Announcements

- I'm back!
- Office Hours
  - 11:30–12:30, Monday and Wednesday
  - Also by appointment
  - EE 324A

Static Single Assignment (SSA)

Use-def chains

- Structure which shows, for each *use* of a variable, which *definitions* could reach it
  - A use may be reached by multiple definitions
- Example:
  - a_5 →
  - b_5 →
  - a_8 →
- Can also build def-use chains

```
1: a = 7;
2: b = 2;
3: if (c)
4:   b = 8;
5: d = a + b;
6: a = 9;
7: while (...) {
8:   d = a + 1;
9:   a = a + 1;
10:}
```

Calculating use-def chains

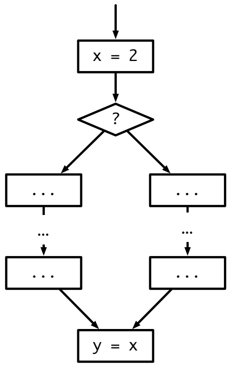
- Easy!
  - Perform a reaching-definitions dataflow analysis
  - At each variable use, look for definitions of that variable that reach the statement
  - Construct use-def chains

Why use-def chains?

- Capture dependence information
  - Use-def chains represent flow of data through program
- Can speed up optimizations
  - Consider constant propagation

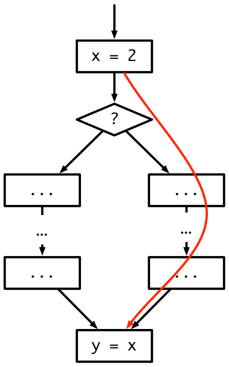
Sparse constant propagation

- Consider what happens when a variable gets updated during constant propagation using worklist algorithm
  - e.g., process x = 2; x moves from ⊥ → 2
- Put all successors of CFG node into worklist
- But what if x isn't used in immediate successor nodes?
  - Spend a lot of time propagating data and processing nodes for no reason
  - Update of x only matters at last node



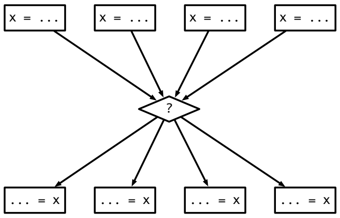
Using use-def chains

- Instead of propagating data along CFG edges, what if we just propagate data along use-def edges?
  - When x is updated, propagate data directly to last node, bypassing all the intermediate nodes!
- Can we run same CP algorithm?
  - Originally initialize with just start node. No uses of definitions → Algorithm terminates early
  - Need to change initialization: Add all statements with constant RHS to initial worklist
- Upshot: original CP algorithm $O(EV^2)$; sparse algorithm $O(N^2V)$
  - N is number of CFG nodes



Problems with u/d chains

- Can be very expensive to represent
  - CFG with N nodes can have N^2 u/d chains
- Each use can have multiple definitions associated with it
  - Can make it difficult to keep u/d information accurate as optimizations are performed and code is transformed
  - Multiple defs can make optimizations harder (will see this when we return to CP)



Solution: SSA

- Static Single Assignment form
- Compact representation of use/def information
- Key feature: No variable is defined more than once (*single* assignment)
  - Eliminates anti/output dependences → more optimizations possible
- SSA enables more efficient versions of optimizations
- Used in many compilers
  - e.g., LLVM

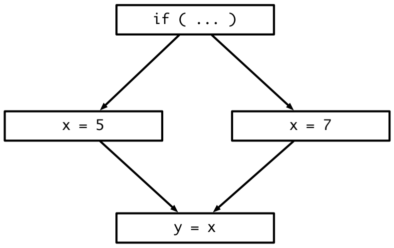
SSA for straight line code

- Each assignment to a variable is given a unique name
- All of the uses reached by that assignment are renamed to match
- Easy for straight line code:

$$\begin{array}{ll} a & = 4; \\ \ldots & = a + 5; \\ a & = 7; \\ \ldots & = a + 6; \end{array} \quad \longrightarrow \quad \begin{array}{ll} a_1 & = 4; \\ \ldots & = a_1 + 5; \\ a_2 & = 7; \\ \ldots & = a_2 + 6; \end{array}$$

