# Analysis of programs with pointers

# Simple example

```
x := 5
ptr := @x
*ptr := 9
y := x
```

$$\begin{tikzcd}
S1 \arrow[dd, bend left] \\
S2 \arrow[d] \\
S3 \arrow[d] \\
S4
\end{tikzcd}$$

program dependences

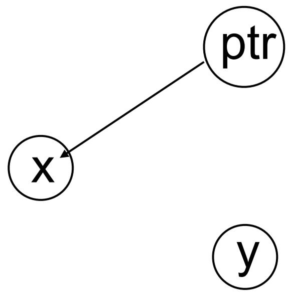
- What are the dependences in this program?
- Problem: just looking at variable names will not give you the correct information
  - After statement S2, program names “x” and “*ptr” are both expressions that refer to the same memory location.
  - We say that ptr points-to x after statement S2.
- In a C-like language that has pointers, we must know the points-to relation to be able to determine dependences correctly

# Program model

- For now, only types are int and int*
- No heap
  - All pointers point to only to stack variables
- No procedure or function calls
- Statements involving pointer variables:
  - address: x := &y
  - copy: x := y
  - load: x := *y
  - store: *x := y
- Arbitrary computations involving ints

# Points-to relation

- Directed graph:
  - nodes are program variables
  - edge (a,b): variable a points-to variable b

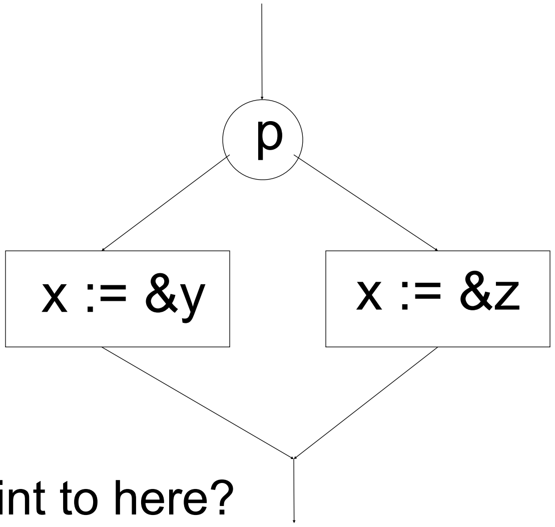


- Can use a special node to represent NULL
- Points-to relation is different at different program points

# Points-to graph

- Out-degree of node may be more than one
  - if points-to graph has edges (a,b) and (a,c), it means that variable a may point to either b or c
  - depending on how we got to that point, one or the other will be true
  - path-sensitive analyses: track how you got to a program point (we will not do this)

```
if (p)
    then x := &y
    else x := &z
.....
```



What does x point to here?

## Ordering on points-to relation

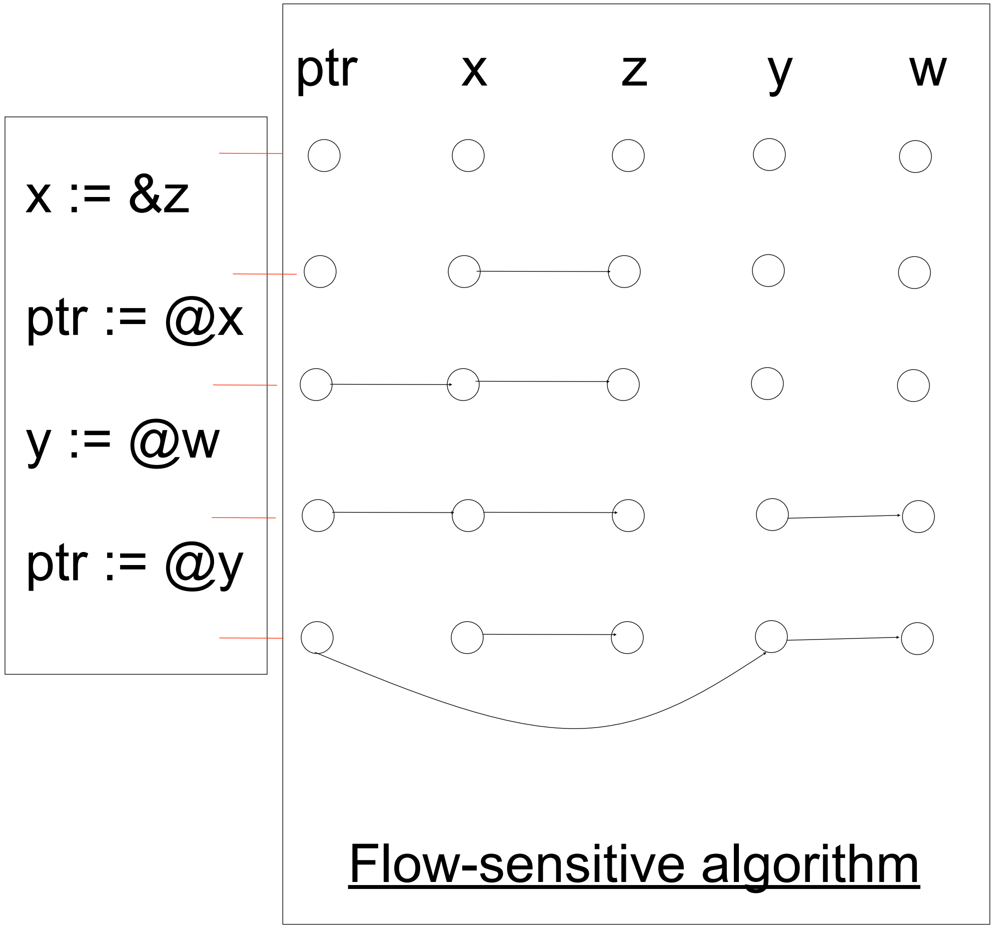
- Subset ordering: for a given set of variables
  - Least element is graph with no edges
  - G1 <= G2 if G2 has all the edges G1 has and maybe some more
- Given two points-to relations G1 and G2
  - G1 U G2: least graph that contains all the edges in G1 and in G2

