Parsers

Agenda

- How to define a language
  - Context-free grammars
- How to recognize a language
  - LL(1) Parsers
  - LR Parsing

Terminology

- Grammar $G = (V_t, V_n, S, P)$
  - V_t is the set of *terminals*
  - V_n is the set of *non-terminals*
  - S is the *start symbol*
  - P is the set of *productions*
    - Each production takes the form: $V_n \rightarrow \lambda \mid (V_n \mid V_t)+$
    - Grammar is *context-free* (why?)
- A simple grammar:

  $G = (\{a, b\}, \{S, A, B\}, \{S \rightarrow A\ B\ \$, A \rightarrow A\ a, A \rightarrow a, B \rightarrow B\ b, B \rightarrow b\}, S)$

Terminology

- V is the *vocabulary* of a grammar, consisting of terminal (V_t) and non-terminal (V_n) symbols
- For our sample grammar
  - $V_n = \{S, A, B\}$
    - Non-terminals are symbols on the LHS of a production
    - Non-terminals are constructs in the language that are recognized during parsing
  - $V_t = \{a, b\}$
    - Terminals are the tokens recognized by the scanner
    - They correspond to symbols in the text of the program

Terminology

- Productions (rewrite rules) tell us how to derive strings in the language
  - Apply productions to rewrite strings into other strings
- We will use the standard BNF form
- P = {

  $S \rightarrow A\ B$

  $A \rightarrow A\ a$

  $A \rightarrow a$

  $B \rightarrow B\ b$

  $B \rightarrow b$

  }

Generating strings

$S \rightarrow A\ B$

$A \rightarrow A\ a$

$A \rightarrow a$

$B \rightarrow B\ b$

$B \rightarrow b$

- Given a start rule, productions tell us how to rewrite a non-terminal into a different set of symbols
- By convention, first production applied has the start symbol on the left, and there is only one such production

To derive the string "a a b b b" we can do the following rewrites:

$S \Rightarrow A\ B \Rightarrow A\ a\ B \Rightarrow a\ a\ B \Rightarrow a\ a\ B\ b \Rightarrow$

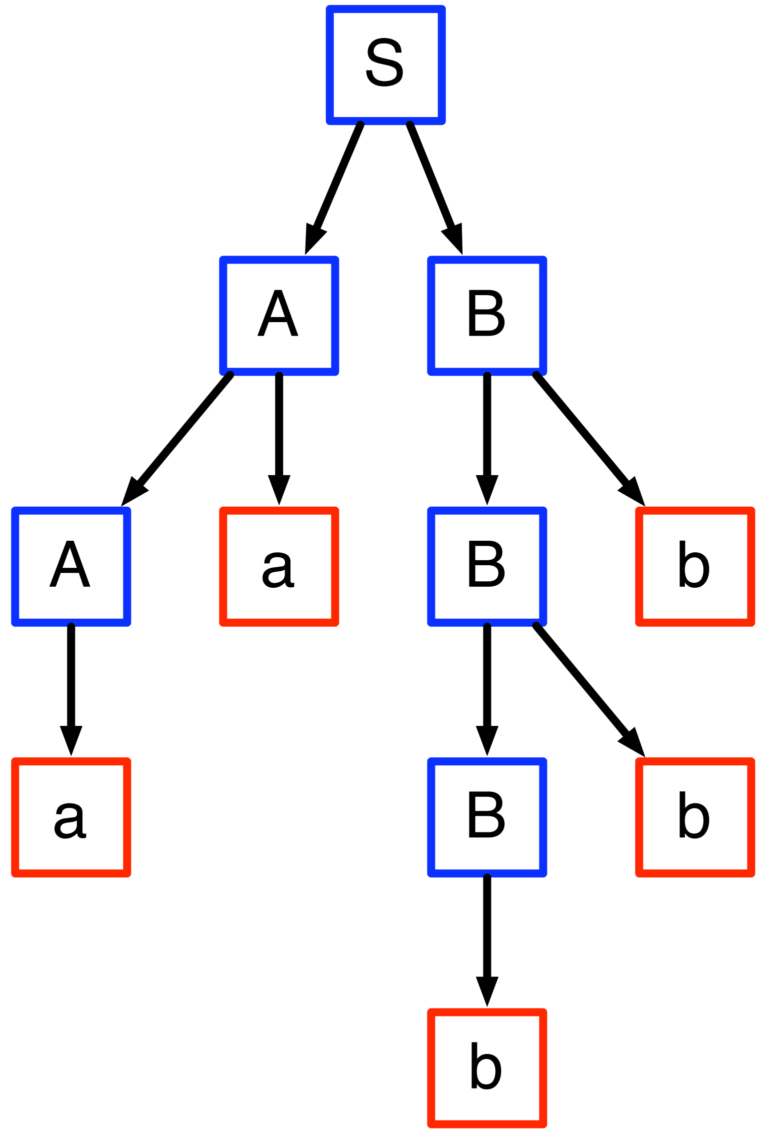
$a\ a\ B\ b\ b \Rightarrow a\ a\ b\ b\ b$

Terminology

- *Strings* are composed of symbols
  - A A a a B b b A a is a string
  - We will use Greek letters to represent strings composed of both terminals and non-terminals
- *L(G)* is the language produced by the grammar *G*
  - All strings consisting of only terminals that can be produced by G
  - In our example, L(G) = a+b+
  - All regular expressions can be expressed as grammars for context-free languages, but not vice-versa
    - Consider: $a^i b^i$ (what is the grammar for this?)