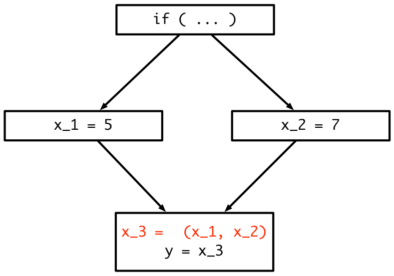
SSA for control flow

- Easy when only one definition reaches a use
- What do we do for code with branches/loops?
  - Multiple definitions reach a single use



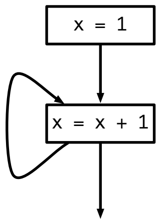
φ functions

- Dummy function that represents merging of two values
  - Part of IR, but not actually emitted as code
- Inserted at merge points to combine two definitions into one



Loops

- How would you put this loop into SSA form?

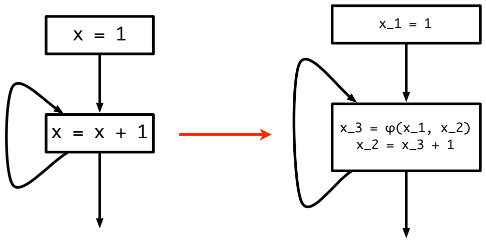


x = 1

x = x + 1

Loops

- How would you put this loop into SSA form?



x = 1

x = x + 1

x_1 = 1

x_3 = φ(x_1, x_2)
x_2 = x_3 + 1

Converting to SSA form

- Two steps to convert a program to SSA form
  - φ function placement
    - Where do we place the φ functions?
  - Variable renaming
    - Rename variable definitions and uses to satisfy single-assignment property

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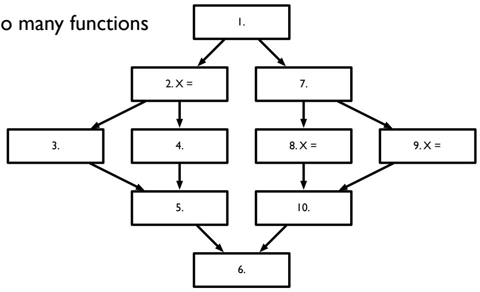
φ function placement

- Need to place φ functions wherever two definitions of a variable might merge
- Safe: place a φ function at every join point in CFG
  - Clearly too many functions



1.

2. X =

7.

3.

4.

8. X =

9. X =

5.

10.

6.

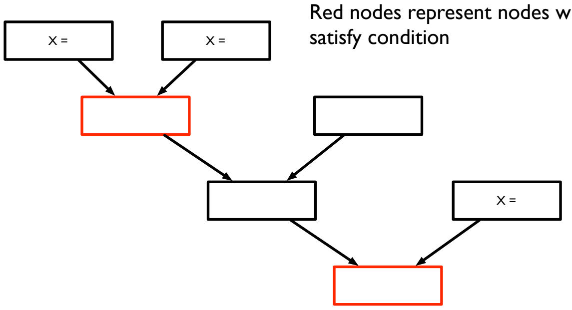
φ function placement

- Condition:
  - If $\exists$ CFG nodes X, Y, Z such that there are paths $X \rightarrow^+ Z$ and $Y \rightarrow^+ Z$ which converge at Z, and X and Y contain assignments to some variable *v* (in the original program), then a φ-node must be inserted in Z (in the new program)
- Options:
  - *minimal*: As few φ-nodes as possible subject to condition
  - *Briggs-minimal*: Do not insert φ-nodes if V is not live across basic blocks
  - *pruned*: Remove "dead" φ-nodes

Minimal placement

- Condition:
  - If $\exists$ CFG nodes X, Y, Z such that there are paths $X \rightarrow^+ Z$ and $Y \rightarrow^+ Z$ which converge at Z, and X and Y contain assignments to some variable *v* (in the original program), then a φ-node must be inserted in Z (in the new program)
- Only want to place φ-nodes wherever the placement condition is true
  - Will be at join points, but not all points
- Want to trace paths from definitions and find *earliest* place those paths merge.

Example



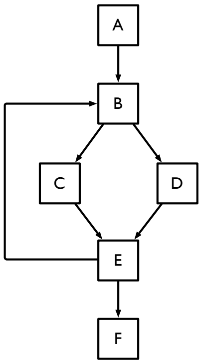
Red nodes represent nodes which satisfy condition

Finding minimal placement

- Could trace every path from assignments to find convergence points
  - This is *expensive*!
- Intuition: what if, for each assignment, we can find the set of nodes which *could* result in a convergence of definitions?
  - Then only need to place φ-nodes there!

