# Dataflow Analysis

# Program optimizations

- So far we have talked about different kinds of optimizations
  - Peephole optimizations
  - Local common sub-expression elimination
  - Loop optimizations
- What about *global optimizations*
  - Optimizations across multiple basic blocks (usually a whole procedure)
    - Not just a single loop

# Useful optimizations

- Common subexpression elimination (global)
    - Need to know which expressions are available at a point
- Dead code elimination
    - Need to know if the effects of a piece of code are never needed, or if code cannot be reached
- Constant folding
    - Need to know if variable has a constant value
- So how do we get this information?

# Dataflow analysis

- Framework for doing compiler analyses to drive optimization
- Works across basic blocks
- Examples
  - Constant propagation: determine which variables are constant
  - Liveness analysis: determine which variables are live
  - Available expressions: determine which expressions are have valid computed values
  - Reaching definitions: determine which definitions could "reach" a use

# Example: constant propagation

- Goal: determine when variables take on constant values
- Why? Can enable many optimizations
  - Constant folding

```
x = 1;
y = x + 2;
if (x > z) then y = 5
... y ...
```

  - Create dead code

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# Example: constant propagation

- Goal: determine when variables take on constant values
- Why? Can enable many optimizations
  - Constant folding

```
x = 1;                              x = 1;
y = x + 2;                  →       y = 3;
if (x > z) then y = 5               if (x > z) then y = 5
... y ...                           ... y ...
```

  - Create dead code

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x = 1;
y = x + 2;                  →
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# Example: constant propagation

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- Why? Can enable many optimizations
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if (x > z) then y = 5                     if (x > z) then y = 5
... y ...                                 ... y ...
```

  - Create dead code

```
x = 1;                                    x = 1;
y = x + 2;                  ——→           y = 3; //dead code
if (y > x) then y = 5                     if (true) then y = 5 //simplify!
... y ...                                 ... y ...
```

# How can we find constants?

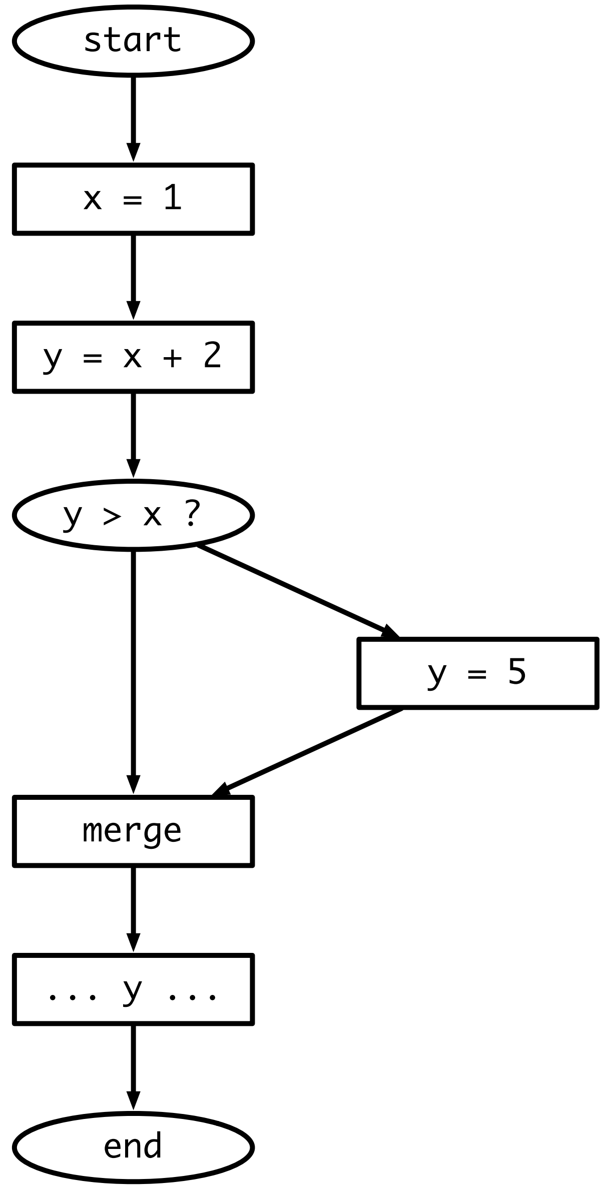
- Ideal: run program and see which variables are constant
  - Problem: variables can be constant with some inputs, not others – need an approach that works for all inputs!
  - Problem: program can run forever (infinite loops?) – need an approach that we know will finish
- Idea: run program *symbolically*
  - Essentially, keep track of whether a variable is constant or not constant (but nothing else)

# Overview of algorithm

- Build control flow graph
  - We'll use statement-level CFG (with merge nodes) for this
- Perform symbolic evaluation
  - Keep track of whether variables are constant or not
- Replace constant-valued variable uses with their values, try to simplify expressions and control flow

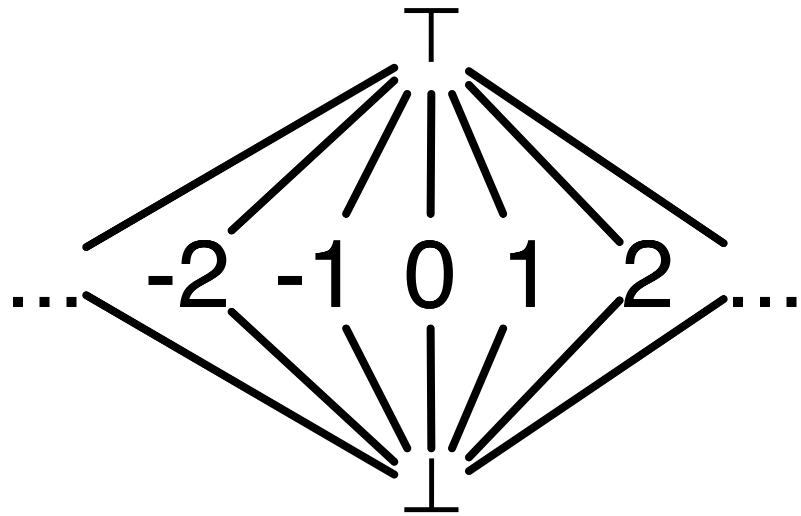
# Build CFG

