# Introduction Scheduling (Part 1) 

## Introduction and Acyclic Scheduling

CS 380C: Advanced Compiler Techniques

Thursday, October 11th 2007

## Lecture Overview

Code Generator

- Back end part of compiler (code generator)
- Instruction scheduling
- Register allocation


## Instruction Scheduling

- Input: set of instructions
- Output: total order on that set


## Lecture Outline

## Lectures

(1) Introduction and acylic scheduling (today)
(2) Software pipelining (Tuesday 23)

## Today

- Definition of instruction scheduling
- Constraints
- Scheduling process
- Acylic scheduling: list scheduling


## Introduction to Instruction Scheduling

## Context

- Backend part of the compiler chain (code generation)
- Inputs: set of instructions (assembly instructions)
- Outputs: a schedule
- Set of scheduling dates (one date per instruction)
- Total order


## Goal

- Minimize the execution time (number of cycles)
- Different possible objective functions to minimize:
- Power consumption
- . . .


## Constraints

- Is it possible to generate any schedule?


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Example:

$$
\begin{aligned}
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
& \mathrm{d}=\mathrm{a}+3 ; \\
& \mathrm{e}=\mathrm{f}+\mathrm{d} ;
\end{aligned}
$$

## Constraints

- Is it possible to generate any schedule?


## Example:

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& \mathrm{a}=\mathrm{b}+\mathrm{c} ; \\
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\end{aligned}
$$

- Possibility to change instruction order?
- No, because of data dependences
- Flow dependences on a and d


## Constraints

- Data dependences enforce a partial order for the final schedule
- Other types of constraints?


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- Other types of constraints?

Example:
$a=b+c ;$
$d=e+f ;$

- Target architecture with 1 ALU


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- Data dependences enforce a partial order for the final schedule
- Other types of constraints?

Example:

$$
\begin{aligned}
& a=b+c ; \\
& d=e+f ;
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- Target architecture with 1 ALU
- Impossible to use the same functional unit concurrently
- Resource constraints


## Constraints

- Data dependences enforce a partial order for the final schedule
- Other types of constraints?

Example:
$\mathrm{a}=\mathrm{b}+\mathrm{c}$;
$\mathrm{d}=\mathrm{e}+\mathrm{f}$;

- Target architecture with 1 ALU
- Impossible to use the same functional unit concurrently
- Resource constraints


## Constraints

- Two types of constraints: data dependences and resource usage


## Constraints influencing Instruction Scheduling

## Constraints

- Data dependences
- Resource constraints

Rule

- The final schedule must respect these constraints


## Dealing with constraints

- How to represent such constraints to deal with during the scheduling process?


## Constraints influencing Instruction Scheduling

## Constraints

- Data dependences
- Resource constraints

Rule

- The final schedule must respect these constraints

Dealing with constraints

- How to represent such constraints to deal with during the scheduling process?
- Data dependences $\rightarrow$ graph
- Resource constraints $\rightarrow$ reservation tables or automaton


## Data Dependence Representation

## Data Dependence Graph (DDG)

- 1 node $\Leftrightarrow 1$ instruction
- 1 edge $\Leftrightarrow 1$ flow dependence (directed graph)
- Edge label = parameters of the dependence
- Latency (\# of cycles)
- Distance (\# of iterations)


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- Latency (\# of cycles)
- Distance (\# of iterations)
- Example (1-cycle latency):

$$
\begin{aligned}
& \mathrm{a}=\mathrm{b}+\mathrm{c} ; / / \text { ADD1 } \\
& \mathrm{d}=\mathrm{a}+3 ; / / \text { ADD2 } \\
& \mathrm{e}=\mathrm{a}+\mathrm{d} ; / / \text { ADD3 }
\end{aligned}
$$

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\end{aligned}
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## Data Dependence Representation - Example 2

- Daxpy loop: double alpha times $X$ plus $Y$
- $y \leftarrow \alpha \times x+y$
- C-like code:

```
for ( i=0; i<N; i++)
    Y[i] = alpha*X[i] + Y[i];
```

- Targeting Itanium ISA:
- LD: Load from memory (latency 6 cycles from L2 cache)
- ST: Store to memory
- FMA: Fuse multiply and add (latency 4 cycles)


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- LD: Load from memory (latency 6 cycles from L2 cache)
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## Data Dependence Representation - Example 3

- Daxpy loop with inter-iteration dependence
- C-like code:

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\begin{aligned}
& \text { for }(i=0 ; i<N ; i++) \\
& \mid Y[i+2]=\text { alpha*X[i] +Y[i] }
\end{aligned}
$$

- Inter-iteration dependence
- Distance of 2


## Data Dependence Representation - Example 3

- Daxpy loop with inter-iteration dependence
- C-like code:

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\end{aligned}
$$

- Inter-iteration dependence
- Distance of 2



## Data Dependence Representation

## Remarks

- Circuits allowed for a distance $>0$
- For basic block, this is only a DAG


## Drawbacks

- One fix digit for latency
- Fixed latencies
- May not be suitable for cache/memory accesses
- One digit for the distance
- Only uniform dependences


## Resource Constraint Representation

## Resources

- Second set of constraints: resource usage/assignment

Overview

- Need to check if two instructions may race for the same resource (functional unit, bus, pipeline stage, ...)
- Can be several cycles ahead (latency $>1$ )


## Resource Constraint Representation

## Resources

- Second set of constraints: resource usage/assignment

Overview

- Need to check if two instructions may race for the same resource (functional unit, bus, pipeline stage, ...)
- Can be several cycles ahead (latency $>1$ )

State-of-the-art

- 2 representations: reservation tables and automaton


## Reservation Tables - Definition

## Reservation tables

- Intuitive way: resource usage of one instruction as a 2D table


## Semantics

- Rows: latency of the instruction (in cycles)
- Columns: number of resources available in the target architecture
- Cell $(i, j)$ is marked $\Leftrightarrow$ instruction requires $i^{\text {th }}$ resource during its $j^{\text {th }}$ cycle of execution
- Binary tables
- Several tables per instruction (alternatives/options)


## Reservation Tables - Example 1

Example with pipelined resources:

- 2 fully pipelined resources (ALU): ALU0 and ALU1
- 2 instructions ADD and MUL
- Constraints:
- ADD can be executed on ALUO or ALU1
- MUL can only be executed on ALU1


## Reservation Tables - Example 1

Example with pipelined resources:

- 2 fully pipelined resources (ALU): ALU0 and ALU1
- 2 instructions ADD and MUL
- Constraints:
- ADD can be executed on ALUO or ALU1
- MUL can only be executed on ALU1

Tables for ADD:

|  | ALUO | ALU1 |
| :---: | :---: | :---: |
| 0 | X |  |
| 0 OR |  |  |
|  | ALU0 | ALU1 |
| 0 |  | X |

Table for MUL:

|  | ALU0 | ALU1 |
| :---: | :---: | :---: |
| 0 |  | X |

## Reservation Tables - Example 1

ADD instruction:

|  | ALUO | ALU1 |
| :---: | :---: | :---: |
| 0 | X |  |

OR

|  | ALU0 | ALU1 |
| :---: | :---: | :---: |
| 0 |  | X |

- Are the following sequences valid? ADD | ADD? ADD | MUL? MUL | MUL ?
ADD ; ADD ?
ADD | MUL ; MUL ?
MUL instruction:

|  | ALU0 | ALU1 |
| :---: | :---: | :---: |
| 0 |  | X |

## Reservation Tables - Example 1

ADD instruction:

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OR

|  | ALU0 | ALU1 |
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| 0 |  | X |

- Are the following sequences valid?

| ADD | ADD | $\sqrt{ }$ |
| :--- | :--- | :--- |
| ADD | MUL | $\sqrt{ }$ |
| MUL | I MUL | $\times$ |
| ADD | ; ADD | $\sqrt{ }$ |
| ADD | MUL ; MUL | $\sqrt{ }$ |

MUL instruction:

|  | ALU0 | ALU1 |
| :---: | :---: | :---: |
| 0 |  | X |

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MUL instruction:

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| ADD | MUL | $\sqrt{ }$ |
| MUL | I MUL | $\times$ |
| ADD | $;$ | ADD |
| ADD | MUL ; MUL | $\sqrt{ }$ |
|  | $\sqrt{ }$ |  |

- Test if instructions can be scheduled together: AND operation
- Update resource usage: OR operation


## Reservation Tables - Example 2

Example with complex resources:

- 2 resources: ALU and LD/ST
- 3 instructions ADD, SUB and LD
- Constraints:
- ADD instructions have a latency of 1 cycle
- SUB instructions have a latency of 2 cycles
- LD uses first the ALU for 1 cycle and then the LD/ST resource for 1 cycle