Parse trees

- Tree which shows how a string was produced by a language
  - Interior nodes of tree: non-terminals
    - Children: the terminals and non-terminals generated by applying a production rule
  - Leaf nodes: terminals



Leftmost derivation

- Rewriting of a given string starts with the leftmost symbol
- Exercise: do a leftmost derivation of the input program

  F(V + V)

  using the following grammar:

E	→	Prefix (E)
E	→	V Tail
Prefix	→	F
Prefix	→	λ
Tail	→	+ E
Tail	→	λ

- What does the parse tree look like?

Rightmost derivation

- Rewrite using the rightmost non-terminal, instead of the left
- What is the rightmost derivation of this string?

F(V + V)

E	→	Prefix (E)
E	→	V Tail
Prefix	→	F
Prefix	→	λ
Tail	→	+ E
Tail	→	λ

Simple conversions

$A \rightarrow B \mid C$ $\longrightarrow$ $A \rightarrow B$, $A \rightarrow C$

$D \rightarrow E\ \{F\}$ $\longrightarrow$ $D \rightarrow E\ Ftail$, $Ftail \rightarrow F\ Ftail$, $Ftail \rightarrow \lambda$

Top-down vs. Bottom-up parsers

- Top-down parsers expand the parse tree in *pre-order*
  - Identify parent nodes before the children
- Bottom-up parsers expand the parse tree in *post-order*
  - Identify children before the parents
- Notation:
  - LL(1): Top-down derivation with 1 symbol lookahead
  - LL(k): Top-down derivation with k symbols lookahead
  - LR(1): Bottom-up derivation with 1 symbol lookahead

What is parsing

- Parsing is recognizing members in a language specified/defined/generated by a grammar
- When a construct (corresponding to a production in a grammar) is recognized, a typical parser will take some action
  - In a compiler, this action generates an intermediate representation of the program construct
  - In an interpreter, this action might be to perform the action specified by the construct. Thus, if *a+b* is recognized, the value of *a* and *b* would be added and placed in a temporary variable

Top-down parsing

Top-down parsing

- Idea: we know sentence has to start with initial symbol
- Build up partial derivations by *predicting* what rules are used to expand non-terminals
  - Often called *predictive parsers*
- If partial derivation has terminal characters, *match* them from the input stream

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

A simple example

$S \rightarrow A\ B\ c\ \$$

special "end of input" symbol

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  $x\ a\ c\ c\ \$$

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: S

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: A B c $

Predict rule

A simple example

$S \rightarrow A\ B\ c\ \$$

Choose based on *first set* of rules

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a A B c $

Predict rule *based on next token*

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a A B c $

Match token

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a A B c $

Match token

A simple example

$S \rightarrow A\ B\ c\ \$$

Choose based on *first set* of rules

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a c B c $

Predict rule *based on next token*

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a c B c $

Match token

A simple example

$\boxed{S \rightarrow A\ B\ c\ \$}$

$A \rightarrow x\ a\ A$

Choose based on *follow set*

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$\boxed{B \rightarrow b}$

$\boxed{B \rightarrow \lambda}$

- A sentence in the grammar:

  x a c c $

Current derivation: x a c λ c $

Predict rule ***based on next token***

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a c c $

Match token

A simple example

$S \rightarrow A\ B\ c\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow c$

$B \rightarrow b$

$B \rightarrow \lambda$

- A sentence in the grammar:

  x a c c $

Current derivation: x a c c $

Match token

First and follow sets

- First(α): the set of terminals (and/or λ) that begin all strings that can be derived from α
  - First(A) = {x, y, λ}
  - First(xaA) = {x}
  - First (AB) = {x, y, b}
- Follow(A): the set of terminals (and/or $, but no λs) that can appear immediately after A in some partial derivation
  - Follow(A) = {b}

$S \rightarrow A\ B\ \$$

$A \rightarrow x\ a\ A$

$A \rightarrow y\ a\ A$

$A \rightarrow \lambda$

$B \rightarrow b$

First and follow sets

- $\text{First}(\alpha) = \{a \in V_t \mid \alpha \Rightarrow^* a\beta\} \cup \{\lambda \mid \text{if } \alpha \Rightarrow^* \lambda\}$
- $\text{Follow}(A) = \{a \in V_t \mid S \Rightarrow^+ \ldots Aa \ldots\} \cup \{\$ \mid \text{if } S \Rightarrow^+ \ldots A\ \$\}$