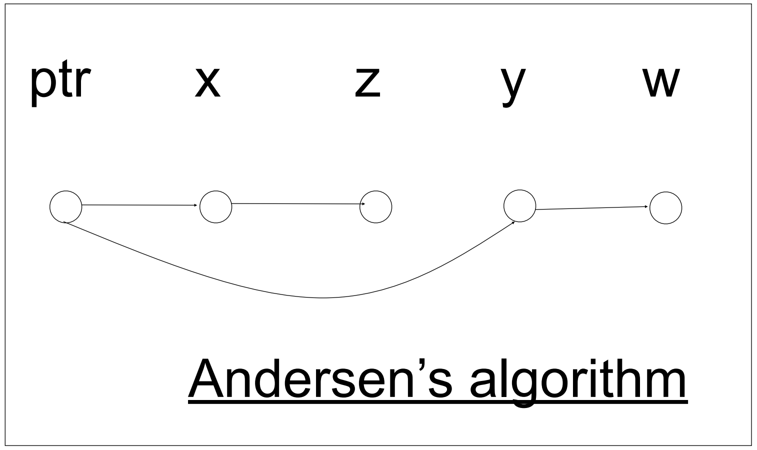
## Overview

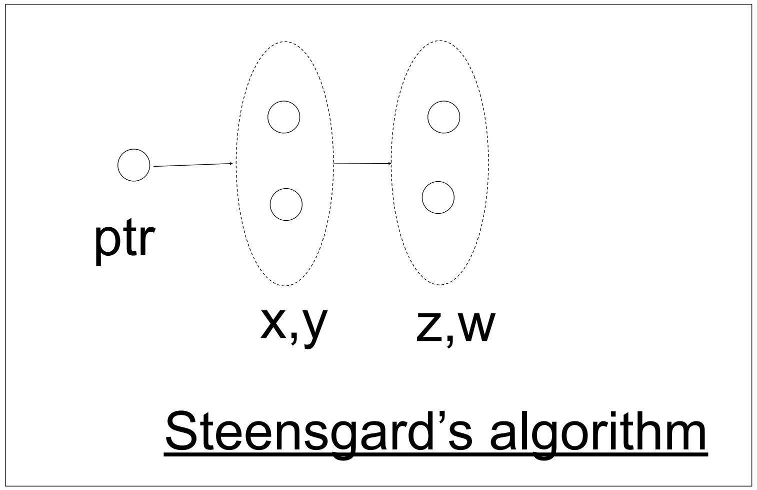
- We will look at three different points-to analyses.
- Flow-sensitive points-to analysis
  - Dataflow analysis
  - Computes a different points-to relation at each point in program
- Flow-insensitive points-to analysis
  - Computes a single points-to graph for entire program
  - Andersen’s algorithm
    - Natural simplification of flow-sensitive algorithm
  - Steensgard’s algorithm
    - Nodes in tree are equivalence classes of variables
      - if x may point-to either y or z, put y and z in the same equivalence class
    - Points-to relation is a tree with edges from children to parents rather than a general graph
    - Less precise than Andersen’s algorithm but faster

## Example

```
x := &z

ptr := @x

y := @w

ptr := @y
```







## Notation

- Suppose S and S1 are set-valued variables.
- S ← S1: strong update
  - set assignment
- S U← S1: weak update
  - set union: this is like S ← S U S1

## Flow-sensitive algorithm

# Dataflow equations

- Forward flow, any path analysis
- Confluence operator: G1 U G2
- Statements

$$\begin{tikzcd} {} \arrow[d, "G"] \\ \boxed{x := \&y} \arrow[d] \\ {} \end{tikzcd}$$

G' = G with pt'(x) ← {y}

$$\begin{tikzcd} {} \arrow[d, "G"] \\ \boxed{x := *y} \arrow[d] \\ {} \end{tikzcd}$$

G' = G with pt'(x) ← U pt(a)
for all a in pt(y)