## Reservation Tables - Example 2

Example with complex resources:

- 2 resources: ALU and LD/ST
- 3 instructions ADD, SUB and LD
- Constraints:
- ADD instructions have a latency of 1 cycle
- SUB instructions have a latency of 2 cycles
- LD uses first the ALU for 1 cycle and then the LD/ST resource for 1 cycle

Table for ADD:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |

Table for SUB:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 | X |  |

Table for LD:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 |  | X |

## Reservation Tables - Example 2

ADD instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |

SUB instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 | X |  |

LD instruction:

- Are the following sequences valid?

| ADD I SUB | $?$ |
| :--- | :--- |
| ADD I ADD | $?$ |
| SUB I LD | $?$ |
| LD ; ADD | $?$ |
| LD ; SUB | $?$ |
| SUB ; LD | $?$ |
| ADD ; SUB ; LD | $?$ |
| LD ; ADD ; SUB ? |  |


|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 |  | X |

## Reservation Tables - Example 2

ADD instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |

SUB instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 | X |  |

LD instruction:

- Are the following sequences valid?

| ADD | SUB | $\times$ |
| :--- | :--- | :--- |
| ADD I ADD | $\times$ |  |
| SUB I LD | $\times$ |  |
| LD ; ADD | $\sqrt{ }$ |  |
| LD ; SUB | $\sqrt{ }$ |  |
| SUB ; LD | $\times$ |  |
| ADD ; SUB ; LD | $\times$ |  |
| LD ; ADD ; SUB | $\sqrt{ }$ |  |


|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 |  | X |

## Reservation Tables - Example 2

ADD instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |

SUB instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 | X |  |

LD instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 |  | X |

- Are the following sequences valid?

| ADD | SUB | $\times$ |
| :--- | :--- | :--- |
| ADD I | ADD | $\times$ |
| SUB I | LD | $\times$ |
| LD ; ADD | $\sqrt{ }$ |  |
| LD ; SUB | $\sqrt{ }$ |  |
| SUB ; LD | $\times$ |  |
| ADD ; SUB ; LD | $\times$ |  |
| LD ; ADD ; SUB | $\sqrt{ }$ |  |

- Test and update according to latencies of instructions


## Reservation Table - Summary

## Use

- AND operation to check if several instruction can be scheduled
- OR operation to update the resource state


## Advantages

- Intuitive representation
- Small storage


## Drawbacks

- Many tests
- Redundant information


## Automaton

## Insight

- Pre-processing of possible resource usages


## Semantics

- 1 state of the automaton $\Leftrightarrow 1$ assignment of resources
- 1 transition of the automaton $\Leftrightarrow$ scheduling of an instruction at the current cycle


## Transition label

- Label of a transition: the instruction to schedule
- Special label: NOP instruction to advance the current cycle


## Automaton - Example 1

ADD instruction:

|  | ALU0 | ALU1 |
| :---: | :---: | :---: |
| 0 | OR |  |


|  | ALU0 | ALU1 |
| :---: | :---: | :---: |
| 0 |  | X |

## Automaton - Example 1

ADD instruction:

|  | ALU0 | ALU1 |
| :---: | :---: | :---: |
| 0 | OR |  |
|  |  |  |
| 0 | ALU0 | ALU1 |
| 0 |  | $X$ |



- 2 fully-pipelined resources $\Rightarrow 2$ bits per state


## Automaton - Example 1



- Are the following sequences valid?

| ADD | ADD | $?$ |
| :--- | :--- | :--- |
| ADD | I MUL | $?$ |
| MUL | I | MUL |

ADD ; ADD
ADD | MUL ; MUL ?

## Automaton - Example 1



- Are the following sequences valid?



## Automaton - Example 2

ADD instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |

## SUB instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 | X |  |

LD instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 |  | X |

## Automaton - Example 2

ADD instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |

SUB instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 | X |  |

LD instruction:

|  | ALU | LD/ST |
| :---: | :---: | :---: |
| 0 | X |  |
| 1 |  | X |



## Automaton - Example 2



- Are the following sequences valid?

| ADD \| SUB ? | LD ; SUB | $?$ |
| :--- | :--- | :--- | :--- |
| ADD \| ADD ? | SUB ; LD | $?$ |
| SUB \| LD ? | ADD ; SUB ; LD | $?$ |
| LD ; ADD ? | LD ; ADD ; SUB ? |  |

