ECE 573
Problem Set 1: Regular expressions and finite automata

1. Give a regular expression that will accept all valid *names*. A name consists of a first name, an optional middle name or initial, and a last name, separated by spaces. First, middle and last names start with a capital letter and are followed by zero or more lowercase letters. If the name has a middle initial, instead of a middle name, it must be a capital letter followed by a period. Examples of valid names include:

   - Joe Public
   - Joe Q. Public
   - Joe Quincy Public

   Examples of invalid names include:

   - Joe P. (no last name)
   - Joe Q Public (middle initial missing a period)
   - Joe Qu. Public (middle initial more than one letter)
   - Joe Quincy Reginald Public (two middle names)
   - Joe Quincy Public, the Third (more than just three names)

   Assume that Σ (the alphabet) for the strings you are accepting is all capital letters, all lowercase letters, and '.'

   Answer: Assume that the character '_' represents a space. Define the following character class (which captures any single name):

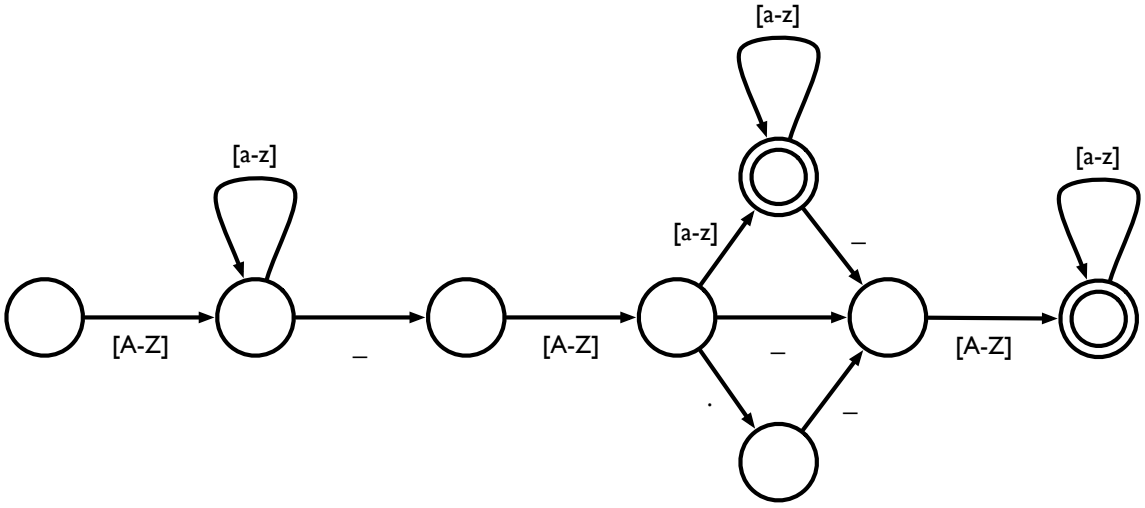
   N = [A-Z][a-z]*

   We can then define the language using the following regular expression:

   N_(N|[A-Z].)?_N

2. Give a DFA for that regular expression.

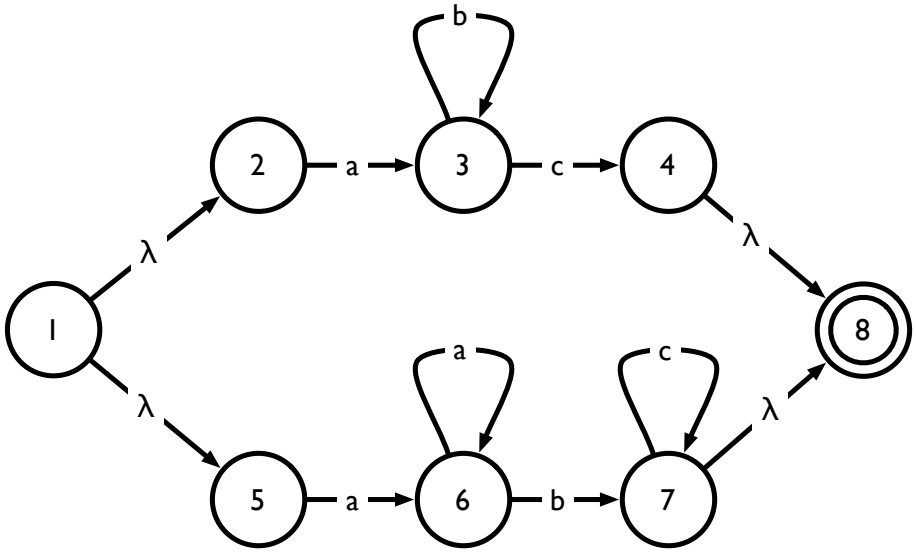
   Answer:



3. Give a non-deterministic FSA for the following regular expression:

$$(ab^*c)|(a^+bc^*)$$

Answer: Here is one possible solution, based on building ab^*c and a^+bc^* first.