S:	start symbol
a:	a terminal symbol
A:	a non-terminal symbol
α,β:	a string composed of terminals and non-terminals (typically, α is the RHS of a production

$\Rightarrow$:	derived in 1 step
$\Rightarrow^*$:	derived in 0 or more steps
$\Rightarrow^+$:	derived in 1 or more steps

Computing first sets

- Terminal: $\text{First}(a) = \{a\}$
- Non-terminal: First(A)
    - Look at all productions for A

      $A \rightarrow X_1 X_2 \ldots X_k$
    - $\text{First}(A) \supseteq (\text{First}(X_1) - \lambda)$
    - If $\lambda \in \text{First}(X_1)$, $\text{First}(A) \supseteq (\text{First}(X_2) - \lambda)$
    - If λ is in $\text{First}(X_i)$ for all i, then $\lambda \in \text{First}(A)$
- Computing $\text{First}(\alpha)$: similar procedure to computing First(A)

Exercise

- What are the first sets for all the non-terminals in following grammar:

$$S \rightarrow A\ B\ \$$$

$$A \rightarrow x\ a\ A$$

$$A \rightarrow y\ a\ A$$

$$A \rightarrow \lambda$$

$$B \rightarrow b$$

$$B \rightarrow A$$

Computing follow sets

- Follow(S) = {}
- To compute Follow(A):
  - Find productions which have A on rhs. Three rules:
    1. $X \rightarrow \alpha A \beta$: Follow(A) $\supseteq$ (First(β) - λ)
    2. $X \rightarrow \alpha A \beta$: If $\lambda \in$ First(β), Follow(A) $\supseteq$ Follow(X)
    3. $X \rightarrow \alpha A$: Follow(A) $\supseteq$ Follow(X)
- Note: Follow(X) never has λ in it.

Exercise

- What are the follow sets for

$$S \rightarrow A\ B\ \$$$

$$A \rightarrow x\ a\ A$$

$$A \rightarrow y\ a\ A$$

$$A \rightarrow \lambda$$

$$B \rightarrow b$$

$$B \rightarrow A$$

Towards parser generators

- Key problem: as we read the source program, we need to decide what productions to use
- Step 1: find the tokens that can tell which production P (of the form $A \rightarrow X_1 X_2 \dots X_m$) applies

$$\mathrm{Predict}(P) =$$

$$\begin{cases} \mathrm{First}(X_1 \dots X_m) & \text{if } \lambda \notin \mathrm{First}(X_1 \dots X_m) \\ (\mathrm{First}(X_1 \dots X_m) - \lambda) \cup \mathrm{Follow}(A) & \text{otherwise} \end{cases}$$

- If next token is in Predict(P), then we should choose this production

Parse tables

- Step 2: build a parse table
  - Given some non-terminal V_n (the non-terminal we are currently processing) and a terminal V_t (the lookahead symbol), the parse table tells us which production P to use (or that we have an error
  - More formally:

    $T: V_n \times V_t \rightarrow P \cup \{Error\}$

Building the parse table

- Start: T[A][t] = //initialize all fields to "error"

  foreach A:

  foreach P with A on its lhs:

  foreach t in Predict(P):

  T[A][t] = P

- Exercise: build parse table for our toy grammar

1. $S \rightarrow A\ B\ \$$
2. $A \rightarrow x\ a\ A$
3. $A \rightarrow y\ a\ A$
4. $A \rightarrow \lambda$
5. $B \rightarrow b$

Stack-based parser for LL(1)

- Given the parse table, a stack-based algorithm is much simpler to generate than a recursive descent parser
- Basic algorithm:

1. Push the RHS of a production onto the stack
2. Pop a symbol, if it is a terminal, match it
3. If it is a non-terminal, take its production according to the parse table and go to 1

- Note: always start with start state

An example

1. $S \rightarrow A\ B\ \$$
2. $A \rightarrow x\ a\ A$
3. $A \rightarrow y\ a\ A$
4. $A \rightarrow \lambda$
5. $B \rightarrow b$

- How would a stack-based parser parse:

x a y a b

Parse stack	Remaining input	Parser action
S	x a y a b $	predict 1

An example

1. $S \rightarrow A\ B\ \$$
2. $A \rightarrow x\ a\ A$
3. $A \rightarrow y\ a\ A$
4. $A \rightarrow \lambda$
5. $B \rightarrow b$

- How would a stack-based parser parse:

  x a y a b

Parse stack	Remaining input	Parser action
S	x a y a b $	predict 1
A B $	x a y a b $	predict 2

An example

- How would a stack-based parser parse:

  x a y a b

1. S → A B $
2. A → x a A
3. A → y a A
4. A → λ
5. B → b

Parse stack	Remaining input	Parser action
S	x a y a b $	predict 1
A B $	x a y a b $	predict 2
x a A B $	x a y a b $	match(x)

