Dataflow Analysis

Program optimizations

- So far we have talked about different kinds of optimizations
  - Peephole optimizations
  - Local common sub-expression elimination
  - Loop optimizations
- What about *global optimizations*
  - Optimizations across multiple basic blocks (usually a whole procedure)
    - Not just a single loop

Useful optimizations

- Common subexpression elimination (global)
  - Need to know which expressions are available at a point
- Dead code elimination
  - Need to know if the effects of a piece of code are never needed, or if code cannot be reached
- Constant folding
  - Need to know if variable has a constant value
- Loop invariant code motion
  - Need to know where and when variables are live
- So how do we get this information?

Dataflow analysis

- Framework for doing compiler analyses to drive optimization
- Works across basic blocks
- Examples
  - Constant propagation: determine which variables are constant
  - Liveness analysis: determine which variables are live
  - Available expressions: determine which expressions are have valid computed values
  - Reaching definitions: determine which definitions could "reach" a use

Example: constant propagation

- Goal: determine when variables take on constant values
- Why? Can enable many optimizations
  - Constant folding

```
x = 1;
y = x + 2;
if (x > z) then y = 5
... y ...
```

  - Create dead code

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x = 1;
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x = 1;
y = x + 2;
if (x > z) then y = 5
... y ...
```

```
x = 1;
y = 3;
if (x > z) then y = 5
... y ...
```

  - Create dead code

```
x = 1;
y = x + 2;
if (y > x) then y = 5
... y ...
```

Example: constant propagation

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- Why? Can enable many optimizations
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x = 1;                          x = 1;
y = x + 2;                      y = 3;
if (x > z) then y = 5   ---->   if (x > z) then y = 5
... y ...                       ... y ...
```

  - Create dead code

```
x = 1;                          x = 1;
y = x + 2;                      y = 3; //dead code
if (y > x) then y = 5   ---->   if (true) then y = 5 //simplify!
... y ...                       ... y ...
```


How can we find constants?

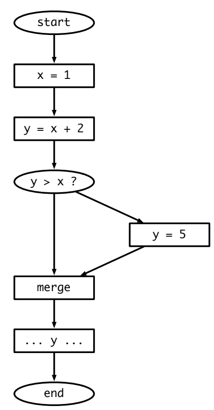
- Ideal: run program and see which variables are constant
  - Problem: variables can be constant with some inputs, not others – need an approach that works for all inputs!
  - Problem: program can run forever (infinite loops?) – need an approach that we know will finish
- Idea: run program *symbolically*
  - Essentially, keep track of whether a variable is constant or not constant (but nothing else)

Overview of algorithm

- Build control flow graph
  - We'll use statement-level CFG (with merge nodes) for this
- Perform symbolic evaluation
  - Keep track of whether variables are constant or not
- Replace constant-valued variable uses with their values, try to simplify expressions and control flow

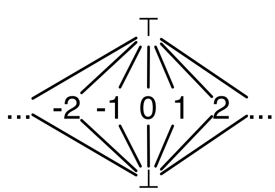
Build CFG

```
x = 1;
y = x + 2;
if (y > x) then y = 5;
... y ...
```



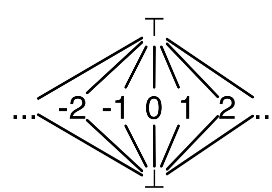
Symbolic evaluation

- Idea: replace each value with a symbol
  - constant (specify which), no information, definitely not constant
- Can organize these possible values in a *lattice*
  - Set of possible values, arranged from least information to most information



Symbolic evaluation

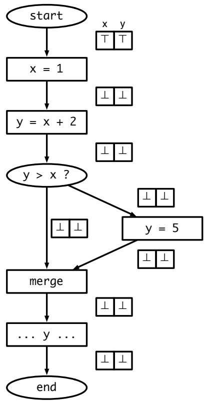
- Evaluate expressions symbolically: eval(e, V_{in})
  - If e evaluates to a constant, return that value. If any input is $\top$ (or $\bot$), return $\top$ (or $\bot$)
    - Why?
- Two special operations on lattice
  - meet(a, b) – highest value less than or equal to both a and b
  - join(a, b) – lowest value greater than or equal to both a and b