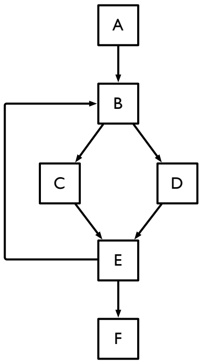
Detour: dominance

- Recall some terms from CFG analysis
- A node X *dominates* a node Y if X appears on all paths from entry to Y
  - $X \in \mathrm{DOM}(Y)$
- A node X *strictly dominates* Y if X DOM Y and $X \neq Y$
  - $X \in \mathrm{DOM!}(Y)$
- A node X is the *immediate dominator* of Y if X is the *closest* dominator of Y
  - $X = \mathrm{IDOM}(Y)$
  - Note: $X = \mathrm{IDOM}(Y) \Rightarrow \forall X' \in \mathrm{DOM}(Y)$, $X' \in \mathrm{DOM}(X)$



Dominance trees

- *Dominance tree* induced by IDOM
  - If $X = \mathrm{IDOM}(Y)$, X is Y's parent in dominance tree



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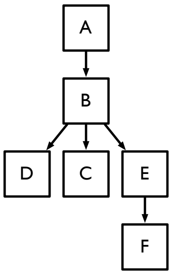
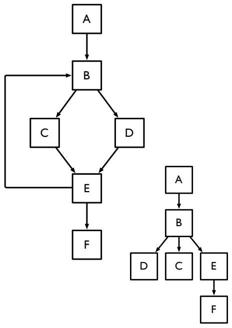


Dominance frontier

- The *dominance frontier* of a node X is the set of nodes DF(X) such that for all $Y \in DF(X)$, X dominates a *predecessor* of Y, but does not *strictly* dominate Y
- What are the dominance frontiers for the nodes in this CFG?



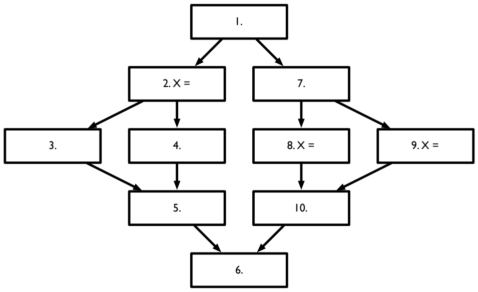
Finding dominance frontiers

- Start by building dominance tree (see algorithm in Cooper *et al.*), then run algorithm:

```
forall v
   if (number of predecessors of v ≥ 2) then
      forall predecessors p of v
         runner = p
         while (runner ≠ IDOM(v))
            add v to DF(runner)
            runner = IDOM(runner)
```

- Intuition:
  - *v* can only be in a DF if it has 2 or more preds
  - Predecessors must have *v* in DF, unless they dominate *v* (by definition).
  - Dominators of predecessors must have *v* in DF, unless they dominate *v*

Example



Iterated dominance frontier

$$DF(\mathcal{L}) = \bigcup_{X\in\mathcal{L}} DF(X)$$

$$\begin{aligned} DF^+(\mathcal{L}) &= \text{limit of sequence} \\ DF_1 &= DF(\mathcal{L}) \\ DF_{i+1} &= DF(\mathcal{L} \cup DF_i) \end{aligned}$$

Theorem:

The set of nodes that need φ-nodes for a variable *v* is the iterated dominance frontier $DF^+(L)$ where L is the set of nodes with assignments to *v*

Inserting φ-nodes

```
foreach variable v
   HasAlready = { }
   EverOnWorklist = { }
   Worklist = { }
   foreach node X containing assignment to v
      EverOnWorklist = EverOnWorklist ∪ {X}
      Worklist = Worklist ∪ {X}
   while Worklist not empty
      remove X from Worklist
      foreach Y ∈ DF(X)
         if Y ∉ HasAlready
            insert φ-node for v at {Y}
            HasAlready = HasAlready ∪ {Y}
            if Y ∉ EverOnWorklist
               Worklist = Worklist ∪ {Y}
               EverOnWorklist = EverOnWorklist ∪ {Y}
```