An example

1. S → A B $
2. A → x a A
3. A → y a A
4. A → λ
5. B → b

- How would a stack-based parser parse:

  x a y a b

Parse stack	Remaining input	Parser action
S	x a y a b $	predict 1
A B $	x a y a b $	predict 2
x a A B $	x a y a b $	match(x)
a A B $	a y a b $	match(a)

An example

1. S → A B $
2. A → x a A
3. A → y a A
4. A → λ
5. B → b

- How would a stack-based parser parse:

  x a y a b

Parse stack	Remaining input	Parser action
S	x a y a b $	predict 1
A B $	x a y a b $	predict 2
x a A B $	x a y a b $	match(x)
a A B $	a y a b $	match(a)
A B $	y a b $	predict 3

An example

1. $S \rightarrow A\ B\ \$$
2. $A \rightarrow x\ a\ A$
3. $A \rightarrow y\ a\ A$
4. $A \rightarrow \lambda$
5. $B \rightarrow b$

- How would a stack-based parser parse:

  x a y a b

Parse stack	Remaining input	Parser action
S	x a y a b $	predict 1
A B $	x a y a b $	predict 2
x a A B $	x a y a b $	match(x)
a A B $	a y a b $	match(a)
A B $	y a b $	predict 3
y a A B $	y a b $	match(y)

An example

1. $S \rightarrow A\ B\ \$$
2. $A \rightarrow x\ a\ A$
3. $A \rightarrow y\ a\ A$
4. $A \rightarrow \lambda$
5. $B \rightarrow b$

- How would a stack-based parser parse:

  x a y a b

Parse stack	Remaining input	Parser action
S	x a y a b \$	predict 1
A B \$	x a y a b \$	predict 2
x a A B \$	x a y a b \$	match(x)
a A B \$	a y a b \$	match(a)
A B \$	y a b \$	predict 3
y a A B \$	y a b \$	match(y)
a A B \$	a b \$	match(a)

An example

1. $S \rightarrow A\ B\ \$$
2. $A \rightarrow x\ a\ A$
3. $A \rightarrow y\ a\ A$
4. $A \rightarrow \lambda$
5. $B \rightarrow b$

- How would a stack-based parser parse:

  x a y a b

Parse stack	Remaining input	Parser action
S	x a y a b $	predict 1
A B $	x a y a b $	predict 2
x a A B $	x a y a b $	match(x)
a A B $	a y a b $	match(a)
A B $	y a b $	predict 3
y a A B $	y a b $	match(y)
a A B $	a b $	match(a)
A B $	b $	predict 4

An example

1. $S \rightarrow A\ B\ \$$
2. $A \rightarrow x\ a\ A$
3. $A \rightarrow y\ a\ A$
4. $A \rightarrow \lambda$
5. $B \rightarrow b$

- How would a stack-based parser parse:

x a y a b

Parse stack	Remaining input	Parser action
S	x a y a b $	predict 1
A B $	x a y a b $	predict 2
x a A B $	x a y a b $	match(x)
a A B $	a y a b $	match(a)
A B $	y a b $	predict 3
y a A B $	y a b $	match(y)
a A B $	a b $	match(a)
A B $	b $	predict 4
B $	b $	predict 5

An example

1. S → A B $
2. A → x a A
3. A → y a A
4. A → λ
5. B → b

- How would a stack-based parser parse:

  x a y a b

Parse stack	Remaining input	Parser action
S	x a y a b $	predict 1
A B $	x a y a b $	predict 2
x a A B $	x a y a b $	match(x)
a A B $	a y a b $	match(a)
A B $	y a b $	predict 3
y a A B $	y a b $	match(y)
a A B $	a b $	match(a)
A B $	b $	predict 4
B $	b $	predict 5
b $	b $	match(b)

An example

1. S → A B $
2. A → x a A
3. A → y a A
4. A → λ
5. B → b

- How would a stack-based parser parse:

  x a y a b

Parse stack	Remaining input	Parser action
S	x a y a b $	predict 1
A B $	x a y a b $	predict 2
x a A B $	x a y a b $	match(x)
a A B $	a y a b $	match(a)
A B $	y a b $	predict 3
y a A B $	y a b $	match(y)
a A B $	a b $	match(a)
A B $	b $	predict 4
B $	b $	predict 5
b $	b $	match(b)
$	$	Done!