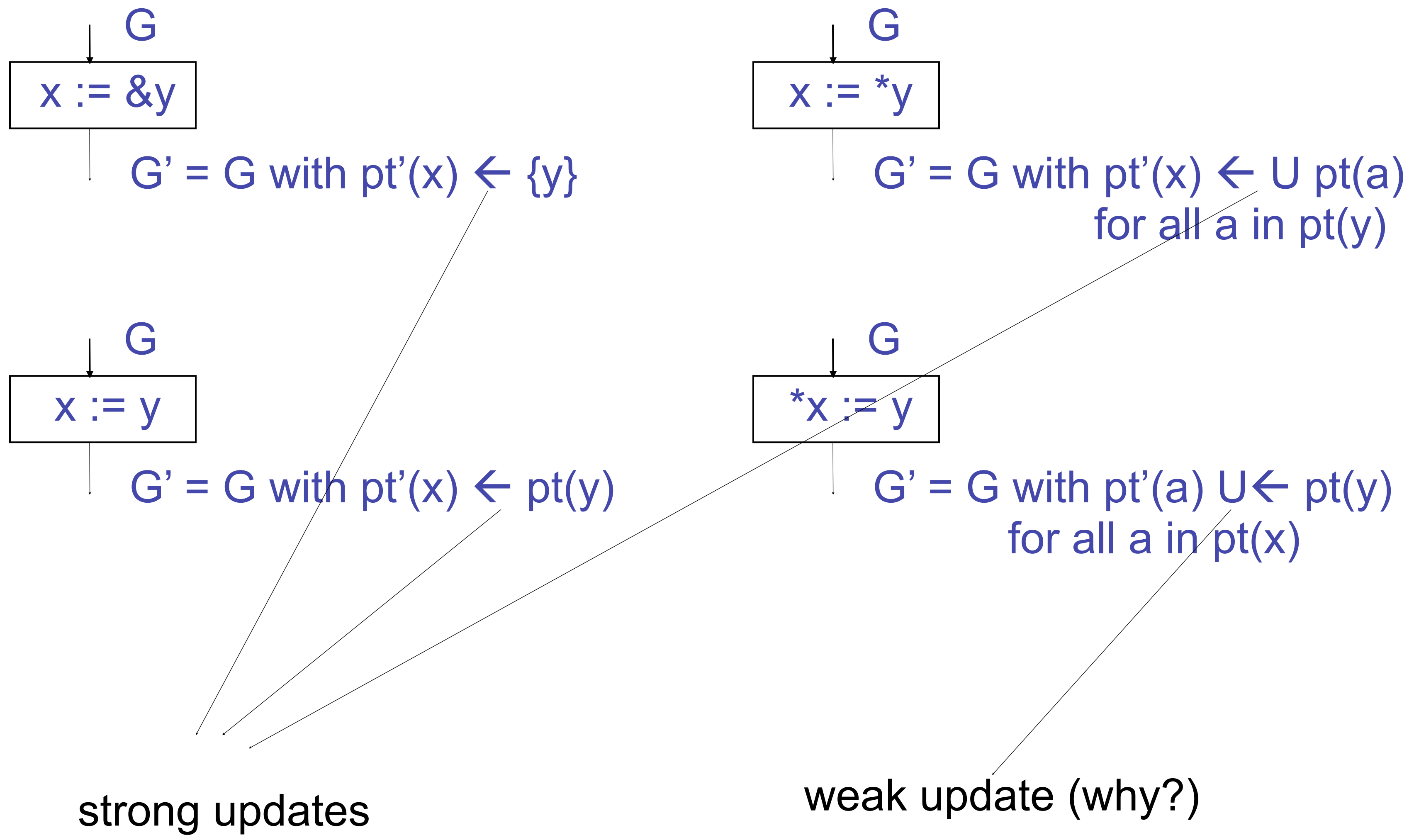
$$\begin{tikzcd} {} \arrow[d, "G"] \\ \boxed{x := y} \arrow[d] \\ {} \end{tikzcd}$$

G' = G with pt'(x) ← pt(y)

$$\begin{tikzcd} {} \arrow[d, "G"] \\ \boxed{*x := y} \arrow[d] \\ {} \end{tikzcd}$$

G' = G with pt'(a) U← pt(y)
for all a in pt(x)

## Dataflow equations (contd.)


G
x := &y
G' = G with pt'(x) ← {y}
G
x := *y
G' = G with pt'(x) ← U pt(a) for all a in pt(y)
G
x := y
G' = G with pt'(x) ← pt(y)
G
*x := y
G' = G with pt'(a) U← pt(y) for all a in pt(x)
strong updates
weak update (why?)