Converting to SSA form

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Variable renaming

- At this point, φ-nodes are of the form *v* = φ(v, v)
  - Need to rename each variable to satisfy SSA criteria
- High level idea:
  - At every φ-node, rename "target" of φ, then replace all names in the block with new name
  - Change names in successor blocks to match new name, unless successor block has a φ-node
    - In which case, generate new name for target, and continue

Algorithms

Stacks: an array of stacks, one for each variable
Counters: an array of counters, one for each variable

```
Procedure Rename(Block X)
    if X visited, return
    foreach φ-node P in X
        GenName(LHS(P))
    foreach statement A in X
        foreach Variable $v \in$ RHS(A)
            replace v with $v_i$ where i = Top(Stacks[v])
        foreach Variable $v \in$ LHS(A) GenName(v)
    foreach Y $\in$ successors(X)
        foreach φ-node P in Y
            replace operands of P according to vars in X
    foreach Y $\in$ successors(X) Rename(Y)
    foreach φ-node or statement A in X
        foreach $v_i \in$ LHS(A)
            Pop(Stacks[v])
```

```
Procedure GenName(Variable v)
    i = Counters[v]++
    replace v with $v_i$
    Push i onto Stacks[v]
```

Start by calling **Rename**(Entry)

Pruning φ-nodes

- Can eliminate φ-nodes that occur because of variables that are not live across basic blocks
    - These “block local” variables won’t be used later, so do not need to be merged
- Can eliminate φ-nodes that are dead
    - Merged variable isn’t used again

Translating out of SSA form

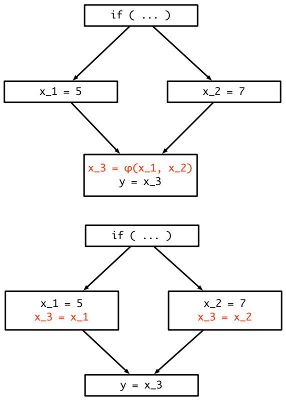
- Cannot just remove φ-nodes and restore variables to original names
- Can mess up optimizations that assume variables use separate storage

```
while (...) do
    read v
    w = v + w
    v = 6
    w = v + w
end
```

```
while (...) do
    $w_3 = \varphi(w_0, w_2)$
    $v_3 = \varphi(v_0, v_2)$
    read $v_1$
    $w_1 = v_1 + w_3$
    $v_2 = 6$
    $w_2 = v_2 + w_1$
```

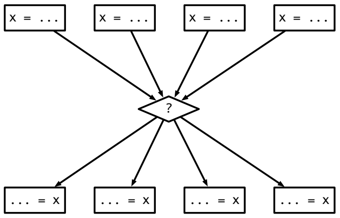
```
$v_2 = 6$
while (...) do
    $w_3 = \varphi(w_0, w_2)$
    $v_3 = \varphi(v_0, v_2)$
    read $v_1$
    $w_1 = v_1 + w_3$
    $w_2 = v_2 + w_1$
```

Translating out of SSA form

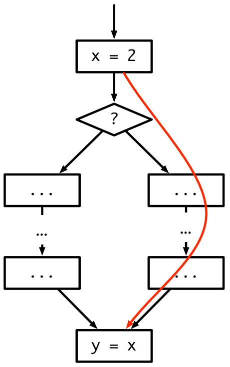


- Eliminate φ-nodes
- Replace with copies in predecessor nodes
- But doesn’t this add a lot of extra copies?
- Solution:
    - Graph coloring with copy/move coalescing!
    - Allows most renamed variables to revert to original name by coalescing with each other
    - If not legal, graph coloring will prevent coalescing

Returning to CP

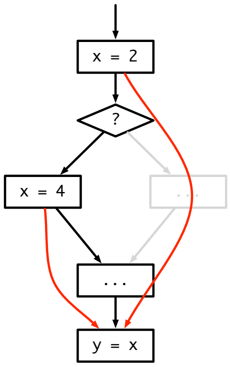


Use-def chains: 16
In SSA form: place φ node in middle



Problems with u/d CP

- What happens if we know which way a branch will resolve?
    - Do not need to propagate information from that branch
    - Easy to do with CFGs
- What does this mean when we’re using u/d chains?
    - Can be very hard to tell which definitions to ignore!



Use/def CP with SSA

- SSA form shortens u/d chains
- Chains terminate at merge points, rather than crossing them
- Can simply ignore information merged from un-taken branches
- Much easier to account for irrelevant information
- Complexity: O(EV)