```
x = 1;
y = x + 2;
if (y > x) then y = 5;
... y ...
```



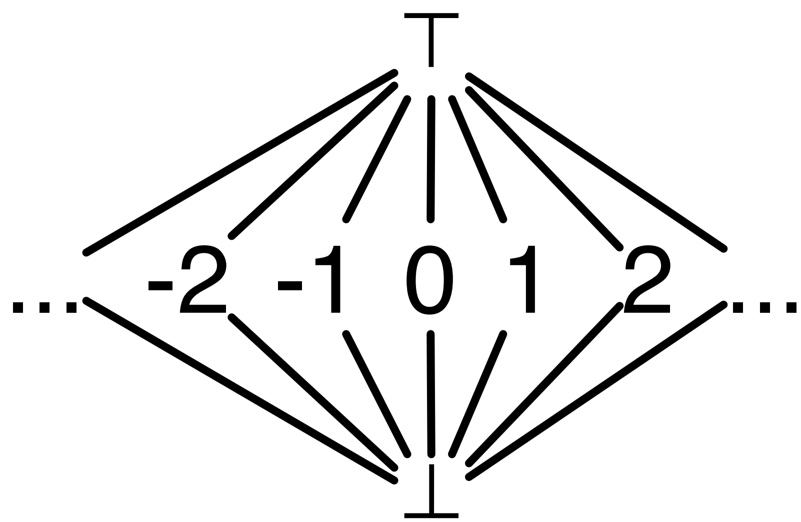
# Symbolic evaluation

- Idea: replace each value with a symbol
  - constant (specify which), no information, definitely not constant
- Can organize these possible values in a *lattice*
  - Set of possible values, arranged from least information to most information



# Symbolic evaluation

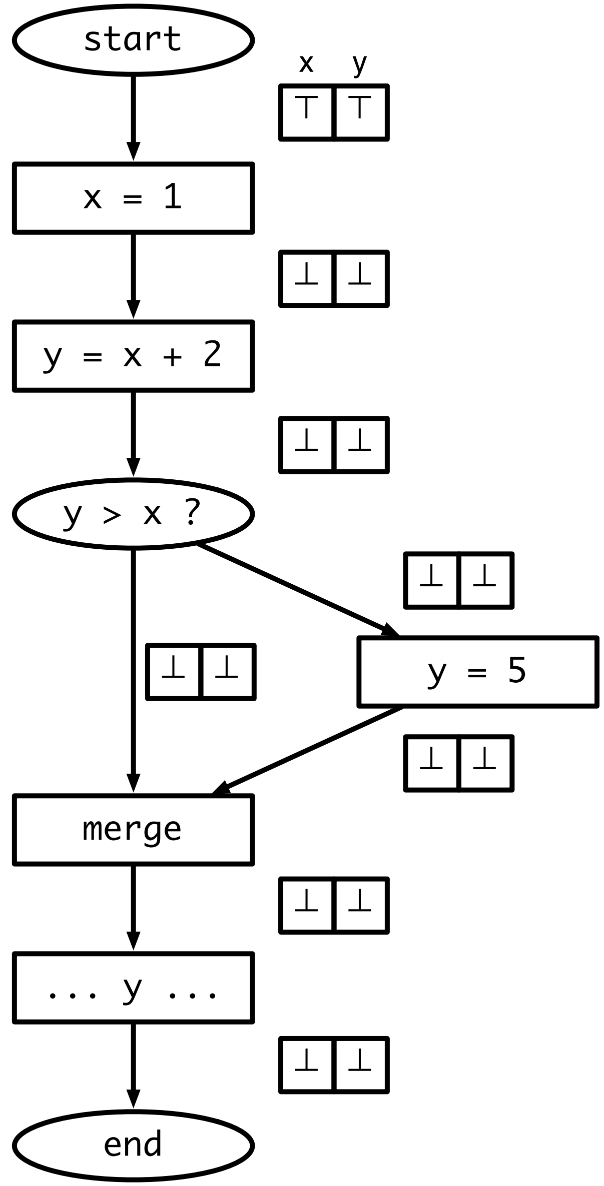
- Evaluate expressions symbolically: eval(e, $V_{in}$)
  - If e evaluates to a constant, return that value. If any input is ⊤ (or ⊥), return ⊤ (or ⊥)
    - Why?
- Two special operations on lattice
  - meet(a, b) – highest value less than or equal to both a and b
  - join(a, b) – lowest value greater than or equal to both a and b




Join often written as a ⊔ b
Meet often written as a ⊓ b

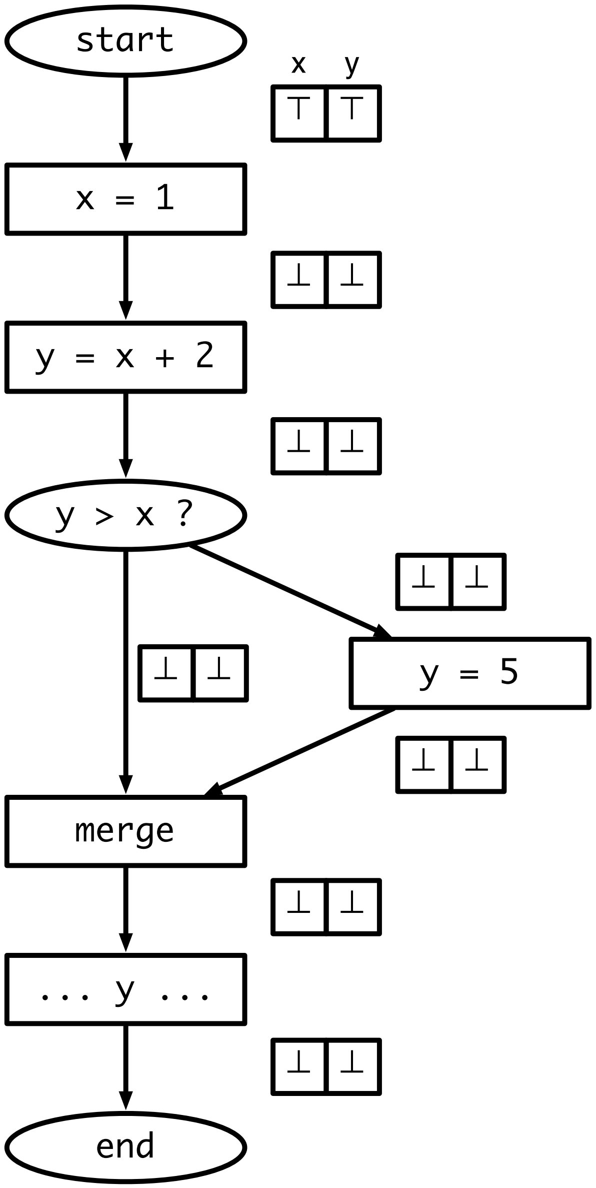
# Putting it together

- Keep track of the symbolic value of a variable at every program point (on every CFG edge)
  - State vector
- What should our initial value be?
  - Starting state vector is all ⊤
    - Can't make any assumptions about inputs – must assume not constant
  - Everything else starts as ⊥, since we have no information about the variable at that point