## Strong vs. weak updates

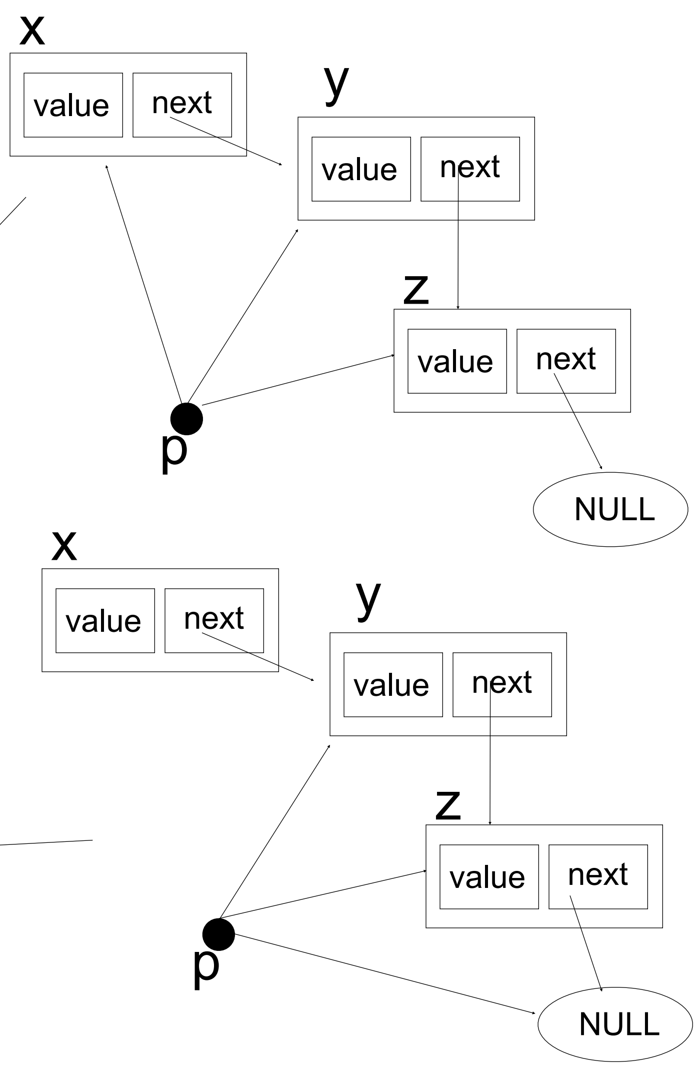
- Strong update:
  - At assignment statement, you know precisely which variable is being written to
  - Example: x := ….
  - You can remove points-to information about x coming into the statement in the dataflow analysis.
- Weak update:
  - You do not know precisely which variable is being updated; only that it is one among some set of variables.
  - Example: *x := …
  - Problem: at analysis time, you may not know which variable x points to (see slide on control-flow and out-degree of nodes)
  - Refinement: if out-degree of x in points-to graph is 1 and x is known not be nil, we can do a strong update even for *x := …

## Structures

- Structure types
  - struct cell {int value; struct cell *left, *right;}
  - struct cell x,y;
- Use a “field-sensitive” model
  - x and y are nodes
  - each node has three internal fields labeled value, left, right
- This representation permits pointers into fields of structures
  - If this is not necessary, we can simply have a node for each structure and label outgoing edges with field name

# Example

```
int main(void)
        { struct cell {int value;
                    struct cell *next;
                    };
         struct cell x,y,z,*p;
         int sum;

         x.value = 5;
         x.next = &y;
         y.value = 6;
         y.next = &z;
         z.value = 7;
         z.next = NULL;

         p = &x;
         sum = 0;
         while (p != NULL) {
                   sum = sum + (*p).value;
                   p = (*p).next;
              }
         return sum;
        }
```



## Flow-insensitive algorithms

# Flow-insensitive analysis

- Flow-sensitive analysis computes a different graph at each program point.
- This can be quite expensive.
- One alternative: flow-insensitive analysis
  - Intuition:compute a points-to relation which is the least upper bound of all the points-to relations computed by the flow-sensitive analysis
- Approach:
  - Ignore control-flow
  - Consider all assignment statements together
    - replace strong updates in dataflow equations with weak updates
  - Compute a single points-to relation that holds regardless of the order in which assignment statements are actually executed

# Andersen’s algorithm

- Statements

weak updates only

G

x := &y

G = G with pt(x) U← {y}

G

x := *y

G = G with pt(x) U← pt(a)
for all a in pt(y)

G

x := y

G = G with pt(x) U← pt(y)

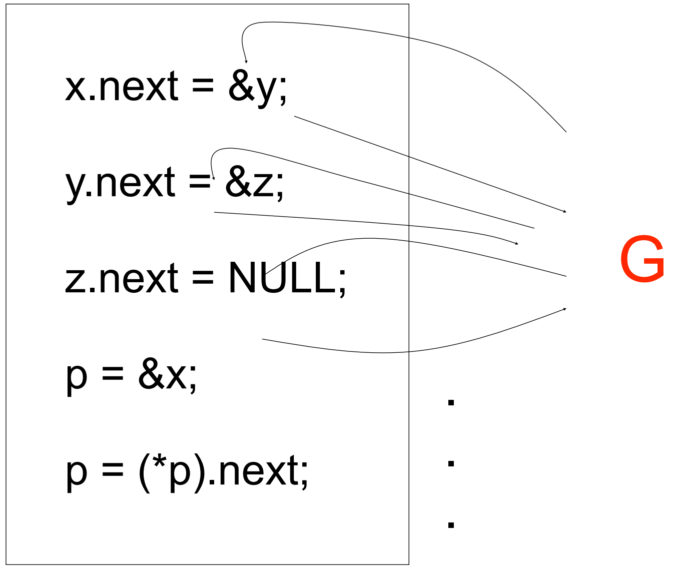
G

*x := y

G = G with pt(a) U← pt(y)
for all a in pt(x)