Dealing with semantic actions

- When a construct (corresponding to a production in a grammar) is recognized, a typical parser will invoke a semantic action
  - In a compiler, this action generates an intermediate representation of the program construct
  - In an interpreter, this action might be to perform the action specified by the construct. Thus, if *a+b* is recognized, the value of *a* and *b* would be added and placed in a temporary variable

Dealing with semantic actions

- We can annotate a grammar with *action symbols*
  - Tell the parser to invoke a semantic action routine
- Can simply push action symbols onto stack as well
- When popped, the semantic action routine is called
  - Routine manipulates *semantic records* on a stack
  - Can generate new records (e.g., to store variable info)
  - Can generate code using existing records
- Example: semantic actions for x = a + 3

```
statement ::= ID = expr #assign
expr ::= term + term #addop
term ::= ID | LITERAL
```

Non-LL(1) grammars

- Not all grammars are LL(1)!
- Consider

  <stmt> → if <expr> then <stmt list> endif

  <stmt> → if <expr> then <stmt list> else <stmt list> endif

- This is not LL(1) (why?)
- We can turn this in to

  <stmt> → if <expr> then <stmt list> <if suffix>

  <if suffix> → endif

  <if suffix> → else <stmt list> endif

Left recursion

- *Left recursion* is a problem for LL(1) parsers
  - LHS is also the first symbol of the RHS
- Consider:

  $E \rightarrow E + T$

- What would happen with the stack-based algorithm?

Removing left recursion

$E \rightarrow E + T$
$E \rightarrow T$

$\longrightarrow$

$E \rightarrow E1\ Etail$
$E1 \rightarrow T$
$Etail \rightarrow + T\ Etail$
$Etail \rightarrow \lambda$

LL(k) parsers

- Can look ahead more than one symbol at a time
  - *k*-symbol lookahead requires extending first and follow sets
  - 2-symbol lookahead can distinguish between more rules:

    $A \rightarrow ax \mid ay$
- More lookahead leads to more powerful parsers
- What are the downsides?

Are all grammars LL(k)?

- No! Consider the following grammar:

$$
\begin{aligned}
S &\rightarrow E \\
E &\rightarrow (E + E) \\
E &\rightarrow (E - E) \\
E &\rightarrow x
\end{aligned}
$$

- When parsing E, how do we know whether to use rule 2 or 3?
    - Potentially unbounded number of characters before the distinguishing '+' or '–' is found
    - No amount of lookahead will help!

In real languages?

- Consider the if-then-else problem
- `if x then y else z`
- Problem: else is optional
- `if a then if b then c else d`
  - Which `if` does the `else` belong to?
- This is analogous to a "bracket language": $[^i\ ]^j$ $(i \geq j)$

$$
\begin{aligned}
S &\rightarrow [\ S\ C \\
S &\rightarrow \lambda \\
C &\rightarrow\ ] \\
C &\rightarrow \lambda
\end{aligned}
$$

[[ ] can be parsed: SSλC or SSCλ
(it's ambiguous!)

Solving the if-then-else problem

- The ambiguity exists at the language level. To fix, we need to define the semantics properly
  - "] matches nearest unmatched ["
  - This is the rule C uses for if-then-else
  - What if we try this?

$$
\begin{aligned}
S &\rightarrow [\ S \\
S &\rightarrow S1 \\
S1 &\rightarrow [\ S1\ ] \\
S1 &\rightarrow \lambda
\end{aligned}
$$

This grammar is still not LL(1) (or LL(k) for any k!)

Two possible fixes

- If there is an ambiguity, prioritize one production over another
  - e.g., if C is on the stack, always match "]" before matching "λ"

$$
\begin{aligned}
S &\rightarrow [\ S\ C \\
S &\rightarrow \lambda \\
C &\rightarrow] \\
C &\rightarrow \lambda
\end{aligned}
$$

- Another option: change the language!
  - e.g., all if-statements need to be closed with an endif

$$
\begin{aligned}
S &\rightarrow \text{if } S\ E \\
S &\rightarrow \text{other} \\
E &\rightarrow \text{else } S \text{ endif} \\
E &\rightarrow \text{endif}
\end{aligned}
$$

Parsing if-then-else

- What if we don't want to change the language?
  - C does not require { } to delimit single-statement blocks
- To parse if-then-else, *we need to be able to look ahead at the entire rhs of a production* before deciding which production to use
  - In other words, we need to determine how many "]" to match before we start matching "["s
- *LR parsers* can do this!

LR Parsers

- Parser which does a Left-to-right, Right-most derivation
  - Rather than parse top-down, like LL parsers do, parse bottom-up, starting from leaves
- Basic idea: put tokens on a stack until an entire production is found
- Issues:
  - Recognizing the endpoint of a production
  - Finding the length of a production (RHS)
  - Finding the corresponding nonterminal (the LHS of the production)

LR Parsers

- Basic idea:
  - **shift** tokens onto the stack. At any step, keep the set of productions that could generate the read-in tokens
  - **reduce** the RHS of recognized productions to the corresponding non-terminal on the LHS of the production. Replace the RHS tokens on the stack with the LHS non-terminal.