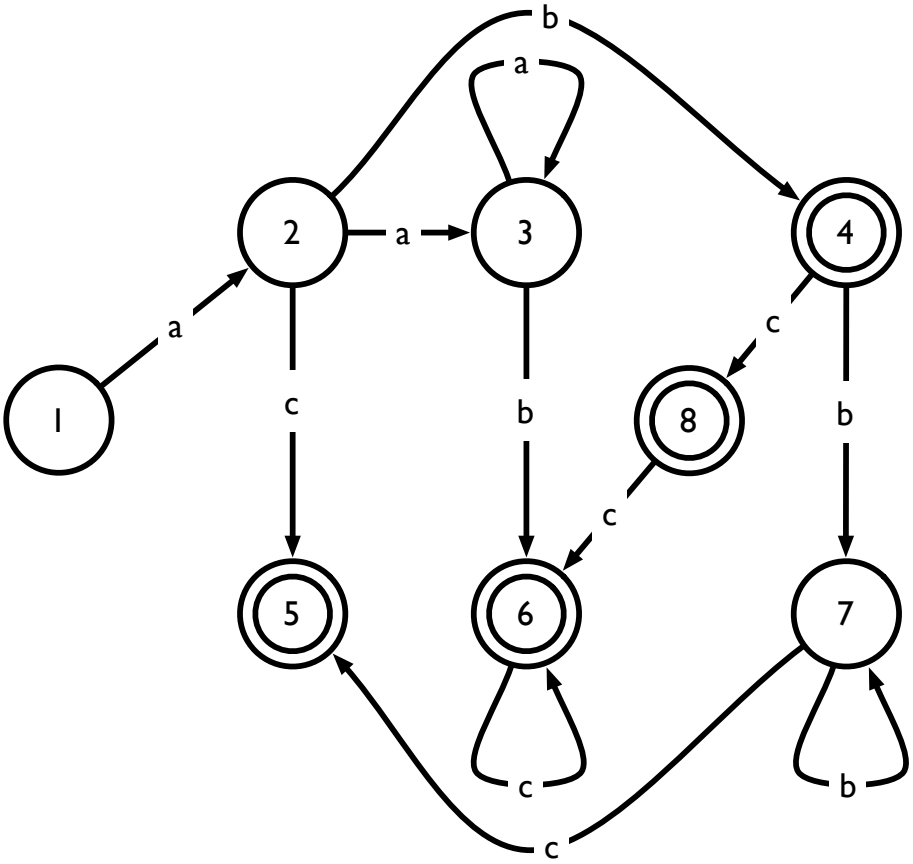


4. Produce the deterministic equivalent of the NFA you built in question 3. Show both the graphical representation and the tabular representation.

Answer: Let us proceed by the subset construction

State(s)	a	b	c	Final?	New state
{1, 2, 5}	{3, 6}	err	err	No	1
{3, 6}	{6}	{3, 7, 8}	{4, 8}	No	2
{6}	{6}	{7, 8}	err	No	3
{3, 7, 8}	err	{3}	{4, 7, 8}	Yes	4
{4, 8}	err	err	err	Yes	5
{7, 8}	err	err	{7, 8}	Yes	6
{3}	err	{3}	{4, 8}	No	7
{4, 7, 8}	err	err	{7, 8}	Yes	8

The graphical representation is given below:



5. Minimize the deterministic FSA.

   The only two states that behave the same way are 6 and 8, so they can be merged.

Note that it doesn't matter that 3 transitions to 6 on b, but not 8. Once you're in 6 or 8, the two states behave the same.

6. Can the language $(^i\ g\ )^i, i \geq 0$ be recognized by an FSA? Why or why not?

   No, it cannot. We need to somehow determine how many '('s are seen before matching the same number of ')'s. This requires a potentially unbounded number of states.

7. Can the language $(^k\ g\ )^k$ for *one particular* k be recognized by an FSA? Why or why not?

   Yes, it can. We can use k states to recognize the '('s, one state to recognize the 'g', and k more states to recognize the ')'s.