## Example

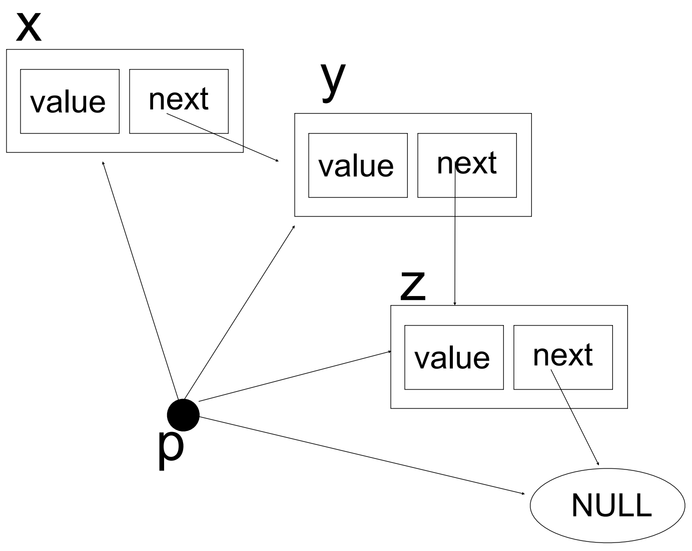
```
int main(void)
        { struct cell {int value;
                    struct cell *next;
                    };
         struct cell x,y,z,*p;
         int sum;

         x.value = 5;
         x.next = &y;
         y.value = 6;
         y.next = &z;
         z.value = 7;
         z.next = NULL;

         p = &x;
         sum = 0;
         while (p != NULL) {
                  sum = sum + (*p).value;
                  p = (*p).next;
              }
         return sum;
        }
```



```
x.next = &y;

y.next = &z;

z.next = NULL;

p = &x;

p = (*p).next;
```

Assignments for flow-insensitive analysis

## Solution to flow-insensitive equations



- Compare with points-to graphs for flow-sensitive solution
- Why does p point-to NULL in this graph?

# Andersen's algorithm formulated using set constraints

- Statements

$$pt : \mathrm{var} \circledR\ 2^{\mathrm{var}}$$

x := &y

$$y \in pt(x)$$

x := *y

$$\forall a \in pt(y) . pt(x) \supseteq pt(a)$$

x := y

$$pt(x) \supseteq pt(y)$$

*x := y

$$\forall a \in pt(x) . pt(a) \supseteq pt(y)$$

# Steensgard's algorithm

- Flow-insensitive
- Computes a points-to graph in which there is no fan-out
  - In points-to graph produced by Andersen's algorithm, if x points-to y and z, y and z are collapsed into an equivalence class
  - Less accurate than Andersen's but faster
- We can exploit this to design an O(N*$\alpha$(N)) algorithm, where N is the number of statements in the program.

# Steensgard's algorithm using set constraints

- Statements

$$pt : \mathrm{var} \text{ ® } 2^{\mathrm{var}}$$

No fan-out $\forall x. \forall y, z \in pt(x). pt(y) = pt(z)$

x := &y

$$y \in pt(x)$$

x := *y

$$\forall a \in pt(y). pt(x) = pt(a)$$

x := y

$$pt(x) = pt(y)$$

*x := y

$$\forall a \in pt(x). pt(a) = pt(y)$$

# Trick for one-pass processing

- Consider the following equations

$$pt(x) = pt(y)$$
$$z \in pt(x)$$

$$dummy \in pt(x)$$
$$pt(x) = pt(y)$$
$$z \in pt(x)$$

- When first equation on left is processed, x and y are not pointing to anything.
- Once second equation is processed, we need to go back and reprocess first equation.
- Trick to avoid doing this: when processing first equation, if x and y are not pointing to anything, create a dummy node and make x and y point to that
  - this is like solving the system on the right
- It is easy to show that this avoids the need for revisiting equations.

# Algorithm

- Can be implemented in single pass through program
- Algorithm uses union-find to maintain equivalence classes (sets) of nodes
- Points-to relation is implemented as a pointer from a variable to a representative of a set
- Basic operations for union find:
  - rep(v): find the node that is the representative of the set that v is in
  - union(v1,v2): create a set containing elements in sets containing v1 and v2, and return representative of that set

# Auxiliary methods

```
class var {
   //instance variables
  points_to: var;
  name: string;

   //constructor; also
  creates singleton set in
  union-find data structure
   var(string);
   //class method; also
  creates singleton set in
  union-find data structure
   make-dummy-var():var;

   //instance methods
  get_pt(): var;
  set_pt(var);//updates rep
}
```

```
rec_union(var v1, var v2) {

    p1 = pt(rep(v1));
    p2 = pt(rep(v2));
    t1 = union(rep(v1), rep(v2));
    if (p1 == p2)
          return;
    else if (p1 != null && p2 != null)
          t2 = rec_union(p1, p2);
    else if (p1 != null) t2 = p1;
    else if (p2 != null) t2 = p2;
    else t2 = null;

    t1.set_pt(t2);
    return t1;
}

pt(var v) {
    //v does not have to be representative
    t = rep(v);
    return t.get_pt();
```