Data structures

- At each state, given the next token,
  - A *goto table* defines the successor state
  - An *action table* defines whether to
    - *shift* – put the next state and token on the stack
    - *reduce* – an RHS is found; process the production
    - *terminate* – parsing is complete

Simple example

1. P → S
2. S → x ; S
3. S → e

		Symbol					
		x	;	e	P	S	Action
State	0	1		3		5	Shift
	1		2				Shift
	2	1		3		4	Shift
	3						Reduce 3
	4						Reduce 2
	5						Accept

Parsing using an LR(0) parser

- Basic idea: parser keeps track, simultaneously, of all possible productions that *could be matched* given what it's seen so far. When it sees a full production, match it.
- Maintain a *parse stack* that tells you what state you're in
  - Start in state 0
- In each state, look up in action table whether to:
  - *shift*: consume a token off the input; look for next state in goto table; push next state onto stack
  - *reduce*: match a production; pop off as many symbols from state stack as seen in production; look up where to go according to non-terminal we just matched; push next state onto stack
  - *accept*: terminate parse

Example

- Parse "x ; x ; e"

Step	Parse Stack	Remaining Input	Parser Action
1	0	x ; x ; e	Shift 1
2	0 1	; x ; e	Shift 2
3	0 1 2	x ; e	Shift 1
4	0 1 2 1	; e	Shift 2
5	0 1 2 1 2	e	Shift 3
6	0 1 2 1 2 3		Reduce 3 (goto 4)
7	0 1 2 1 2 4		Reduce 2 (goto 4)
8	0 1 2 4		Reduce 2 (goto 5)
9	0 5		Accept

LR(k) parsers

- LR(0) parsers
  - No lookahead
  - Predict which action to take by looking only at the symbols currently on the stack
- LR(k) parsers
  - Can look ahead *k* symbols
  - Most powerful class of deterministic bottom-up parsers
  - LR(1) and variants are the most common parsers

Terminology for LR parsers

- Configuration: a production augmented with a "•"

  $A \rightarrow X_1 \ldots X_i \bullet X_{i+1} \ldots X_j$

- The "•" marks the point to which the production has been recognized. In this case, we have recognized $X_1 \ldots X_i$

- Configuration set: all the configurations that can apply at a given point during the parse:

  $A \rightarrow B \bullet CD$

  $A \rightarrow B \bullet GH$

  $T \rightarrow B \bullet Z$

- Idea: every configuration in a configuration set is a production that we could be in the process of matching

Configuration closure set

- Include all the configurations necessary to recognize the next symbol after the •
- For each configuration in set:
  - If next symbol is terminal, no new configuration added
  - If next symbol is non-terminal X, for each production of the form X → α, add configuration X → •α

S → E $
E → E + T | T
T → ID | (E)

closure0({S → • E $}) = {
    S → • E $
    E → • E + T
    E → • T
    T → • ID
    T → • (E)
}

Successor configuration set

- Starting with the initial configuration set

  $s0 = closure0(\{S \rightarrow \bullet\ \alpha\ \$\})$

  an LR(0) parser will find the successor given the next symbol X

- X can be either a terminal (the next token from the scanner) or a non-terminal (the result of applying a reduction)

- Determining the successor s' = go_to0(s, X):

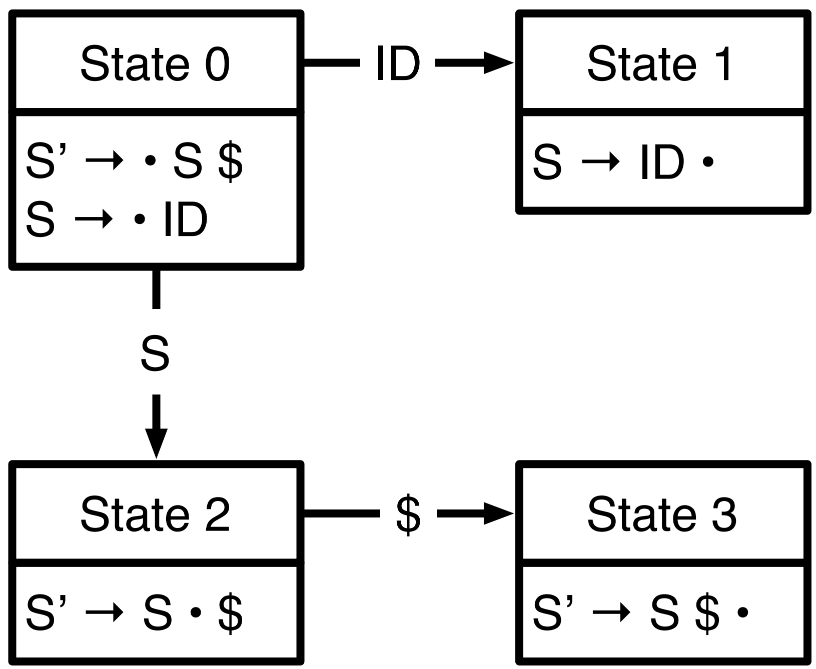
  - For each configuration in s of the form $A \rightarrow \beta \bullet X\ \gamma$ add $A \rightarrow \beta\ X \bullet \gamma$ to t

  - s' = closure0(t)

CFSM

- CFSM = Characteristic Finite State Machine
- Nodes are configuration sets (starting from s0)
- Arcs are go_to relationships