## Automaton - Example 2



- Are the following sequences valid?

| ADD | SUB | $\times$ | LD ; SUB | $\sqrt{ }$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| ADD | ADD | $\times$ | SUB ; LD | $\times$ |
| SUB | LD | $\times$ | ADD ; SUB ; LD | $\times$ |
| LD ; ADD | $\sqrt{ }$ | LD ; ADD ; SUB | $\sqrt{ }$ |  |

## Automaton - Summary

## Use

- An instruction can be currently scheduled if there is an output arc from the current state labeled with this instruction
- Update the state by following this arc


## Advantages

- Low query time: table lookup


## Drawbacks

- Huge computational time (offline)
- Large storage
$\Rightarrow$ split into several automata
- Not very flexible
- e.g. hard to schedule instructions not cycle-wise


## Scheduling Process

## Scheme of a classical scheduler

- High-level part: main heuristic taken care of the data dependences and driving the scheduling process
- Low-level part: storage of the resource usages and updates of the global assignments


## Scheduling Process

## Scheme of a classical scheduler

- High-level part: main heuristic taken care of the data dependences and driving the scheduling process
- Low-level part: storage of the resource usages and updates of the global assignments


## Scheduling process

- Process begins in the high-level part
- Pick up the next instruction to insert in the partial schedule
- Query the low-level part for resource assignements:
- If okay, then goes on with another instruction
- Otherwise backtrack


## Acyclic Scheduling: List Scheduling

## Context

- Schedule a basic block $\Rightarrow$ acyclic scheduling
- Goal: minimize the length of the generated code
- Must respect data dependences and resource constraints


## Example

- Sum the first element of 3 vectors $X, Y$ and $Z$ in the first cell of array A:

$$
\mathrm{A}[0]=\mathrm{X}[0]+\mathrm{Y}[0]+\mathrm{Z}[0] ;
$$

- 3 instructions: ADD, LD, ST (1-cycle latency)
- 3 fully-pipelined resources: ALU, LD0 and LD/ST1 units


## Acyclic Scheduling - Example

DDG?

## Acyclic Scheduling - Example

## Reservation tables:

DDG:


## Acyclic Scheduling - Example

Reservation tables:
ADD instruction:

|  | ALU | LD0 | LD/ST1 |
| :---: | :---: | :---: | :---: |
| 0 | X |  |  |



LD instruction:

|  | ALU | LD0 | LD/ST1 |
| :--- | :---: | :---: | :---: |
| 0 |  | X |  |


|  | ALU | LD0 | LD/ST1 |
| :---: | :---: | :---: | :---: |
| 0 |  |  | X |

ST instruction:

|  | ALU | LD0 | LD/ST1 |
| :---: | :---: | :---: | :---: |
| 0 |  |  | X |

## Acyclic Scheduling - Example

Reservation tables:
ADD instruction:

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A possible schedule?

## Acyclic Scheduling - Example

- A possible schedule respecting both constraints and minimizing the total length:

```
LD(X) | LD(Y) ; // Cycle 1
ADD1 | LD(Z) ; // Cycle 2
ADD2 ; // Cycle 3
ST ; // Cycle 4 = length
```


## Acyclic Scheduling - Example

- A possible schedule respecting both constraints and minimizing the total length:

```
LD(X) | LD(Y) ; // Cycle 1
ADD1 | LD(Z) ; // Cycle 2
ADD2 ; // Cycle 3
ST ; // Cycle 4 = length
```

- Good the execute as much instructions as possible
- Pick up the good instruction is crucial (LD (X) and LD (Y) before LD (Z))
- Be careful of explicit resource assignments through reservation tables:
- Only one valid combination to execute a ST and a LD at the same cycle


## List Scheduling

## Principle

- List scheduling algorithm is based on this approach
- Sort the instruction according to priority based on data dependences
- Pick up one ready instruction in priority order
- Until every instruction has been scheduled


## Priority

- Many priority schemes exist
- We will use the height-based priority:
- Priority of a node is the longest path from that node to the furthest leaf
- The path is weighted by latencies


## Conclusion

## Instruction scheduling

- Generate a total order of a set of instructions


## Constraints

- Data dependences
- Represented as a graph: DDG
- Resource usages
- Represented as reservation tables or automaton

Acyclic scheduling

- List scheduling
- Assign priority to instructions according to their contribution to the critical path