# Algorithm

Initialization: make each program variable into an object of type var and enter object into union-find data structure

```
for each statement S in the program do
    S is x := &y:  {if (pt(x) == null)
                        x.set-pt(rep(y));
                    else rec-union(pt(x),y);
                    }
    S is x := y: {if (pt(x) == null and pt(y) == null)
                       x.set-pt(var.make-dummy-var());
                  y.set-pt(rec-union(pt(x),pt(y)));
                 }
     S is x := *y:{if (pt(y) == null)
                      y.set-pt(var.make-dummy-var());
                   var a := pt(y);
                   if(pt(a) == null)
                      a.set-pt(var.make-dummy-var());
                   x.set-pt(rec-union(pt(x),pt(a)));
                   }
 S is *x := y:{if (pt(x) == null)
                      x.set-pt(var.make-dummy-var());
                   var a := pt(x);
                   if(pt(a) == null)
                      a.set-pt(var.make-dummy-var());
                   y.set-pt(rec-union(pt(y),pt(a)));
                   }
```

## Inter-procedural analysis

- What do we do if there are function calls?

```
x1 = &a
y1 = &b
swap(x1, y1)
```

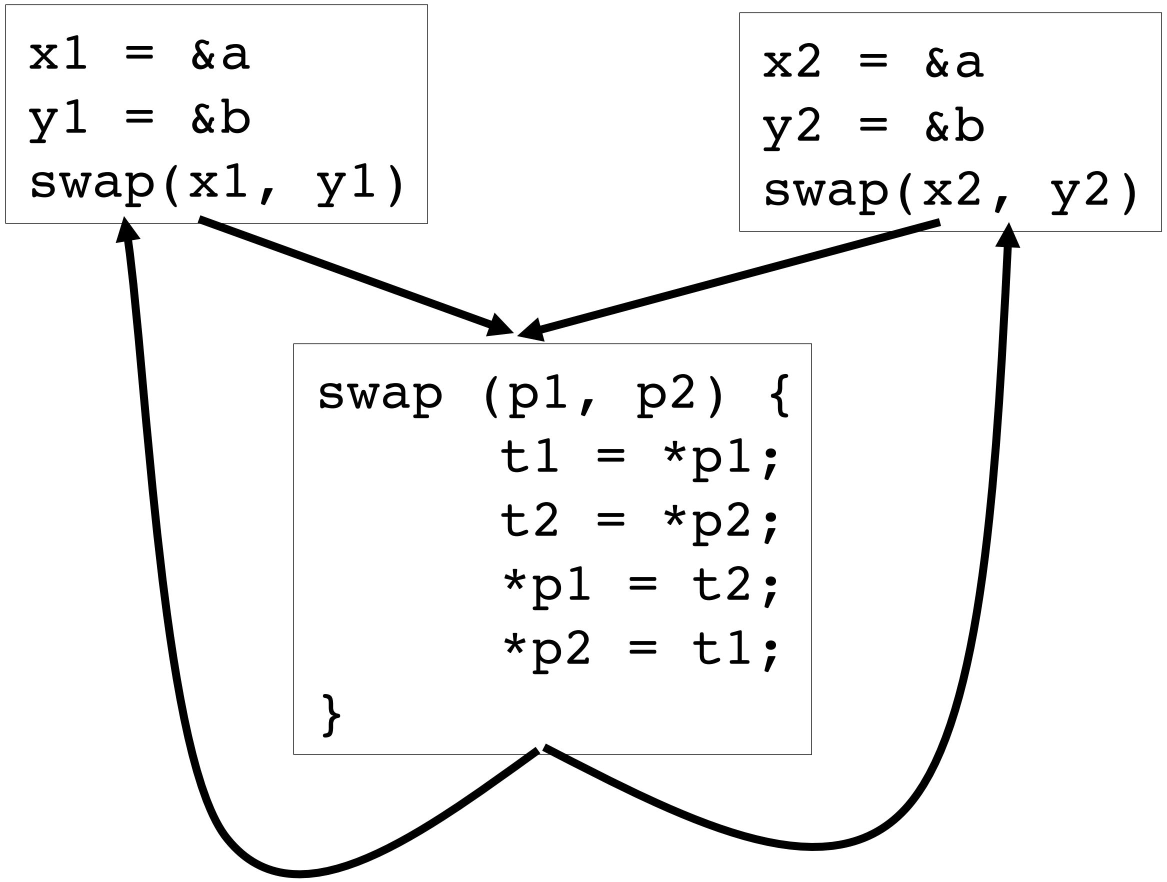
```
x2 = &a
y2 = &b
swap(x2, y2)
```

```
swap (p1, p2) {
      t1 = *p1;
      t2 = *p2;
      *p1 = t2;
      *p2 = t1;
}
```

## Two approaches

- Context-sensitive approach:
  - treat each function call separately just like real program execution would
  - problem: what do we do for recursive functions?
    - need to approximate
- Context-insensitive approach:
  - merge information from all call sites of a particular function
  - in effect, inter-procedural analysis problem is reduced to intra-procedural analysis problem
- Context-sensitive approach is obviously more accurate but also more expensive to compute

