ECE 468
Problem Set 1 **Solutions**: Regular expressions and finite automata

1. Write a regular expression that captures the set of strings composed of 'a', 'b', and 'c', where any string uses *at most* two of the three letters (for example,"abbab" is a valid string, or "bccbb", or "ccacaa", but not "abccba"; strings that contain only one of the three letters are also fine).

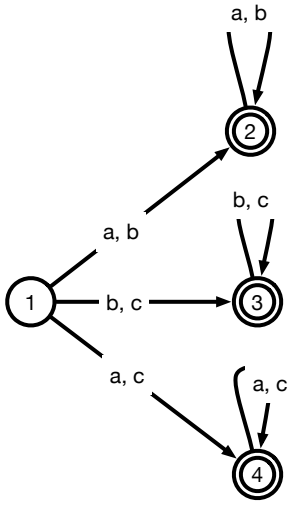
   Answer:

   $(a|b)^+|(a|c)^+|(b|c)^+$

2. Give a *non-deterministic* finite automaton that captures the regular expression from above. Show the automaton in graphical form.

   Answer:

   There are many possible ways to build an NFA for this regular expression. Here is one possibility:



3. Using the construction described in class, give a *deterministic* version of the automaton. You only need to show the transition table.

   Answer:

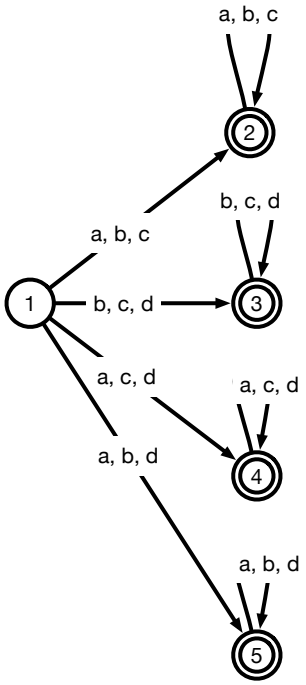
State	a	b	c	Final?
1	2, 4	2, 3	3, 4	No
2, 4	2, 4	2	4	Yes
2, 3	2	2, 3	3	Yes
3, 4	4	3	3, 4	Yes
2	2	2	Error	Yes
4	4	Error	4	Yes
3	Error	3	3	Yes

Note that the DFA for this regular expression required 7 states, while the NFA needed 4.

4. Repeat the previous three steps for strings composed of 'a', 'b', 'c', and 'd', where any string uses at most *three* of the *four* letters.

Answer:

Here's the NFA:



And here's the transition table:

State	a	b	c	d	Final?
1	2, 4, 5	2, 3, 5	2, 3, 4	3, 4, 5	No
2, 4, 5	2, 4, 5	2, 5	2, 4	4, 5	Yes
2, 3, 5	2, 5	2, 3, 5	2, 3	3, 5	Yes
2, 3, 4	2, 4	2, 3	2, 3, 4	3, 4	Yes
3, 4, 5	4, 5	3, 5	3, 4	3, 4, 5	Yes
2, 5	2, 5	2, 5	2	5	Yes
2, 4	2, 4	2	2, 4	4	Yes
4, 5	4, 5	5	4	4, 5	Yes
2, 3	2	2, 3	2, 3	3	Yes
3, 5	5	3, 5	3	3, 5	Yes
3, 4	4	3	3, 4	3, 4	Yes
2	2	2	2	Error	Yes
5	5	5	Error	5	Yes
4	4	Error	4	4	Yes
3	Error	3	3	3	Yes

Note that this DFA has 15 states while the NFA has 5 states. In general, for regular expressions of this type, which start with N possible letters, the NFA will have $N+1$ states while the DFA will have $2^N - 1$ states.