S’ → S $
S → ID



Building the goto table

- We can just read this off from the CFSM

		Symbol		
		ID	$	S
State	0	1		2
	1			
	2		3	
	3			

Building the action table

- Given the configuration set s:
  - We shift if the next token matches a terminal after the • in some configuration

    $A \rightarrow \alpha \bullet a \beta \in s$ and $a \in V_t$, else error

  - We reduce production P if the • is at the end of a production

    $B \rightarrow \alpha \bullet \in s$ where production P is $B \rightarrow \alpha$

  - Extra actions:
    - shift if goto table transitions between states on a non-terminal
    - accept if we have matched the goal production

Action table

State	0	Shift
	1	Reduce 2
	2	Shift
	3	Accept

Conflicts in action table

- For LR(0) grammars, the action table entries are unique: from each state, can only shift or reduce
- But other grammars may have conflicts
  - Reduce/reduce conflicts: multiple reductions possible from the given configuration
  - Shift/reduce conflicts: we can either shift or reduce from the given configuration

Shift/reduce conflict

- Consider the following grammar:

  $S \rightarrow A\ y$

  $A \rightarrow x \mid xx$

- This leads to the following configuration set (after shifting one "x":

  $A \rightarrow x \bullet x$

  $A \rightarrow x \bullet$

- Can shift or reduce here

Shift/reduce example (2)

- Consider the following grammar:

  $S \rightarrow A\ y$

  $A \rightarrow \lambda \mid x$

- This leads to the following initial configuration set:

  $S \rightarrow \bullet A\ y$

  $A \rightarrow \bullet\ x$

  $A \rightarrow \lambda\ \bullet$

- Can shift or reduce here

Lookahead

- Can resolve reduce/reduce conflicts and shift/reduce conflicts by employing *lookahead*
  - Looking ahead one (or more) tokens allows us to determine whether to shift or reduce
  - (*cf* how we resolved ambiguity in LL(1) parsers by looking ahead one token)

Semantic actions

- Recall: in LL parsers, we could integrate the semantic actions with the parser
  - Why? Because the parser was *predictive*
- Why doesn't that work for LR parsers?
  - Don't know which production is matched until parser reduces
- For LR parsers, we put semantic actions at the end of productions
  - May have to rewrite grammar to support all necessary semantic actions

Parsers with lookahead

- Adding lookahead creates an LR(1) parser
  - Built using similar techniques as LR(0) parsers, but uses lookahead to distinguish states
  - LR(1) machines can be much larger than LR(0) machines, but resolve many shift/reduce and reduce/reduce conflicts
  - Other types of LR parsers are SLR(1) and LALR(1)
    - Differ in how they resolve ambiguities
    - yacc and bison produce LALR(1) parsers

LR(1) parsing

- Configurations in LR(1) look similar to LR(0), but they are extended to include a lookahead symbol

  $A \rightarrow X_1 \ldots X_i \bullet X_{i+1} \ldots X_j$, l (where $l \in V_t \cup \lambda$)

- If two configurations differ only in their lookahead component, we combine them

  $A \rightarrow X_1 \ldots X_i \bullet X_{i+1} \ldots X_j$, $\{l_1 \ldots l_m\}$

Building configuration sets

- To close a configuration

  $B \to \alpha \bullet A\ \beta, l$

- Add all configurations of the form $A \to \bullet\ \gamma, u$ where $u \in$ First(βl)
- Intuition: the lookahead symbol for any configuration is the terminal we expect to see *after the configuration has been matched*
  - The parse could apply the production for A, and the lookahead after we apply the production should match the next token that would be produced by B

Example

closure1({S → • E $, {λ}}) =

S → E $
E → E + T | T
T → ID | (E)

Example

closure1({S → • E $, {λ}}) =
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closure1({S → • E $, {λ}}) =
S → • E $, {λ}
E → • E + T, {$}

S → E $
E → E + T | T
T → ID | (E)

Example

closure1({S → • E $, {λ}}) =
S → • E $, {λ}
E → • E + T, {$}
E → • T, {$}

S → E $
E → E + T | T
T → ID | (E)

Example

closure1({S → • E $, {λ}}) =
S → • E $, {λ}
E → • E + T, {$}
E → • T, {$}
T → • ID, {$}

S → E $
E → E + T | T
T → ID | (E)

Example

closure1({S → • E $, {λ}}) =
S → • E $, {λ}
E → • E + T, {$}
E → • T, {$}
T → • ID, {$}
T → • (E), {$}

S → E $
E → E + T | T
T → ID | (E)

Example

$S \rightarrow E\ \$$
$E \rightarrow E + T \mid T$
$T \rightarrow ID \mid (E)$

closure1({$S \rightarrow \bullet\ E\ \$$, {λ}}) =
$S \rightarrow \bullet\ E\ \$$, {λ}
$E \rightarrow \bullet\ E + T$, {\$}
$E \rightarrow \bullet\ T$, {\$}
$T \rightarrow \bullet\ ID$, {\$}
$T \rightarrow \bullet\ (E)$, {\$}
$E \rightarrow \bullet\ E + T$, {+}