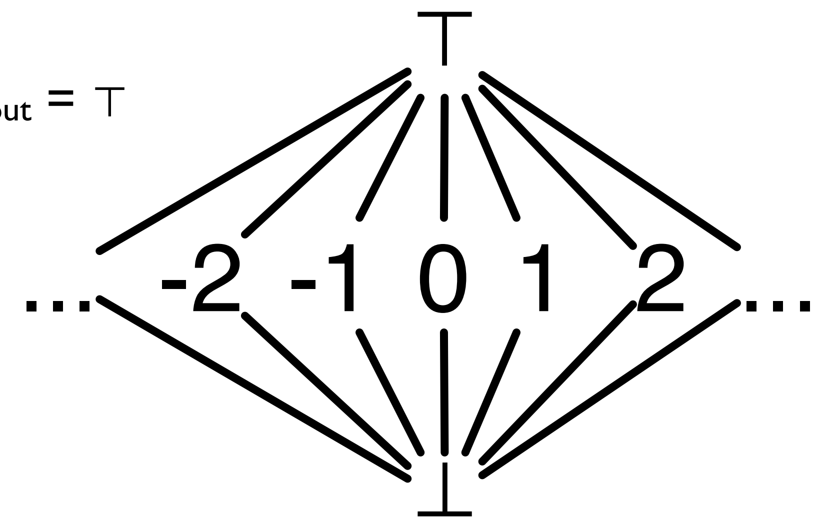
# Executing symbolically

- For each statement `t = e` evaluate `e` using $V_{in}$, update value for `t` and propagate state vector to next statement
- What about switches?
    - If `e` is true or false, propagate $V_{in}$ to appropriate branch
    - What if we can't tell?
        - Propagate $V_{in}$ to both branches, and symbolically execute both sides
- What do we do at merges?



# Handling merges

- Have two different $V_{in}$s coming from two different paths
- Goal: want new value for $V_{in}$ to be *safe* (shouldn't generate wrong information), and we don't know which path we actually took
- Consider a single variable. Several situations:
  - $V_1 = \bot, V_2 = * \rightarrow V_{out} = *$
  - $V_1$ = constant x, $V_2$ = x $\rightarrow V_{out}$ = x
  - $V_1$ = constant x, $V_2$ = constant y $\rightarrow V_{out} = \top$
  - $V_1 = \top, V_2 = * \rightarrow V_{out} = \top$
- Generalization:
  - $V_{out} = V_1 \sqcup V_2$



# Result: worklist algorithm

- Associate state vector with each edge of CFG, initialize all values to ⊥, worklist has just start edge
  - While worklist not empty, do:

```
Process the next edge from worklist

Symbolically evaluate target node of edge using input state vector

If target node is assignment (x = e), propagate Vin[eval(e)/x] to
output edge

If target node is branch (e?)

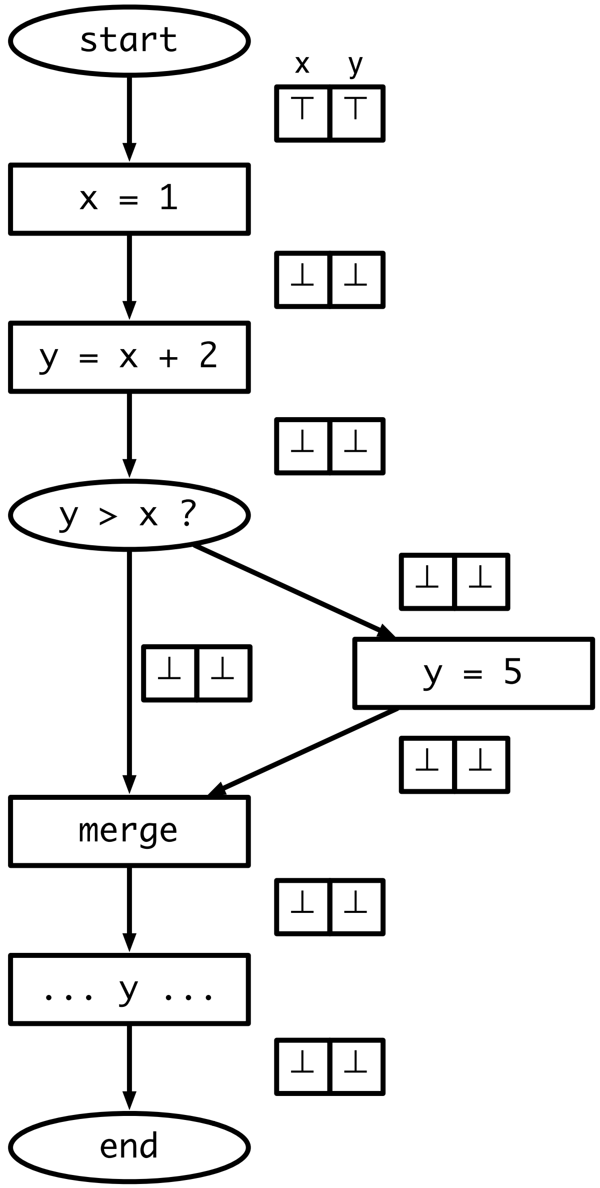
   If eval(e) is true or false, propagate Vin to appropriate
   output edge