Join often written as a $\sqcup$ b
Meet often written as a $\sqcap$ b

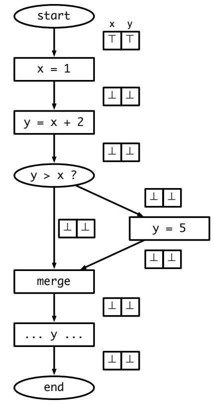
Putting it together

- Keep track of the symbolic value of a variable at every program point (on every CFG edge)
  - State vector
- What should our initial value be?
  - Starting state vector is all $\top$
    - Can't make any assumptions about inputs – must assume not constant
  - Everything else starts as $\bot$, since we have no information about the variable at that point



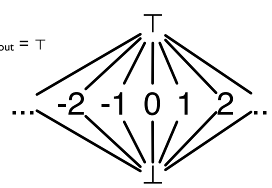
Executing symbolically

- For each statement t = e evaluate e using V_{in}, update value for t and propagate state vector to next statement
- What about switches?
  - If e is true or false, propagate V_{in} to appropriate branch
  - What if we can't tell?
    - Propagate V_{in} to both branches, and symbolically execute both sides
- What do we do at merges?



Handling merges

- Have two different V_{in}s coming from two different paths
- Goal: want new value for V_{in} to be *safe* (shouldn't generate wrong information), and we don't know which path we actually took
- Consider a single variable. Several situations:
  - $V_1 = \bot, V_2 = * \rightarrow V_{out} = *$
  - V_1 = constant x, V_2 = x $\rightarrow V_{out}$ = x
  - V_1 = constant x, V_2 = constant y $\rightarrow V_{out} = \top$
  - $V_1 = \top, V_2 = * \rightarrow V_{out} = \top$
- Generalization:
  - $V_{out} = V_1 \sqcup V_2$



Result: worklist algorithm

- Associate state vector with each edge of CFG, initialize all values to $\bot$, worklist has just start edge
  - While worklist not empty, do:

    Process the next edge from worklist

    Symbolically evaluate target node of edge using input state vector

    If target node is assignment (x = e), propagate V_{in}[eval(e)/x] to output edge

    If target node is branch (e?)

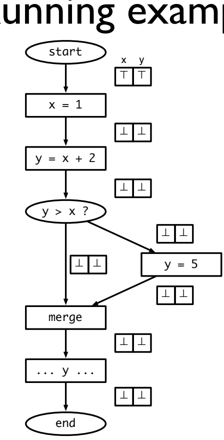
      If eval(e) is true or false, propagate V_{in} to appropriate output edge

      Else, propagate V_{in} along both output edges

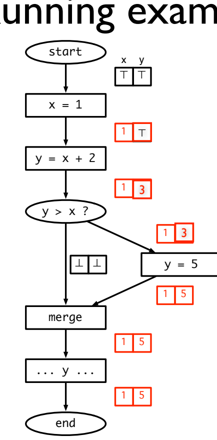
    If target node is merge, propagate join(all V_{in}) to output edge

    If any output edge state vector has changed, add it to worklist

Running example



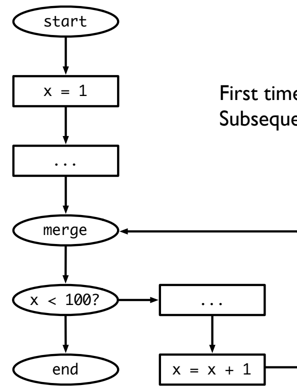
Running example



What do we do about loops?

- Unless a loop never executes, symbolic execution looks like it will keep going around to the same nodes over and over again
- Insight: if the input state vector(s) for a node don't change, then its output doesn't change
  - If input stops changing, then we are done!
- Claim: input will eventually stop changing. Why?

Loop example



First time through loop, x = 1
Subsequent times, x = $\top$

Complexity of algorithm

- V = # of variables, E = # of edges
- Height of lattice = 2 → each state vector can be updated at most 2 *V times.
- So each edge is processed at most 2 *V times, so we process at most 2 * E *V elements in the worklist.
- Cost to process a node: O(V)
- Overall, algorithm takes $O(EV^2)$ time

Question

- Can we generalize this algorithm and use it for more analyses?

Constant propagation

- Step 1: choose lattice (which values are you going to track during symbolic execution)?
  - Use constant lattice
- Step 2: choose direction of dataflow (if executing symbolically, can run program backwards!)
  - Run forward through program
- Step 3: create *transfer functions*
  - How does executing a statement change the symbolic state?
- Step 4: choose *confluence operator*
  - What do do at merges? For constant propagation, use join