Example

$S \rightarrow E\ \$$
$E \rightarrow E + T \mid T$
$T \rightarrow ID \mid (E)$

closure1({S → • E $, {λ}}) =
$S \rightarrow \bullet\ E\ \$, \{\lambda\}$
$E \rightarrow \bullet\ E + T, \{\$\}$
$E \rightarrow \bullet\ T, \{\$\}$
$T \rightarrow \bullet\ ID, \{\$\}$
$T \rightarrow \bullet\ (E), \{\$\}$
$E \rightarrow \bullet\ E + T, \{+\}$
$E \rightarrow \bullet\ T, \{+\}$

Example

closure1({$S \rightarrow \bullet\ E\ \$$, {λ}}) =
$S \rightarrow \bullet\ E\ \$$, {λ}
$E \rightarrow \bullet\ E + T$, {\$}
$E \rightarrow \bullet\ T$, {\$}
$T \rightarrow \bullet\ ID$, {\$}
$T \rightarrow \bullet\ (E)$, {\$}
$E \rightarrow \bullet\ E + T$, {+}
$E \rightarrow \bullet\ T$, {+}
$T \rightarrow \bullet\ ID$, {+}

$S \rightarrow E\ \$$
$E \rightarrow E + T \mid T$
$T \rightarrow ID \mid (E)$

Example

closure1({$S \rightarrow \bullet E\ \$$, {λ}}) =
$S \rightarrow \bullet E\ \$$, {λ}
$E \rightarrow \bullet E + T$, {\$}
$E \rightarrow \bullet T$, {\$}
$T \rightarrow \bullet ID$, {\$}
$T \rightarrow \bullet (E)$, {\$}
$E \rightarrow \bullet E + T$, {+}
$E \rightarrow \bullet T$, {+}
$T \rightarrow \bullet ID$, {+}
$T \rightarrow \bullet (E)$, {+}

$S \rightarrow E\ \$$
$E \rightarrow E + T \mid T$
$T \rightarrow ID \mid (E)$

Building goto and action tables

- The function goto1(configuration-set, symbol) is analogous to goto0(configuration-set, symbol) for LR(0)
  - Build goto table in the same way as for LR(0)
- Key difference: the action table.

  action[s][x] =

  - reduce when • is at end of configuration *and* x ∈ lookahead set of configuration

    $A \rightarrow \alpha \bullet, \{\ldots x \ldots\} \in s$

  - shift when • is before x

    $A \rightarrow \beta \bullet x \gamma \in s$

Example

- Consider the simple grammar:

  <program> → begin <stmts> end $

  <stmts> → SimpleStmt ; <stmts>

  <stmts> → begin <stmts> end ; <stmts>

  <stmts> → λ

Action and goto tables

	begin	end	;	SimpleStmt	$	<program>	<stmts>
0	S / 1						
1	S / 4	R4		S / 5			S / 2
2		S / 3					
3					A		
4	S / 4	R4		S / 5			S / 7
5			S / 6				
6	S / 4	R4		S / 5			S / 10
7		S / 8					
8			S / 9				
9	S / 4	R4		S / 6			S / 11
10		R2					
11		R3					

<program> → begin <stmts> end $

<stmts> → SimpleStmt ; <stmts>

<stmts> → begin <stmts> end ; <stmts>

<stmts> → λ

Example

- Parse: begin SimpleStmt ; SimpleStmt ; end $

Step	Parse Stack	Remaining Input	Parser Action
1	0	begin S ; S ; end $	Shift 1
2	0 1	S ; S ; end $	Shift 5
3	0 1 5	; S ; end $	Shift 6
4	0 1 5 6	S ; end $	Shift 5
5	0 1 5 6 5	; end $	Shift 6
6	0 1 5 6 5 6	end $	Reduce 4 (goto 10)
7	0 1 5 6 5 6 10	end $	Reduce 2 (goto 10)
8	0 1 5 6 10	end $	Reduce 2 (goto 2)
9	0 1 2	end $	Shift 3
10	0 1 2 3	$	Accept

Problems with LR(1) parsers

- LR(1) parsers are very powerful ...
  - But the table size is much larger than LR(0) — as much as a factor of $|V_t|$ (why?)
  - Example: Algol 60 (a simple language) includes several thousand states!
- Storage efficient representations of tables are an important issue

Solutions to the size problem

- Different parser schemes
  - SLR (simple LR): build an CFSM for a language, then add lookahead wherever necessary (*i.e.*, add lookahead to resolve shift/reduce conflicts)
    - What should the lookahead symbol be?
    - To decide whether to reduce using production $A \rightarrow \alpha$, use Follow(A)
  - LALR: merge LR states when they only differ by lookahead symbols