# STABILITY AND ACCURACY OF FINITE ELEMENT SOLUTIONS FOR OVERLAND FLOW PROBLEMS

**Rabi H. Mohtar, Project Leader**

## Cooperators:
Fouad Jaber (*University of Florida*)
Jagadeesh Anmala (*ABE, Purdue*)

## Goals:
To improve the numerical solution of the kinematic wave equation for overland flow by identifying stable and accurate numerical schemes and developing time steps criteria for solution accuracy.

The modeling challenge will explore the effect of the discretization and the scale availability of the data on the numerical solution of the more physically-based modeling approach of the overland flow and sediment transport. Numerous hydrologic models will use the guidelines developed in this study for better decision-making.

## Recent Publications:


## Statement of Problem:
Estimating surface water flow after a storm is critical in our humid regions for flood analysis and water quality predictions. As this life-critical resource becomes endangered with pollution, we look for the science and engineering of water flow paths to help us better provide sustainable and clean water for human needs.

Besides being a vital resource, water is the major carrier to surface and subsurface pollutants from sediments to agricultural and hazardous chemicals. Accurate knowledge of water flow paths is very critical to the environmental assessment of watershed systems.

The products of the proposed research include better knowledge, technologies and strategies to manage water and soil resources at a watershed scale. The expected benefit of the project is the long-term sustainability of our watersheds to support their residents, agriculture and the environment.

## Current Activities:
Kinematic wave theory is widely used in modeling a variety of hydrologic processes. Solving the kinematic wave equations for overland flow using the conventional consistent Galerkin finite element scheme is known to result in numerical oscillations. The upwind finite element scheme and the lumped finite element scheme are tested as alternatives to the consistent method. New accuracy-based dynamic time step estimate for the 1-D and 2-D overland flow kinematic wave solution is developed. The newly developed dynamic time step estimates are functions of the mesh size, watershed slope, roughness, excess rainfall, and time of concentration. The new criteria are developed by comparing each numerical solution of the kinematic wave equation to the characteristic method-based analytical solution, using different time steps and meshes. For each simulation, characterized by boundary and initial conditions and mesh size, an optimal time step that integrates the problem within 5% error is determined. The series of mesh sizes and corresponding optimal time steps is used to develop the dynamic time step. The time step criteria is tested on a variety of problems, including a steady state and time varying rainfall scenarios, and proved to be adequate for accurate and stable results within an efficient computational time. The criteria can be easily integrated in flow routing models to select the optimal time step with minimal user input. An ARCVIEW GIS interface to the model is developed and used to develop and evaluate the dynamic time step criteria for 2-D problems.
The methodology to develop dynamic time step criteria has been established and tested for 1-D problems. The lumped scheme has been identified as the most efficient finite element scheme for 1-D problems and dynamic time steps have been developed for it. The 2-D phase is now being implemented. A stable and accurate scheme is being identified and dynamic time steps are being developed for it. A GIS interface that has been developed for the 2-D model is a major tool in this work.

**Screening of Finite Element Schemes**

**Figure 1.** Error variation with time step for the consistent –C, the limped-L, the upwind 1.0 consistent –CU, and the

![Graph showing error variation with time step for different schemes](image1.png)

**Figure 2.** Arcview interface to the two-dimensional overland flow model.

![Arcview interface](image2.png)

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