# Context-insensitive approach

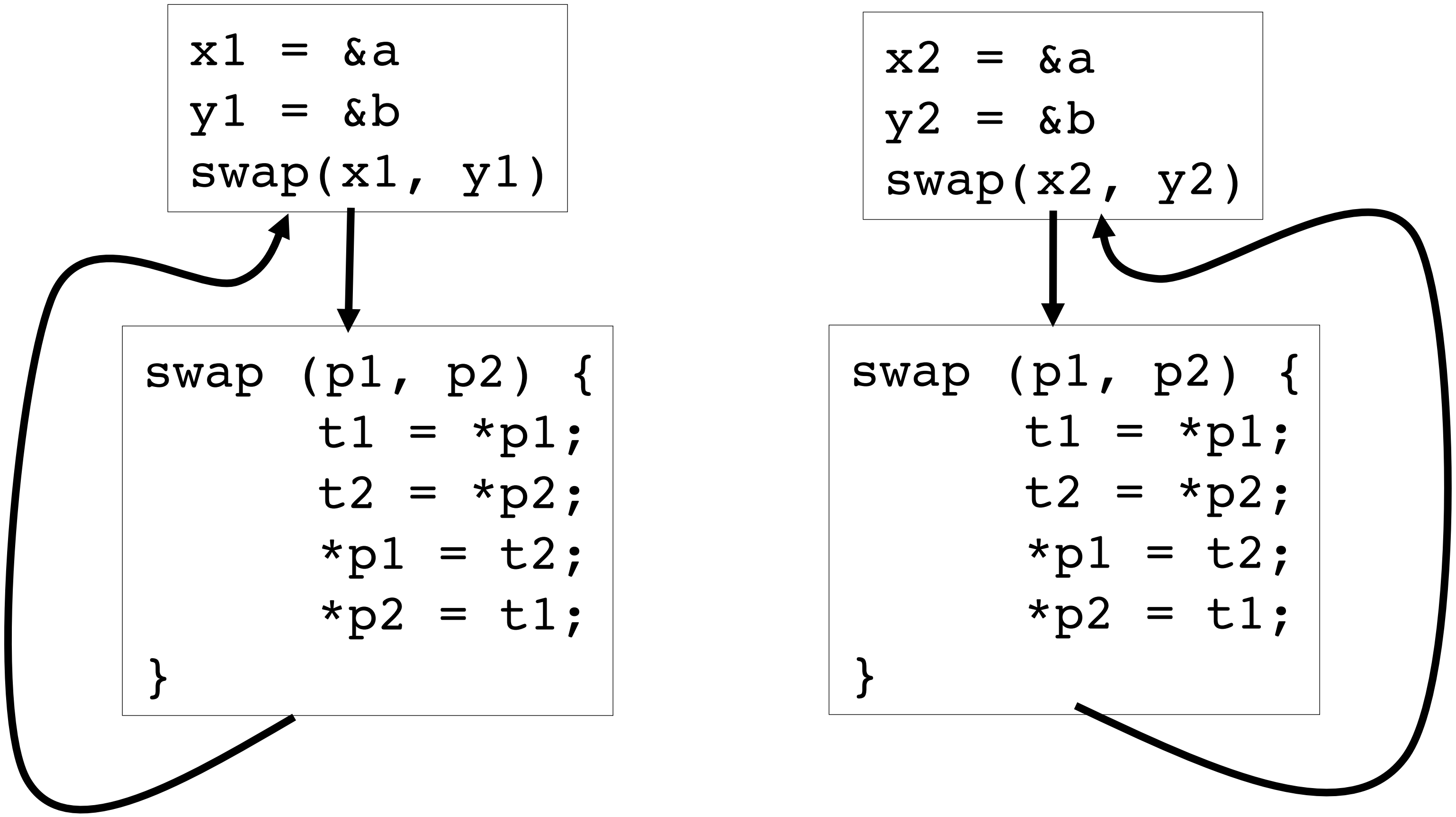


```
x1 = &a
y1 = &b
swap(x1, y1)
```

```
x2 = &a
y2 = &b
swap(x2, y2)
```

```
swap (p1, p2) {
      t1 = *p1;
      t2 = *p2;
      *p1 = t2;
      *p2 = t1;
}
```

# Context-sensitive approach



```
x1 = &a
y1 = &b
swap(x1, y1)
```

```
swap (p1, p2) {
        t1 = *p1;
        t2 = *p2;
        *p1 = t2;
        *p2 = t1;
}
```

```
x2 = &a
y2 = &b
swap(x2, y2)
```

```
swap (p1, p2) {
        t1 = *p1;
        t2 = *p2;
        *p1 = t2;
        *p2 = t1;
}
```

# Context-insensitive/Flow-insensitive Analysis

- For now, assume we do not have function parameters
  - this means we know all the call sites for a given function
- Set up equations for binding of actual and formal parameters at each call site for that function
  - use same variables for formal parameters for all call sites
- Intuition: each invocation provides a new set of constraints to formal parameters

## Swap example

```
x1 = &a
y1 = &b
p1 = x1
p2 = y1
```

```
x2 = &a
y2 = &b
p1 = x2
p2 = y2
```

```
t1 = *p1;
t2 = *p2;
*p1 = t2;
*p2 = t1;
```

## Heap allocation

- Simplest solution:
  - use one node in points-to graph to represent all heap cells
- More elaborate solution:
  - use a different node for each malloc site in the program
- Even more elaborate solution: shape analysis
  - goal: summarize potentially infinite data structures
  - but keep around enough information so we can disambiguate pointers from stack into the heap, if possible

