete RVs

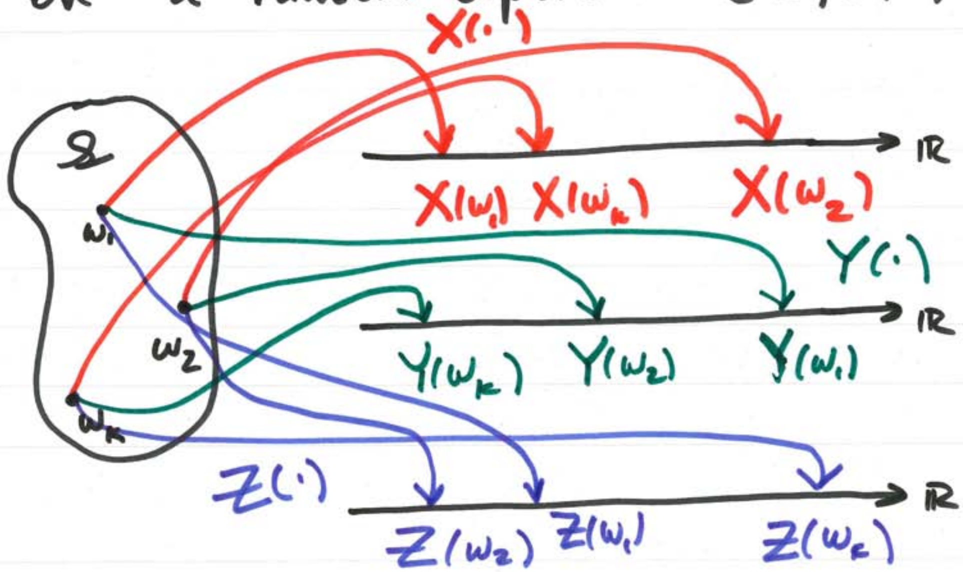
Recall: A discrete RV takes on values from a discrete (finite or countable) set.

A continuous RV takes on values from an uncountable set.

2.18

Multiple RVs:

We can define multiple RVs on a random experiment $(\mathcal{S}, \mathcal{F}, P)$:



I will assume you know how to characterize the joint probabilistic behavior of jointly distributed multiple RVs using joint pdfs and joint pmfs.

We can also define a countable sequence of RVs

$$X_1(\omega),\ X_2(\omega),\ \dots,\ X_n(\omega),\ \dots$$

on a random experiment $(\mathcal{S}, \mathcal{F}, P)$

(n.b. If each RV X_n is measurable, the sequence is measurable as well.)

Random sequences and their convergence are fundamental to information theory.