   Else, propagate Vin along both output edges

If target node is merge, propagate join(all Vin) to output edge

If any output edge state vector has changed, add it to worklist
```

# Running example



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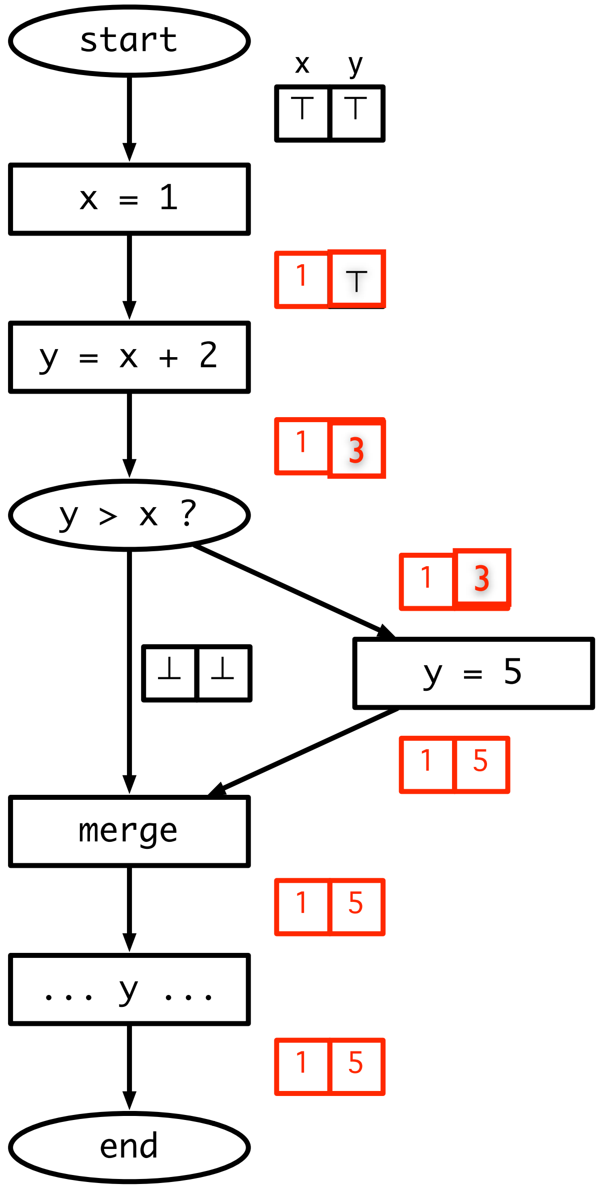
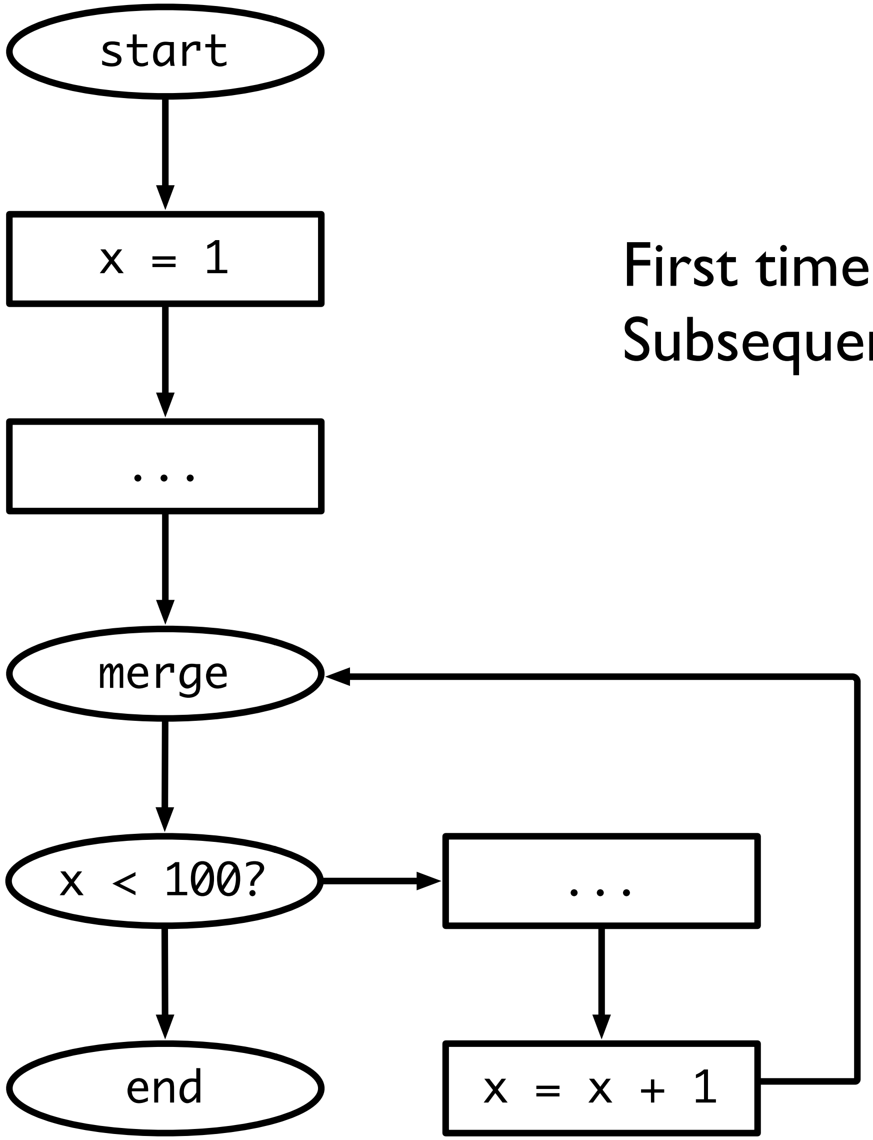


# What do we do about loops?

- Unless a loop never executes, symbolic execution looks like it will keep going around to the same nodes over and over again
- Insight: if the input state vector(s) for a node don't change, then its output doesn't change
    - If input stops changing, then we are done!
- Claim: input will eventually stop changing. Why?

# Loop example




First time through loop, x = 1
Subsequent times, x = $\top$

# Complexity of algorithm

- V = # of variables, E = # of edges
- Height of lattice = 2 → each state vector can be updated at most 2 * V times.
- So each edge is processed at most 2 * V times, so we process at most 2 * E * V elements in the worklist.
- Cost to process a node: O(V)
- Overall, algorithm takes $O(EV^2)$ time

# Question

- Can we generalize this algorithm and use it for more analyses?

# Constant propagation

- Step 1: choose lattice (which values are you going to track during symbolic execution)?
  - Use constant lattice
- Step 2: choose direction of dataflow (if executing symbolically, can run program backwards!)
  - Run forward through program
- Step 3: create *transfer functions*
  - How does executing a statement change the symbolic state?
- Step 4: choose *confluence operator*
  - What do do at merges? For constant propagation, use join