# Summary

| Less precise | More precise |
|---|---|
| Equality-based | Subset-based |
| Flow-insensitive | Flow-sensitive |
| Context-insensitive | Context-sensitive |

No consensus about which technique to use
Experience: if you are context-insensitive, you might as well be flow-insensitive

# History of points-to analysis

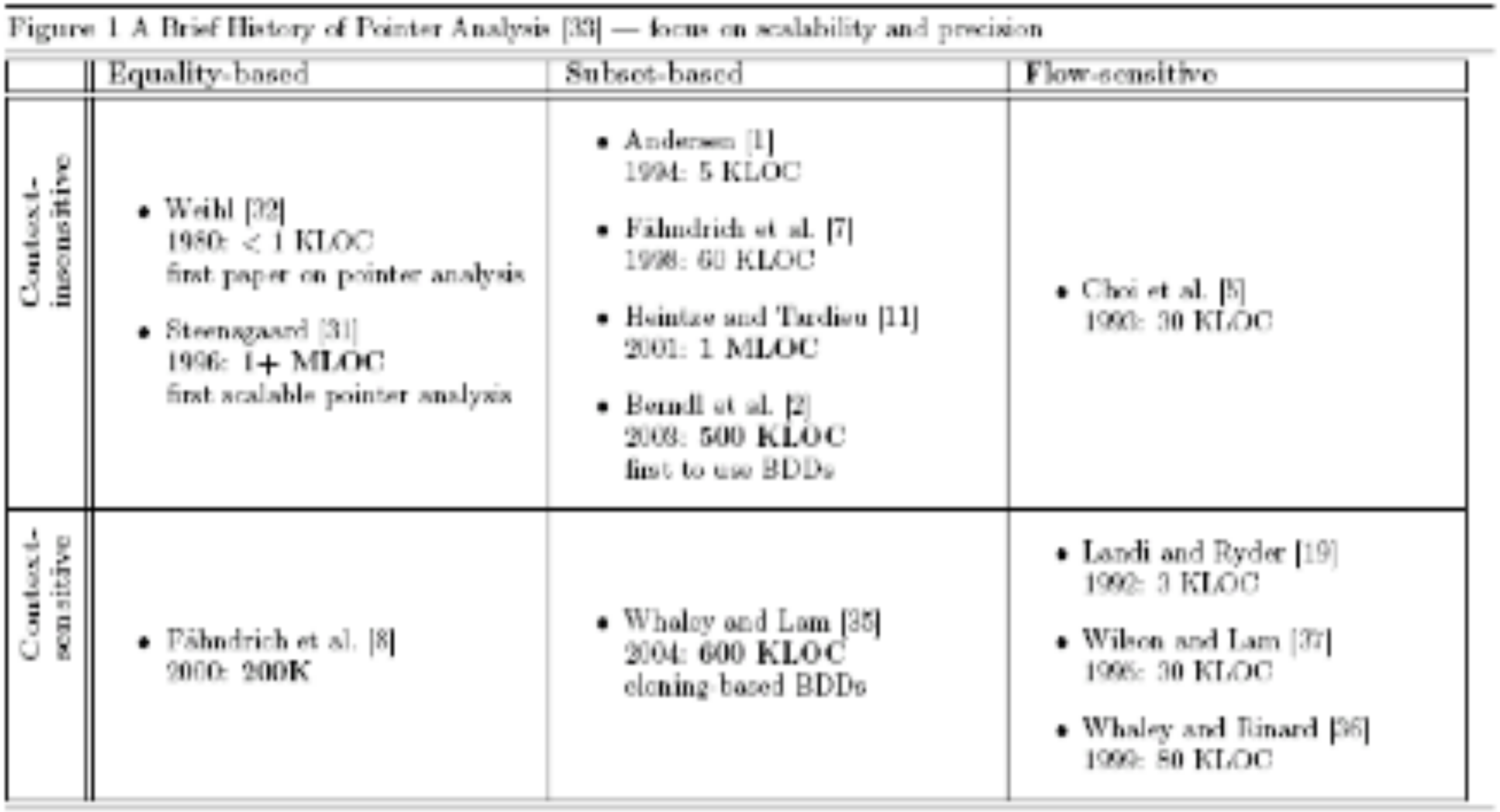


Figure 1 A Brief History of Pointer Analysis [33] — focus on scalability and precision

| | Equality-based | Subset-based | Flow-sensitive |
|---|---|---|---|
| Context-insensitive | • Weihl [32] 1980: < 1 KLOC first paper on pointer analysis<br>• Steensgaard [31] 1996: 1+ MLOC first scalable pointer analysis | • Andersen [1] 1994: 5 KLOC<br>• Fähndrich et al. [7] 1998: 60 KLOC<br>• Heintze and Tardieu [11] 2001: 1 MLOC<br>• Berndl et al. [2] 2003: 500 KLOC first to use BDDs | • Choi et al. [5] 1993: 30 KLOC |
| Context-sensitive | • Fähndrich et al. [8] 2000: 200K | • Whaley and Lam [35] 2004: 600 KLOC cloning based BDDs | • Landi and Ryder [19] 1992: 3 KLOC<br>• Wilson and Lam [37] 1995: 30 KLOC<br>• Whaley and Rinard [36] 1999: 80 KLOC |

from Ryder and Rayside